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Electric field computation inside a rectangular petrol tank

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ABSTRACT

In the paper was developed a hybrid method (analytic–numeric) for the electric field computation inside a rectangular petrol tank. For this problem it will be assumed a static state and all the computations will be developed in a Cartesian coordinate system. Two regions will be delimitated: first one is the free space and second one is the liquid electro-statically charged (petrol) inside the tank.

For solving the Laplace equation in the first region, variable separation method will be applied and the electric potential will be determined. In the second region the Poisson equation will be solved, assuming a particular solution and then the electric potential will be obtained. To determine the constants from the potential expressions the zero potential condition at the tank surface and the continuity conditions at the separation surface was considered. Using mathematical manipulations (series development, numerical approximations etc.) an ill conditioned matrix system (23×23) resulted. This system will be numerically solved/discussed using a developed MathCad algorithm. The condition number of the coefficients matrix is being computed and the regularization methods applied to the system will be discussed.

1. Introduction. Electrostatic charge in industrial processes

At the displacement of the liquid inside a metallic pipeline, the thin layer of liquid in direct contact with the metal, that stays fixed during the movement, is the absorbed layer and is separated through the sliding surface. Usually, the sliding surface is inside the diffusion layer and therefore the liquid that reaches the recipient is electrically charged. The level of electrostatic charge of the displacement liquid is function of the traveling speed, the roughness of the interior surface of the pipeline, type of movement, electric permittivity and conductivity [1–3].

If electrostatically charged liquid is sent to an earthed tank, electrostatic charging of the tank to potentially dangerous values is possible. Static electricity level of the tank is determined both by the convection current due to electric charges driven by the liquid, but also by the non-zero insulation resistance of the container with earth [4].

Due to the fact that electrostatic charge phenomenon is extremely complex an universally accepted relationship for the computation of the electrostatic charge level could not be established. Under these conditions, the relations derived are indicative; the analysis of the real phenomenon is based only on experimental data [5].

The transfusion of the flammable liquids with superior properties is followed by the apparition of an important quantity of electric charge when loading and unloading the tanks. In certain conditions, the electric charge amassed may lead to dangerous loading values of the pipeline, tank or other earthed metal objects.

2. The electric field inside a parallelepiped gas tank considering a bidimensional variation

A rectangular shaped tank is at least as common in practice as the cylinder tank. The dimensions of the tank will replicate the ones used in ports. Thus the cross section through such a tank will be a rectangle with the length $2a = 4$ m and the height $b = 10$ m, Fig. 1. The depth of the parallelepiped has a size greater than 15 m that allows considering a plane parallel problem. In Cartesian axis system, tank size on z axis is much larger than the size on the O_x axis Fig. 2.

In the standards of use of these tanks is prohibited their filling more than 95% of the height. Since the danger of explosion is proportional to the maximum value of the electric field in the air and knowing that this value is proportional to the volume density of the electric charge (which depends on many factors difficult to control) it can be concluded that there may be problems even if the height of petrol in the tank is less than that prescribed in the regulations. These observations justify the need of an electric potential and electric field study function of the gas layer height.

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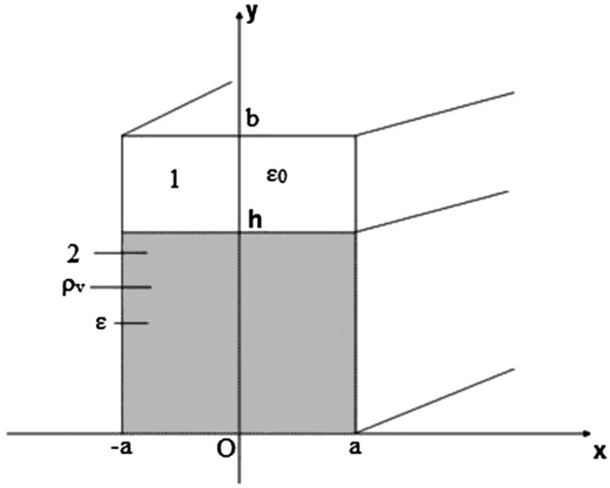

Fig. 1. The cross section through the tank. *axb* – the dimensions of the rectangular tank; *h* – the height of the liquid in the tank. ε_0 – the permittivity of the vacuum (medium 1); ε – the permittivity of the gasoline (medium 2).

