8. Statistical properties of grayscale images

8.1. Introduction

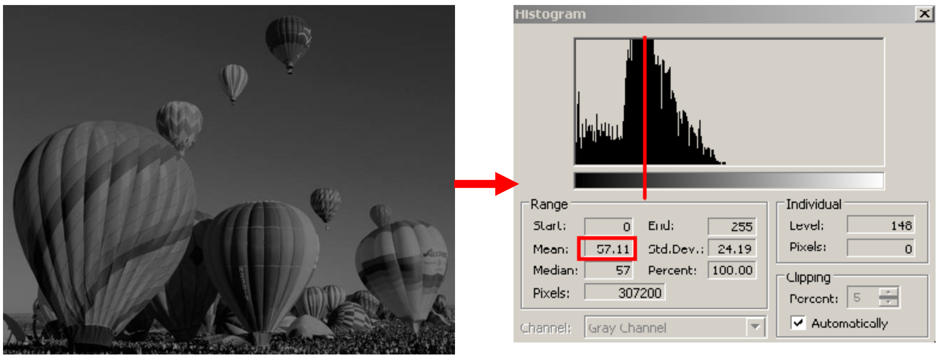
This laboratory work presents the main statistic features that characterize the distribution of intensity levels in a grayscale image or in an area / region of interest (ROI) of the image. These statistic features can be applied similarly to color images, on each color component.

The following notation will be used throughout this lab:

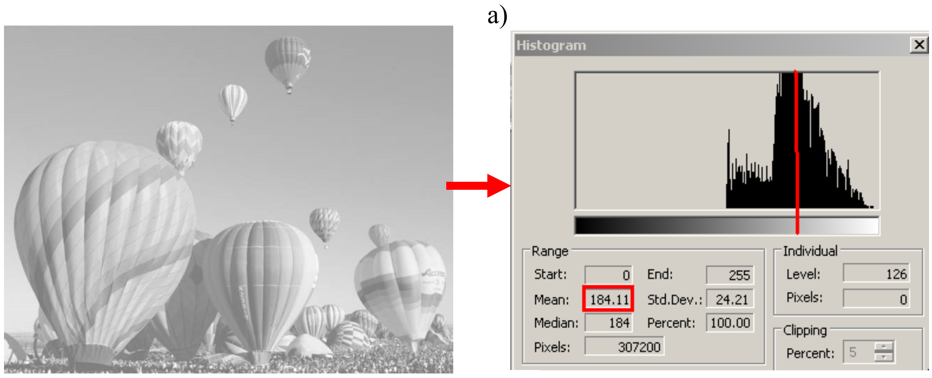
- *L=255* highest intensity level
- *h(g)* histogram function, counts the number of pixels with gray level *g*
- *M=H*W*, number of pixels in the image
- *p(g)=h(g)/M* gray level probability distribution function (PDF).

8.2. The mean value of intensity levels

The mean value of intensity levels is a measure of the mean intensity of the given image or of the region of interest. A dark image has a low mean value (Fig. 8.1a), and a bright image has a high mean value (Fig. 8.1b).



a)



b)

Fig. 8.1 The position of the histogram and the mean value of the intensity levels for a dark image (a) and a bright image (b)

