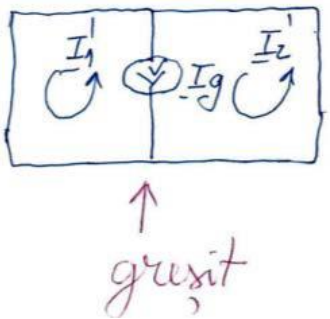
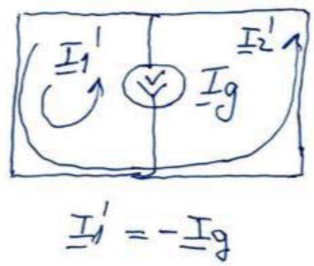
Method of loop currents

To solve a circuit using the **method of loop currents we take the following steps**:

1. We check whether the circuit contains current sources. If so, the loop currents are chosen so that only one loop current passes through the branch with the current source. Its value will be equal to the value of the current source, if the two currents (loop crt. and the one through the source) have the same direction, or it will be equal to minus the value of the current source, if the two currents do not have the same direction.





2. The system of equations corresponding to the method is formed. It contains a number of equations equal to the number of loop currents (the number of independent loops of the circuit). Since the loop current through the current source (if any) is known, the equation corresponding to this current (this loop) is no longer necessary, the system of equations formed will therefore have one less equation.

The system of equations specific to the method, in which the right-hand member represents the sum of the electromotive voltage sources belonging to the respective loop (the sign of the source can be + or - , depending on its meaning with respect to the meaning of the chosen loop current) is:

$$\begin{cases} \underline{J}_1 \cdot \underline{Z}_{11} + \underline{J}_2 \cdot \underline{Z}_{12} + \cdots + \underline{J}_n \cdot \underline{Z}_{1n} = \sum \underline{E}_1 \\ \underline{J}_1 \cdot \underline{Z}_{21} + \underline{J}_2 \cdot \underline{Z}_{22} + \cdots + \underline{J}_n \cdot \underline{Z}_{2n} = \sum \underline{E}_2 \\ \ldots \\ \underline{J}_1 \cdot \underline{Z}_{n1} + \underline{J}_2 \cdot \underline{Z}_{n2} + \cdots + \underline{J}_n \cdot \underline{Z}_{nn} = \sum \underline{E}_n \end{cases}$$

When determining the impedances that occur in the system, the following algebraic rules are taken into account:

To determine **the self impedance of the loop *k***:

$$\underline{Z}_{kk} = \sum_{\ell \in (k)} \underline{Z}_\ell + \sum_{\substack{L_p \in (k) \\ L_q \in (k)}} \varepsilon_{pq} \cdot 2j\omega L_{pq}$$

where $\varepsilon_{pq} = \begin{cases} 0, & \text{if there } \textit{are no two mutually coupled coils} \text{ in loop k;} \\ 1, & \text{if } \underline{J}_k \text{ it has the same direction with respect to the marked} \\ & \text{terminals of the coils Lp and Lq, belonging to the k loop;} \\ -1, & \text{otherwise} \end{cases}$

Where ℓ represents all the branches that belong to loop *k*.

Joint impedance between loops *k* and *j*:

$$\underline{Z}_{kj} = \sum_{\substack{\ell \in (k) \\ \ell \in (j)}} \alpha_{kj} \cdot \underline{Z}_{\ell} + \sum_{\substack{L_m \in (k) \\ L_n \in (j)}} \eta_{kj} \cdot j\omega L_{mn} \quad \text{where}$$

$\alpha_{kj} =$
- 1, if $\underline{J}_k$ and $\underline{J}_j$ have the same direction through the commomn branch ℓ between *k* and *j*;
- 0, if there is no impedance on the common branch or there is no common branch between the two loops;
- −1, otherwise

$\eta_{kj} =$
- 1, if $\underline{J}_k$ and $\underline{J}_j$ have the same orientation with respect to the marked terminals of the coils L_m and L_n;
- −1, otherwise;
- 0, if there are no mutually coupled coils in loops *k* and *j*

(j), (k) - represent any loops of the circuit, and ℓ their common branch.