2.1. Theoretical determination of the electric field and potential in the tank

Two regions have been determined: 1 with air ($\varepsilon = \varepsilon_0$) and 2 with electrostatic charged liquid (gas). In the first region although exist liquid vapors, the zone is considered as vacuum (air) with $\varepsilon = \varepsilon_0$. The computations will be performed in the Cartesian coordinate system:

In region 1 the Laplace equation was considered : $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ (1)

In region 2 the Poison equation was considered : $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\rho v}{\varepsilon}$ (2)

For solving Laplace equation in vacuum (region 1) variable separation method is applied:

$$V(x,y) = P(x)\,Q(y) \tag{3}$$

$$\text{It results}: \quad \frac{1}{P(x)}\frac{\partial^2 P(x)}{\partial x^2} = -\frac{1}{Q(y)}\frac{\partial^2 Q(y)}{\partial y^2} = k^2 \tag{4}$$

the solution of Laplace equation is:

$$V^{(1)}(x,y) = \sum_{k=1}^{\infty}(M_k chkx + N_k shkx)(R_k \cos ky + S_k \sin ky) \tag{5}$$

h =	V1(h) =
1	$1.306\cdot10^6$
2	$2.388\cdot10^6$
3	$3.231\cdot10^6$
4	$3.817\cdot10^6$
5	$4.123\cdot10^6$
6	$4.12\cdot10^6$
7	$3.768\cdot10^6$
8	$3.016\cdot10^6$
9	$1.793\cdot10^6$
10	0

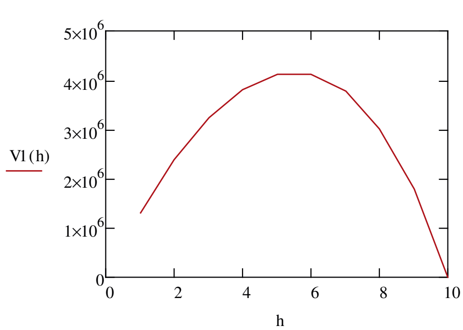

Fig. 2. Potential variation at the separation surface on the symmetry axis.

For solving Poison equation in the gas area (region 2) a particular solution form will be found:

$$V_p(x,y) = -\frac{\rho_v}{2\varepsilon}x^2 + mx \tag{6}$$

or

$$V_p(x,y) = -\frac{\rho_v}{2\varepsilon}y^2 + qy \tag{7}$$

Using Eq. (6), results:

$$V^{(2)}(x,y) = \sum_{k=1}^{\infty}(A_k chkx + B_k shkx)\,(C_k \cos ky + D_k \sin ky) - \frac{\rho_v}{2\varepsilon}x^2 + mx \tag{8}$$

using Eq. (7) again the following form has been obtained:

$$V^{(2)}(x,y) = \sum_{k=1}^{\infty}(A_k chkx + B_k shkx)\,(C_k \cos ky + D_k \sin ky) - \frac{\rho_v}{2\varepsilon}y^2 + qy \tag{9}$$

Constants determination is based on the following conditions: a) Symmetry condition: $V(x,y) = V(-x,y)$, Eqs. (5), (8), (9) became:

$$V^{(1)}(x,y) = \sum_{k=1}^{\infty} chkx\,(T_k \cos ky + W_k \sin ky) \tag{10}$$

$$V^{(2)}(x,y) = \sum_{k=1}^{\infty} chkx\,(E_k \cos ky + F_k \sin ky) - \frac{\rho_v}{2\varepsilon}x^2 \tag{11}$$

or

$$V^{(2)}(x,y) = \sum_{k=1}^{\infty} chkx\,(E_k \cos ky + F_k \sin ky) - \frac{\rho_v}{2\varepsilon}y^2 + qy \tag{12}$$

where $T_k = M_k R_k$, $W_k = M_k S_k$, $E_k = A_k C_k$ and $F_k = A_k D_k$

b) Tank walls have null potential so the zero potential condition on the basis from Eq. (12) has been used:

$$V^{(2)}(x,0) = 0 \tag{13}$$

leading to the conclusion that the particular form of the Poisson equation is best suited, relations (9) and (12) respectively:

$$V^{(2)}(x,0) = \sum_{k=1}^{\infty} E_k chkx = 0 \Rightarrow E_k = 0 \tag{14}$$

$$V^{(2)}(x,y) = \sum_{k=1}^{\infty} F_k chkx \ \sin ky - \frac{\rho_v}{2\varepsilon}y^2 + qy \tag{15}$$

condition b) has been accomplished by Eq. (15).

