Seminar 2

Numere complexe

Probleme propuse

2.1. Să se determine partea reală şi imaginară a numerelor complexe:

$a)\ z = (-1 + i\sqrt{3})^{2015}$ $\qquad b)\ z = e^{2+i\frac{7\pi}{6}}$

$c)\ z = \dfrac{\cos i}{3 - i}$ $\qquad d)\ z = \cos\left(i\dfrac{1}{2}\ln 2 - \dfrac{3\pi}{4}\right)$

$e)\ z = (1 - i)^{1+i}.$ $\qquad f)\ z = (-1)^i$

2.2. Să se rezolve în $\mathbb{C}$ ecuaţiile:

$a)\ z^3 = 1$ $\qquad b)\ z^4 = 1$

$c)\ z^3 = i$ $\qquad d)\ z^{2n+1} - 1 = 0$

$e)\ (z + 1)^n + (z - 1)^n = 0$ $\qquad f)\ \sqrt{3}\sin z + i\cos z = 1$

$g)\ \sin z = -2.$

Indicaţii la problemele propuse

2.1. a) $z = -2^{2014}(1 + i\sqrt{3})$ b) $z = -\frac{e^2}{2}(\sqrt{3} + i)$ c) $z = \frac{3(e^2+1)}{20e} + i\frac{e^2+1}{20e}$ d) $z = -\frac{3}{4} + \frac{i}{4}$ e) $z = \sqrt{2}e^{-\frac{7\pi}{4}-2k\pi}\left[\cos\left(\frac{7\pi}{4} + \ln\sqrt{2}\right) + i\sin\left(\frac{7\pi}{4} + \ln\sqrt{2}\right)\right]$, $k \in \mathbb{Z}$ f) $z = e^{-\pi(2k+1)}$, $k \in \mathbb{Z}$

2.2. a) $z_1 = 1,\ z_{2,3} = (-1 \pm i\sqrt{3})/2$ b) $z_{1,2} = \pm 1,\ z_{3,4} = \pm i$ c) $z_{1,2} = \pm\frac{\sqrt{3}}{2} + \frac{i}{2}$ $z_3 = -i$ d) $z_k = e^{\frac{2k\pi i}{2n+1}},\ k \in \{0, 1, \ldots, 2n\}$ e) $z_k = i\operatorname{ctg}\frac{\pi(2k+1)}{2n},\ k \in \{0, 1, \ldots, n-1\}$ f) $z_k = \frac{\pi}{4} + 2k\pi - i\ln\frac{\sqrt{6}+\sqrt{2}}{2}$ şi $z_k = \frac{3\pi}{4} + 2k\pi - i\ln\frac{\sqrt{6}+\sqrt{2}}{2}$, $k \in \mathbb{Z}$ g) Soluţia în $\mathbb{C}$ este $z_k = \frac{3\pi}{2} + 2k\pi - i\ln(2 \pm \sqrt{3})$, $k \in \mathbb{Z}$