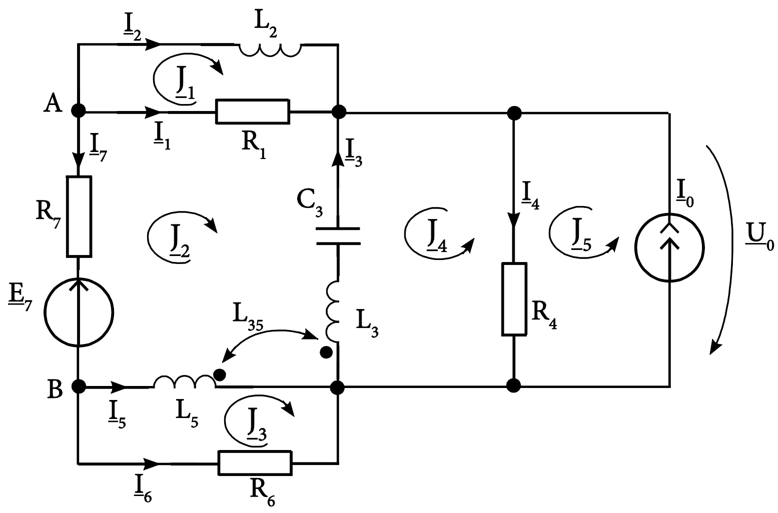
It is noted that for both impedances $\underline{Z}_{kk}$ and $\underline{Z}_{kj}$, the second term of the sum that composes them is due to the mutual magnetic couplings between the coils.

3. We solve the system of equations and compute the loop currents.
4. All the loop currents that run along the branch for which the current is calculated are added together. The sign of the loop current is (+) if it runs along the branch in the same direction as the branch current and (-) otherwise .

Example:

The circuit in the figure is considered, with the following numerical data: R1 = 2[Ω], ωL2 = 2[Ω], ωL3 = 2[Ω], $\frac{1}{\omega C_3}$ = 1[Ω], R4 = 1[Ω] , ωL5 = 1[Ω], R6 = 1[Ω], R7 = 1[Ω], ωL35 = 1Ω[], $\underline{I0}$ = 20 – 10j [A] and $\underline{E7}$ = 20 + 10j[V].

a) To be solved by the cyclic current method.
b) To determine the current through the source $\underline{E7}$ with the help of Thevenin's theorem.



Solution:

1. We check if the circuit contains current sources. We select the loop currents so that a single loop current passes through the branch with the current source. Its value will be equal to the value of the current source: $\underline{J5} = \underline{I0}$
2. The system of equations corresponding to the method is formed. It contains a number of equations equal to the number of cyclic currents (the number of independent meshes of the circuit). Since the loop current $\underline{J5}$ is known, the 5th equation (corresponding to this current) is no longer necessary, so the system of equations formed will be with 4 equations and 4 unknowns.

$$\begin{cases} \underline{J}_1 \cdot \underline{Z}_{11} + \underline{J}_2 \cdot \underline{Z}_{12} + \underline{J}_3 \cdot \underline{Z}_{13} + \underline{J}_4 \cdot \underline{Z}_{14} + \underline{J}_5 \cdot \underline{Z}_{15} = 0 \\ \underline{J}_1 \cdot \underline{Z}_{21} + \underline{J}_2 \cdot \underline{Z}_{22} + \underline{J}_3 \cdot \underline{Z}_{23} + \underline{J}_4 \cdot \underline{Z}_{24} + \underline{J}_5 \cdot \underline{Z}_{25} = \underline{E}_7 \\ \underline{J}_1 \cdot \underline{Z}_{31} + \underline{J}_2 \cdot \underline{Z}_{32} + \underline{J}_3 \cdot \underline{Z}_{33} + \underline{J}_4 \cdot \underline{Z}_{34} + \underline{J}_5 \cdot \underline{Z}_{35} = 0 \\ \underline{J}_1 \cdot \underline{Z}_{41} + \underline{J}_2 \cdot \underline{Z}_{42} + \underline{J}_3 \cdot \underline{Z}_{43} + \underline{J}_4 \cdot \underline{Z}_{44} + \underline{J}_5 \cdot \underline{Z}_{45} = 0 \end{cases}$$