c) From the continuity condition of the potential on the separation surface $V^{(1)}(x, h) = V^{(2)}(x, h)$, with $V^{(1)}$ and $V^{(2)}$ necessary exists for the same values of k we have:

$$\sum_{k=1}^{\infty} chkx[T_k \cos kh + (W_k - F_k)\sin kh] = -\frac{\rho_v}{2\varepsilon}h^2 + qh \tag{16}$$

as *chkx* is variable function of *x*, it has been imposed:

$$[T_k \cos kh + (W_k - F_k)\sin kh] = 0 \text{ and } -\frac{\rho_v}{2\varepsilon}h^2 + qh = 0 \tag{17}$$

$$\Rightarrow T_k = 0; W_k = F_k; q = \frac{\rho_v}{2\varepsilon}h \tag{18}$$

Results that the relation (18) solve c) condition, that is equivalent to the conservation of the electric field tangential component on the separation surface. Results:

$$V^{(1)}(x,y) = \sum_{k=1}^{\infty} F_k chkx \sin ky \tag{19}$$

and

$$V^{(2)}(x,y) = \sum_{k=1}^{\infty} F_k chkx \sin ky - \frac{\rho_v}{2\varepsilon} y^2 + \frac{\rho_v}{2\varepsilon} hy \tag{20}$$

2.2. System of hybrid equations (analytic–numeric)

After some numerical trials it was proved that is very convenient to work in the following steps with $z_k = F_k \frac{ch(ka)}{10^5}$.

d) Null potential condition on the tank lid results in:

$$V^1(x,b) = \sum_{k=1}^{\infty} 10^5 \frac{chkx}{chka} \frac{chkaF_k}{10^5} \sin kb = 0; \quad x\varepsilon(0,a) \tag{21}$$

e) From the conservation condition of the normal component of the electric field induction on the separation surface

$$\varepsilon_0 \varepsilon_r \frac{\partial V^{(2)}(x,y)}{\partial y}\bigg|_{y=h} = \varepsilon_0 \frac{\partial V^{(1)}(x,y)}{\partial y}\bigg|_{y=h} \tag{22}$$

$$\text{results}: \sum_{k=1}^{\infty} kF_k ch\, kx \cos kh = \frac{\varepsilon_r}{\varepsilon_r - 1}\left(\frac{\rho_v}{\varepsilon} y - \frac{\rho_v}{\varepsilon}\frac{h}{2}\right)\bigg|_{y=h} \tag{23}$$

where $\rho_v = 10^{-5} \mathrm{C/m^3}$ was chosen taking into account some experimental results [5], so the condition which will be numerically used is:

$$\sum_{k=1}^{\infty} k \frac{chkaF_k}{10^5} \frac{chkx}{chka} \cos kh = \frac{1.8\pi}{\varepsilon_r - 1} h \tag{24}$$

f) Null potential condition on the vertical tank wall in the gas region is

$$V^2(a,y) = \sum_{k=1}^{\infty} F_k \frac{chka}{10^5} \sin ky - \frac{1.8\pi}{\varepsilon_r}\left(y^2 - yh\right) = 0; \quad \forall y \in (0,h) \tag{25}$$

and in the air region is

$$V^1(a,y) = \sum_{k=1}^{\infty} F_k \frac{chka}{10^5} \sin ky = 0; \quad \forall y \in (h,b) \tag{26}$$

For the (25) formula was taken into account that:

$$\frac{\rho_v}{2\varepsilon}\left(y^2 - yh\right) = \frac{1.8\pi}{\varepsilon_r}\left(y^2 - yh\right) \tag{27}$$

After the unknown determination $z_k = F_k \frac{ch(ka)}{10^5}$, the formula of the electric potential in air is:

$$V^1(x,y) = \sum_{k=1}^{\infty} F_k \frac{chka}{10^5} \frac{10^5}{chka} chkx \sin ky \tag{28}$$

Numerically, it has been chosen:

$m = 4 \Rightarrow m+1 = 5$ points corresponding to O_x axis
$t = 8 \Rightarrow t = 8$ points corresponding to O_y axis (inside the gas)
$p = 6 \Rightarrow p-1 = 5$ points in the tank air on top of the gas

2.3. Numeric calculus for the electric potential and electric field in the tank. Mathcad implementation

Numerical data for Mathcad computations are: $\varepsilon_r = 2$, $a = 2$, $b = 10$, $h = 8$, $m = 4$ (5 equations); $t = 8$ (8 equations); $p = 6$ (5 equations), $n = 2m + p + t + 1 = 23$ equations [6].