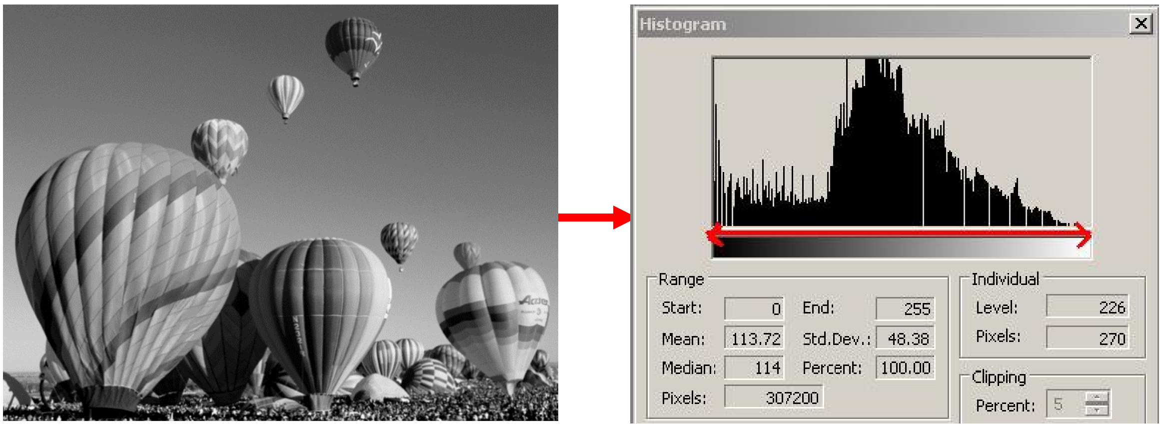
The mean intensity value is computed as follows:

$$\overline{g} = \mu = \int_{-\infty}^{+\infty} g \cdot p(g) dg = \sum_{g=0}^{L} g \cdot p(g) = \frac{1}{M} \sum_{g=0}^{L} g \cdot h(g) \tag{8.1}$$

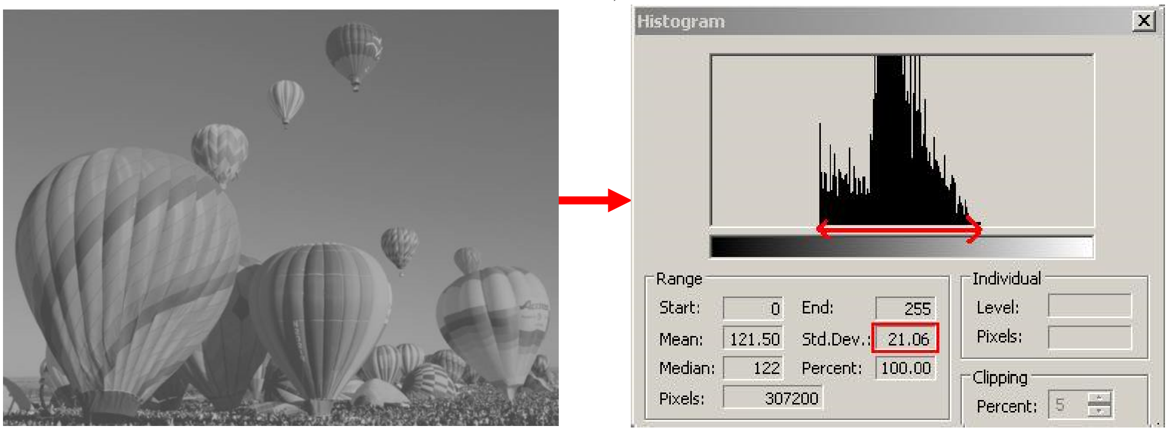
$$\overline{g} = \mu = \frac{1}{M} \sum_{i=0}^{H-1} \sum_{j=0}^{W-1} I(i,j) \tag{8.2}$$

8.3. The standard deviation of the intensity levels

The standard deviation of the intensity levels represents a measure of the contrast of an image (region of interest). It characterizes the dispersion (spreading) of the intensity levels with respect to the mean value. An image having a high contrast will have a large standard deviation (Fig. 8.2a – the histogram is spread on the entire range of intensity levels), and an image having a low contrast will be characterized by a small standard deviation (Fig. 8.2b – the histogram is restricted to some intensity levels located around the mean value).



a)



b)

Fig. 8.2 The position of the histogram and of the standard deviation (2σ) of the intensity levels for an image of high contrast (a) and an image of low contrast (b).