The impedance values for the circuit related to the problem are:

$$\underline{Z}_{11} = R_1 + j\omega L_2 = 2 + 2j[\Omega];$$

$$\underline{Z}_{12} = \underline{Z}_{21} = -R_1 = -2[\Omega];$$

$$\underline{Z}_{13} = \underline{Z}_{31} = 0[\Omega];$$

$$\underline{Z}_{14} = \underline{Z}_{41} = 0[\Omega];$$

$$\underline{Z}_{15} = 0[\Omega];$$

$$\underline{Z}_{22} = R_7 + R_1 + \frac{1}{j\omega C_3} + j\omega L_3 + j\omega L_5 - 2j\omega L_{35} = 3[\Omega];$$

$$\underline{Z}_{32} = \underline{Z}_{23} = -j\omega L_5 + j\omega L_{35} = 0[\Omega];$$

$$\underline{Z}_{42} = \underline{Z}_{24} = j\omega L_3 + \frac{1}{j\omega C_3} - j\omega L_{35} = 0[\Omega];$$

$$\underline{Z}_{25} = 0[\Omega];$$

$$\underline{Z}_{33} = j\omega L_5 + R_6 = 1 + j[\Omega];$$

$$\underline{Z}_{34} = \underline{Z}_{43} = j\omega L_{35} = j[\Omega];$$

$$\underline{Z}_{35} = 0[\Omega];$$

$$\underline{Z}_{44} = R_4 + \frac{1}{j\omega C_3} + j\omega L_3 = 1 + j[\Omega];$$

$$\underline{Z}_{45} = -R_4 = -1[\Omega].$$

The system becomes: $\begin{cases} \underline{J}_1 \cdot (2 + 2j) - 2\underline{J}_2 = 0 \\ -2\underline{J}_1 + 3\underline{J}_2 = 20 + 10j \\ (1 + j)\underline{J}_3 + j \cdot \underline{J}_4 = 0 \\ j \cdot \underline{J}_3 + (1 + j)\underline{J}_4 - 1 \cdot (20 - 10j) = 0 \end{cases}$

with the solutions: $\underline{J}_1 = 5 - 5j$[A]; $\underline{J}_2 = 10$[A]; $\underline{J}_3 = -10$[A]; $\underline{J}_4 = 10 - 10j$[A].

3. The currents on the branches are determined as follows:

$\underline{I}_1 = \underline{J}_2 - \underline{J}_1 = 5 + 5j$ [A]

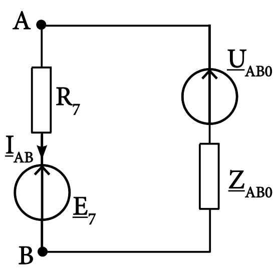
$\underline{I}_2 = \underline{J}_1 = 5 - 5j$ [A]

$\underline{I}_5 = \underline{J}_3 - \underline{J}_2 = -20$ [A]

$\underline{I}_6 = -\underline{J}_3 = 10$ [A]

$\underline{I}_3 = -\underline{J}_2 - \underline{J}_4 = -20 + 10j$ [A] $\underline{I}_7 = -\underline{J}_2 = -10$ [A]
$\underline{I}_4 = \underline{J}_5 - \underline{J}_4 = 10$ [A]

b) Finding the current through one branch of the circuit by Thévenin's theorem implies the replacement of that circuit (except the targeted side) with an equivalent voltage generator.
For the present problem, the electrical diagram below is represented:



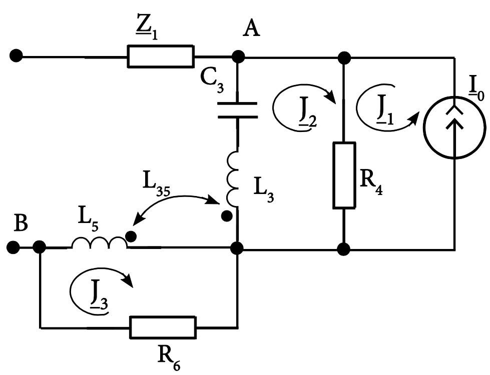
So: $\underline{I}_7 = \underline{I}_{AB} = \dfrac{\underline{U}_{AB_0} - \underline{E}_7}{R_7 + \underline{Z}_{AB_0}}$;

with the observation that the sign of $\underline{E}_7$ is taken with + if it coincides with the meaning of $\underline{I}_{AB}$ and with – otherwise.

The voltage $\underline{U}_{AB0}$ (open circuit between terminals A and B) is determined from the adjacent circuit, where $\underline{Z}_1$ represents the impedance of the parallel circuit R_1, L_2:

$$\underline{Z}_1 = \frac{R_1 \cdot j\omega L_2}{R_1 + j\omega L_2} = 1 + j[\Omega].$$

It is found that this impedance is not crossed by the current (the circuit is interrupted between A and B) as a result there is no voltage drop on this impedance. Point A therefore moves according to the figure:



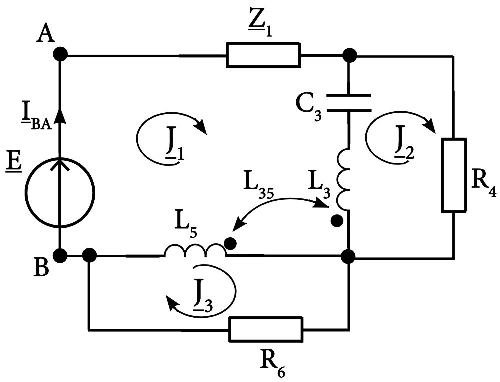
Next, the problem is solved using the loop current method:

$$\begin{cases} \underline{J}_1 = \underline{I}_0 = 20\text{-}10j \\ R_4 \cdot \underline{J}_1 + \left(R_4 + j\,\omega L_3 + \dfrac{1}{j\omega C_3}\right) \cdot \underline{J}_2 - j\,\omega L_{35} \cdot \underline{J}_3 = 0 \\ -j\omega L_{35} \cdot \underline{J}_2 + (R_6 + j\omega L_5) \cdot \underline{J}_3 = 0 \end{cases}$$

$$\begin{cases} \underline{J}_1 = 20 - 10j \\ \underline{J}_1 + (1 + j) \cdot \underline{J}_2 - j \cdot \underline{J}_3 = 0 \\ -j \cdot \underline{J}_2 + (1 + j) \cdot \underline{J}_3 = 0 \end{cases}$$

With the solution $\underline{J}_1 = 20 - 10j$ [A]; $\underline{J}_2 = -10 + 10j$ [A]; $\underline{J}_3 = -10$ [A], the current through the resistance R_4 is therefore $\underline{J}_1 + \underline{J}_2$ and through R6 it is $\underline{J}_3$.
So: $\underline{U}_{AB_0} = \underline{U}_{R_4} + \underline{U}_{R_6} = R_4(\underline{J}_1 + \underline{J}_2) + R_6 \cdot \underline{J}_3 = 0[V]$.

To determine the $\underline{Z}_{AB0}$ impedance (the internal impedance of the circuit with all sources st to zero – i.e. with the voltage sources replaced by a wire, and the current sources by an interruption) the following artifice is used: between terminals A and B any voltage source is considered connected, so that the impedance between the terminals can be calculated as the ratio between the voltage of the source and the current that travels through it (see figure below).



$$\underline{Z}_{AB_0} = \frac{\underline{E}}{\underline{I}_{BA}} = \frac{\underline{U}_{AB}}{\underline{I}_{BA}}$$