The height of the gas in the tank has been set to 80% of the maximum value. This value has been chosen based on the fact that is the critical height, explosion danger at this height has not been specified in the instructions. The construction manner of the equations for the case of the $n = 23$ equations considered is being presented. The first $m + 1 = 5$ equations represent the potential expression in 5 equidistant points located on the tank lid. The first location is the symmetry axis and the last one is the lateral wall of the tank.

$$j := 1..n \quad i := 1..m+1$$
$$A_{i,j} := \frac{10^5 \cdot ch\left[j\cdot(i-1)\cdot\frac{a}{m}\right]\cdot\sin(j\cdot b)}{ch(j\cdot a)} \tag{29}$$

The next $t = 8$ equations represent the null potential condition on the lateral walls of the tank inside the gas layer.

$$i := m+2..m+t+1 \quad A_{i,j} := \sin\left[j\cdot(i-m-1)\cdot\frac{h}{t}\right] \tag{30}$$

The $p - 1 = 5$ equations represent the null potential on the lateral wall of the tank in air.

$$i := m+t+2..m+t+p$$
$$A_{i,j} := \sin\left[j\cdot\left[h + \left[(i-m-1)\cdot\frac{b-h}{p}\right]\right]\right] \tag{31}$$

The last $m + 1 = 5$ equations represent the conservation of the normal component of electric flux density at the separation surface between air and gas.

$$i := m+t+p+1..m+t+p+m+1$$
$$A_{i,j} = \frac{j\, ch\left(j\cdot(i-m-t-p-1)\cdot\frac{a}{m}\right)\cdot\cos(j\cdot h)}{ch(j\cdot a)} \tag{32}$$

Adding all the equations together the 23 equations system results. The free term has been built according to the observations made in the first paragraph of this paper.

$$i := 1..m+1; \quad T_{i,1} := 0$$
$$i := m+2..m+t+1$$
$$T_{i,1} := \frac{1.8\cdot\pi\cdot(i-m-1)\cdot\frac{h}{t}\cdot[-h+(i-m-1)]}{er} \tag{33}$$
$$i := m+t+2..m+t+p \quad T_{i,1} := 0$$
$$i := m+t+p+1..m+t+p+m+1; \quad T_{i,1} := \frac{1.8\cdot\pi\cdot h}{er-1}$$

The unknown column vector has 23 rows; on the k line the unknown was noted with $z_k = \frac{chka}{10^5}F_k$. For solving the system of 23×23 it was applied the method of regularization of degenerate systems of equations (binomial coefficients method) [7,8].

Solving the determined system with 23 equations column vector z ($z_1 = -1.393\cdot10^4, z_2 = -1.553\cdot10^5$ etc.) For the computation of the potential in air, where the point has x, y coordinates, the numeric form deduced from the relation (28) has been used:

$$V^{(1)}(x,y) = \sum_{k=1}^{23} z_k \frac{chkx}{chka} 10^5 \sin ky \tag{34}$$

Then, this calculation has been repeated for a vertical line situated parallel to the first at a distance of $a/m + 1 = a/5 = 0.4$ m. At last, the field on the horizontal direction was calculated.

For example:

$$E_y(0,h) = \frac{V(0,h) - V\left(0, h + \frac{b-h}{p}\right)}{\frac{b-h}{p}} = \frac{1.688 \cdot 10^6 - 3.906 \cdot 10^5}{1/3} = 38.924 \frac{\text{KV}}{\text{cm}} \tag{35}$$

$$E_x(0,h) = \frac{V(0,h) - V\left(\frac{a}{m+1}, h\right)}{\frac{a}{m+1}} = \frac{1.688 \cdot 10^6 - 1.389 \cdot 10^6}{2/5} = 7.475 \frac{\text{KV}}{\text{cm}} \tag{36}$$

$$E(0,h) = \sqrt{E_x(0,h)^2 + E_y(0,h)^2} = 39.635 \text{KV/cm} \tag{37}$$

It is noticeable that for given data, this field produces for certain an explosion, due to the fact that the maximum limit of dielectric rigidity of the air (30 kV/cm) has been exceeded.

It was represented also the potential variation at the separation surface on the symmetry axis in function of height of the liquid in the tank.

3. Conclusions

In this paper the authors developed a hybrid method for the electric field computation inside a rectangular tank partially filled with gasoline. Regarding the case of cylindrical tank in which the solution it could be found applying the standard method of variable separation method and in the solution Bessel functions appear, in our case (rectangular thank) this standard method could not be applied and we used a hybrid method.

The theoretical determinations have shown that the electric field is directly proportional to the volume density of the electric charge. So, the first method limits the electric field from the tank in order to prevent fuel particles electrified by friction to enter the tank. To that effect a reliable connection to earth of the metal pipe joints needs to be realized. Also, a flow rate that does not favor the workload has to be chosen accordingly.

It was selected two regions: first one is the free space and second one is the liquid electro-statically charged inside the tank. It was solved analytically the Laplace equation in the first region, and Poisson equation in the second region, assuming a particular solution. Using several mathematical manipulations (series development, numerical approximations etc.) an ill conditioned matrix system (23 × 23) resulted and was numerically solved using regularization methods.

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