The standard deviation of the intensity levels is given by:

$$\sigma = \sqrt{\sum_{g=0}^{L} (g-\mu)^2 \cdot p(g)} \tag{8.3}$$

$$\sigma = \sqrt{\frac{1}{M} \sum_{i=0}^{H-1} \sum_{j=0}^{W-1} \left(I(i,j) - \mu \right)^2} \tag{8.4}$$

8.4. Basic global thresholding algorithm

This thresholding algorithm is suitable for grayscale images having a bimodal histogram. A bimodal histogram is characterized by two dominant modes, thus one threshold (*T*) is enough for image segmentation.

Algorithm

1. Initialization step:
   - Compute the image histogram h
   - Find the maximum intensity I_{max} and the minimum intensity I_{min}
   - Take an initial value for threshold *T*:

$$T = \left(I_{max} + I_{min}\right)/2$$

2. Segment the image after *T* by dividing the image pixels in 2 groups
   - compute μ_{G_1} for $G_1 : I(i,j) \le T$
   - compute μ_{G_2} for $G_2 : I(i,j) > T$

   Efficient implementation: compute the means μ_{G_1} and μ_{G_2} using the initial histogram

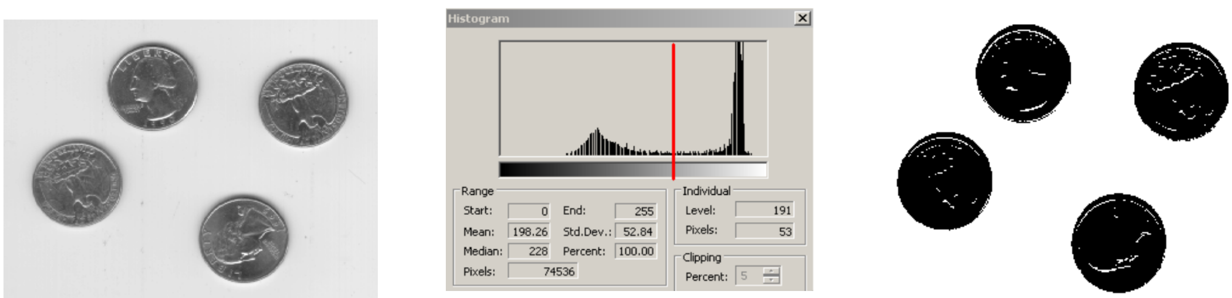
$$\mu_{G_1} = \frac{1}{N_1} \sum_{g=I_{min}}^{g=T} g \cdot h(g) \text{ where } N_1 = \sum_{g=I_{min}}^{g=T} h(g)$$

$$\mu_{G_2} = \frac{1}{N_2} \sum_{g=T+1}^{g=I_{max}} g \cdot h(g) \text{ where } N_2 = \sum_{g=T+1}^{g=I_{max}} h(g)$$

3. Update the threshold value: $T = \left(\mu_{G_1} + \mu_{G_2}\right)/2$

4. Repeat 2-3 until $\left|T_k - T_{k-1}\right| < error$ (where *error* is a positive value)

5. Threshold the image using *T*



a. Original image b. Image histogram c. Binary image after thresholding with $T = 165$ ($error = 0.1$)

Fig. 8.3. Segmentation result using the computed threshold

8.5. Analytical histogram transformation functions

In Fig. 8.4 are shown some typical transformation functions of the intensity values, which can be expressed in an analytical form:

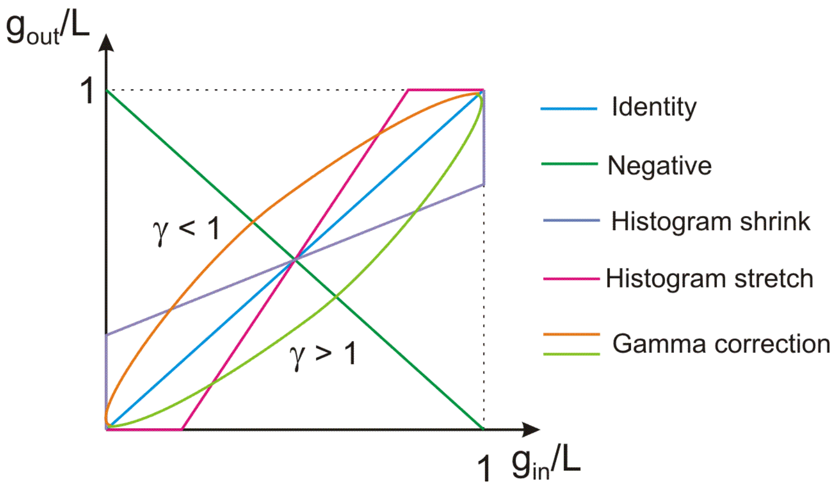


Fig. 8.4 Typical gray levels transformation functions

8.5.1. Identity function (no effect):

$$g_{out} = g_{in} \tag{8.5}$$

8.5.2. Image negative:

$$g_{out} = L - g_{in} = 255 - g_{in} \tag{8.6}$$

8.5.3. Histogram stretching / shrinking:

$$g_{out} = g_{out}^{MIN} + (g_{in} - g_{in}^{MIN}) \frac{g_{out}^{MAX} - g_{out}^{MIN}}{g_{in}^{MAX} - g_{in}^{MIN}} \tag{8.7}$$

Where:

$$\frac{g_{out}^{MAX} - g_{out}^{MIN}}{g_{in}^{MAX} - g_{in}^{MIN}} = \begin{cases} >1 & \Rightarrow stretch \\ <1 & \Rightarrow shrink \end{cases} \tag{8.8}$$

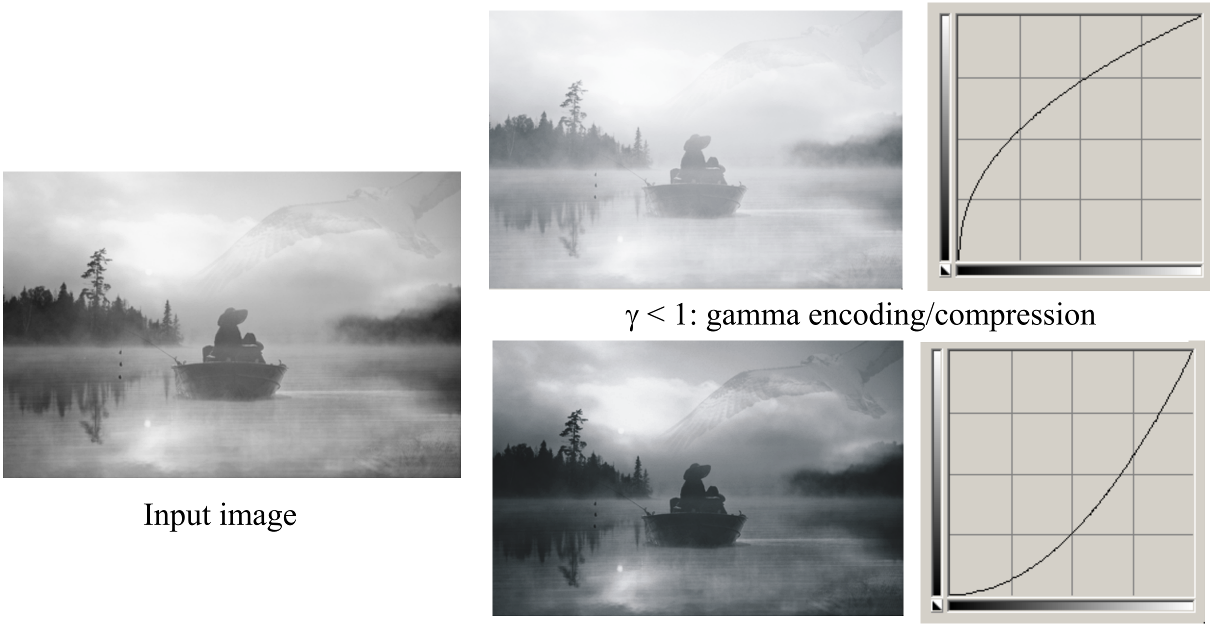
8.5.4. Gamma correction:

$$g_{out} = L\left(\frac{g_{in}}{L}\right)^{\gamma} \tag{8.9}$$

Where:

γ is a positive coefficient: < 1 (gamma encoding/compression) or > 1 (gamma decoding / decompression)

Attention: always check that: 0 <= g_{out} <=255. If outside the domain, values should be saturated!!!



Input image

$\gamma < 1$: gamma encoding/compression

$\gamma > 1$: gamma decoding/expansion

Fig. 8.5 Results of gamma correction operations

8.5.5. Brightness changing (histogram slide)

$$g_{out} = g_{in} + offset \tag{8.10}$$

Attention: always the following checking will be done: 0 <= g_{out} <=255. If an overflow beyond these limits appears, output values will be truncated or scaled!!!

8.6. Histogram equalization

Histogram equalization is a transform which allows us to obtain an output image with a quasi-uniform histogram/PDF, regardless the shape of the histogram/PDF of the input image. For that purpose, the following transform will be used (see lecture notes for more details):

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \quad , \quad k = 0...L \tag{8.11}$$

Where:

r_k – normalized intensity level of the input image corresponding to the (un-normalized) intensity level k: $r_k = \dfrac{k}{L}$, ($0 \le r_k \le 1$ and $0 \le k \le L$)

s_k – corresponding normalized intensity level of the output image;

$p_C(r_k)$ – cumulative probability density function (CPDF) of the input image

$$p_C(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{g=0}^{k} \frac{h_r(g)}{M} \tag{8.12}$$

r_j – normalized intensity level of the input image corresponding to the (un-normalized) intensity level j: $r_j = \dfrac{j}{L}$.

8.6.1. Histogram equalization algorithm

1. Compute the histogram or the PDF of the input image (as a 256 elements vector)
2. Compute the CPDF of the input image (8.20), as a vector of 256 elements.
3. Compute the transformation for the histogram equalization according to (8.20). Because the s_k values obtained from (8.19) are normalized intensity values, it is necessary to transform the normalized intensity values s_k back to un-normalized ones by multiplication with L (the highest intensity value: 255 for 8 bits/pixel images):

$$g_{out} = Ls_k = \frac{L}{M}\sum_{g=0}^{g_{in}} h(g) \quad , \quad k = g_{in} \tag{8.13}$$

This transformation function can be written as an equivalence table (vector):

$$g_{out} = tab(g_{in}) = 255 \cdot p_C(g_{in}) \tag{8.14}$$

4. The intensity values of the output (equalized) image are computed using the equivalence table:

$$Dst(i, j) = tab\left(Src(i, j)\right) \tag{8.15}$$

8.7. Practical work

1. Compute and display the mean and standard deviation, the histogram and the cumulative histogram of the image intensity levels. For the histogram use the *ShowHistogram* function from OpenCV Application (see also L3).
2. Implement the automatic threshold computation (section 8.4) and threshold the images according to this threshold.
3. Implement the histogram transformation functions (section 8.5) for histogram stretching/shrinking, gamma correction, histogram slide. Input the limits $g_{out}^{MIN}, g_{out}^{MAX}$, the gamma coefficient and the brightness increase value from the console. After each processing display the histograms of the source and destination images.
4. Implement the histogram equalization algorithm (section 8.6). Display the histograms of the source and destination images.
5. **Save your work. Use the same application in the next laboratories. At the end of the image processing laboratory you should present your own application with the implemented algorithms.**

Bibliography

[1]. R.C.Gonzales, R.E.Woods, *Digital Image Processing. 2-nd Edition*, Prentice Hall, 2002.