To determine the IAB current, the cyclic current method is again applied:

$$\begin{cases} \underline{J}_1 \cdot \left(\underline{Z}_1 + \frac{1}{j\omega C_3} + j\omega L_3 + j\omega L_5 - 2j\omega L_{35} \right) + \left(-j\omega L_3 - \frac{1}{j\omega C_3} + j\omega L_{35} \right) \cdot \underline{J}_2 + \\ \qquad + \left(-j\omega L_5 + j\omega L_{35} \right) \cdot \underline{J}_3 = \underline{E} \\ \underline{J}_1 \cdot \left(-j\omega L_3 - \frac{1}{j\omega C_3} + j\omega L_{35} \right) + \left(R_4 + \frac{1}{j\omega C_3} + j\omega L_3 \right) \cdot \underline{J}_2 - j\omega L_{35} \cdot \underline{J}_3 = 0 \\ \underline{J}_1 \cdot (-j\omega L_5 + j\omega L_{35}) + \underline{J}_2 \cdot (-j\omega L_{35}) + \underline{J}_3 (j\omega L_5 + R_6) = 0 \end{cases}$$

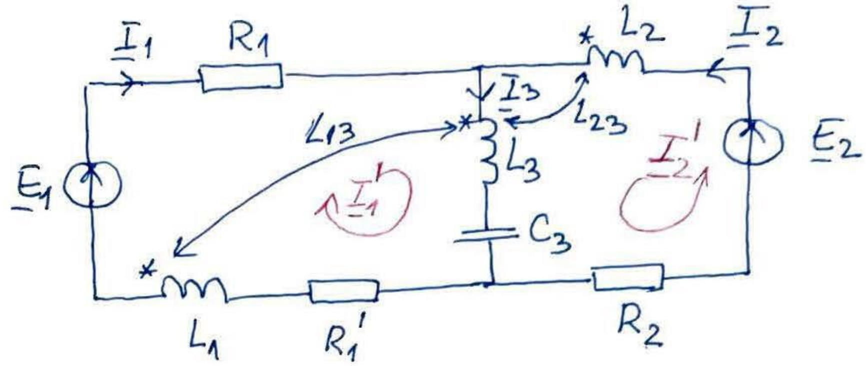
$$\begin{cases} \underline{J}_1 \,(1 - j) + \underline{J}_2 \,(-2j) + \underline{J}_3 (0) = \underline{E} \\ \underline{J}_1 \,(-2j) + \underline{J}_2 \,(1 + j) - j\, \underline{J}_3 = 0 \\ \underline{J}_1 \cdot (0) + \underline{J}_2 \,(-j) + \underline{J}_3 (1 + j) = 0 \end{cases}$$

The system is solved, the term $\underline{E}$ is simplified and the following is obtained:

$$\underline{Z}_{AB_0} = \frac{\underline{E}}{\underline{I}_{BA}} = \frac{\underline{E}}{\underline{J}_1} = 1 + j[\Omega].$$

Exemplul 2

Să se rezolve circuitul prin metoda curenților ciclici:



$$\begin{cases} \underline{Z}_{11}' \cdot \underline{I}_1' + \underline{Z}_{12}' \cdot \underline{I}_2' = \underline{E}_1 \\ \underline{Z}_{21}' \cdot \underline{I}_1' + \underline{Z}_{22}' \cdot \underline{I}_2' = \underline{E}_2 \end{cases}$$

$$\underline{Z}_{11}' = R_1 + R_1' + j\omega L_1 + j\omega L_3 + \frac{1}{j\omega C_3} - 2j\omega L_{13}$$

$$\underline{Z}_{12}' = \underline{Z}_{21}' = j\omega L_3 + \frac{1}{j\omega C_3} - j\omega L_{13} - j\omega L_{23}$$

$$\underline{Z}_{22}' = R_2 + j\omega L_2 + j\omega L_3 + \frac{1}{j\omega C_3} - 2j\omega L_{23}$$

$$\begin{cases} \underline{I}_1 = \underline{I}_1' \\ \underline{I}_3 = \underline{I}_1' + \underline{I}_2' \\ \underline{I}_2 = \underline{I}_2' \end{cases}$$

The method of node voltages,

When applying **the method of node voltages,** the following steps are followed:

1. A node of the circuit is chosen as the potential origin (it is assigned the value of 0[V]). We check whether the circuit contains an ideal voltage source. If so, the node that represents the origin of potential is chosen so that it is one the nodes adjacent to the branch containing the ideal voltage source. The voltage of the other node adjacent to the ideal voltage source is computed directly using KVL.

2. A system of equations is formed to determine the voltages of the other nodes of the circuit, (the number of equations is equal to the number of nodes minus 1). The equation corresponding to the node for which we already computed the voltage value at step 1(if any) is eliminated from the system of equations.

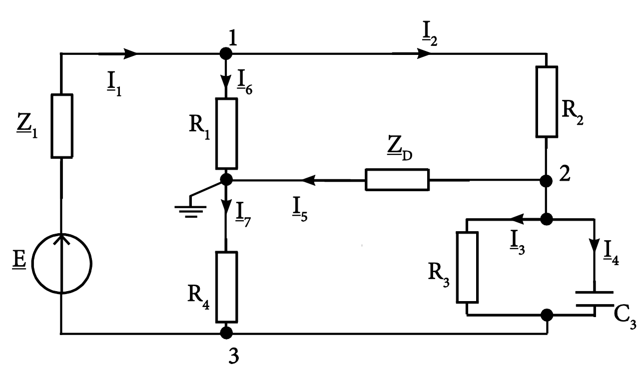
$$\begin{cases} \underline{V_1} \cdot \underline{Y_{1}}_1 + \underline{V_2} \cdot \underline{Y_{1}}_2 + \cdots + \underline{V_n} \cdot \underline{Y_{1}}_n = \sum \underline{I}_{sc_1} \\ \dots \\ \underline{V_1} \cdot \underline{Y_{n}}_1 + \underline{V_2} \cdot \underline{Y_{n}}_2 + \cdots + \underline{V_n} \cdot \underline{Y_{n}}_n = \sum \underline{I}_{sc_n} \end{cases}$$

The terms $\underline{Y}kk$ represent the sum, with the sign (+), of the admitances of the branches adjacent to the node (k).
The terms $\underline{Y}_{kj}$ represent the sum, with the sign (-), of the admittances of the branches connecting the nodes (k) and (j).
The right hand side terms of the system of equations represent the sum of the short circuit currents of the branches adjacent to each node of the circuit. The short-circuit currents of the branches are calculated by short-circuiting their ends with a wire. For the branches that do not contain sources, the short-circuit currents are zero. If the branch adjacent to the node contains a source directed towards it (the node), the short-circuit current is taken with the sign (+) and with (-) otherwise.

3. We solve the system of equations and compute the node voltages.
4. We calculate the currents of the branches, knowing the node voltageg and applying KVL.

Examples:

1. The circuit shown in the figure below is a Wien bridge. Write the equations for solving the circuit by the method of node potentials.



Solution:

A node of the circuit is chosen as the potential origin (it is assigned the value of 0[V]);
The system of equations is formed (the number of equations, in the case of this, is 3).

$$\begin{cases} \underline{V}_1 \cdot \underline{Y}_{11} + \underline{V}_2 \cdot \underline{Y}_{12} + \underline{V}_3 \cdot \underline{Y}_{13} = \sum \underline{I}_{sc_1} \\ \underline{V}_1 \cdot \underline{Y}_{21} + \underline{V}_2 \cdot \underline{Y}_{22} + \underline{V}_3 \cdot \underline{Y}_{23} = \sum \underline{I}_{sc_2} \\ \underline{V}_1 \cdot \underline{Y}_{31} + \underline{V}_2 \cdot \underline{Y}_{32} + \underline{V}_3 \cdot \underline{Y}_{33} = \sum \underline{I}_{sc_3} \end{cases}$$

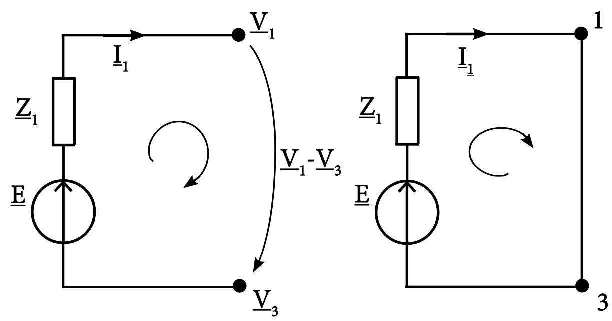
$$\underline{Y}_{11} = \frac{1}{R_1} + \frac{1}{\underline{Z}_1} + \frac{1}{R_2}; \qquad \underline{Y}_{22} = \frac{1}{\underline{Z}_D} + \frac{1}{R_2} + \frac{1}{R_3} + j\omega C_3;$$

$$\underline{Y}_{12} = -\frac{1}{R_2} = \underline{Y}_{21}; \qquad \underline{Y}_{23} = \underline{Y}_{32} = -\left(\frac{1}{R_3} + j\omega C_3\right);$$

$$\underline{Y}_{13} = \underline{Y}_{31} = -\frac{1}{\underline{Z}_1}; \qquad \underline{Y}_{33} = \frac{1}{R_4} + \frac{1}{R_3} + j\omega C_3 + \frac{1}{\underline{Z}_1}.$$

$$\sum \underline{I}_{sc_1} = \frac{\underline{E}}{\underline{Z}_1}; \qquad \sum \underline{I}_{sc_2} = 0; \qquad \sum \underline{I}_{sc_3} = -\frac{\underline{E}}{\underline{Z}_1}.$$

Note:



For example, if the side between nodes 1 and 3 is short-circuited:

$$\underline{I}_1 = \frac{\underline{E}}{\underline{Z}_1}.$$

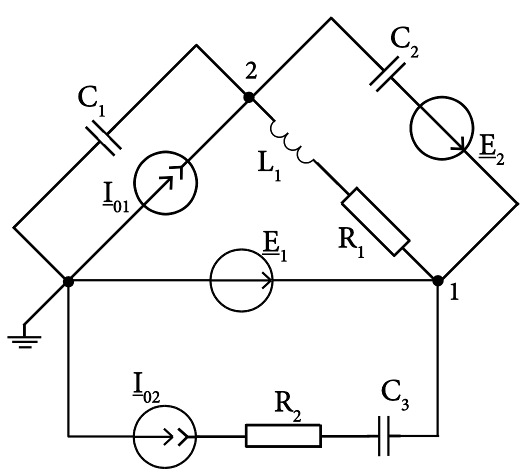
Solving the system leads to the determination of the unknown potentials $\underline{V}_1$, $\underline{V}_2$ and $\underline{V}_3$.

Calculate the currents of the sides:

$$\underline{Z}_1 \cdot \underline{I}_1 + \underline{V}_1 - \underline{V}_3 = \underline{E} \quad \Rightarrow \quad \underline{I}_1 = \frac{\underline{E} + \underline{V}_3 - \underline{V}_1}{\underline{Z}_1}; \underline{I}_2 = \frac{\underline{V}_1 - \underline{V}_2}{R_2}; \underline{I}_3 = \frac{\underline{V}_2 - \underline{V}_3}{R_3}; \ \underline{I}_4 = j\omega C_3 \left(\underline{V}_2 - \underline{V}_3\right);$$

$$\underline{I}_5 = \frac{\underline{V}_2}{\underline{Z}_D}; \ \underline{I}_6 = \frac{\underline{V}_1}{R_1} \text{ and } \underline{I}_7 = -\frac{\underline{V}_3}{R_4}.$$

2. Write the system of equations corresponding to the node potential method for the circuit in the adjacent figure.



Solution:

It is noted that the circuit contains an ideal voltage source ($\underline{E}_1$). In this case, the node chosen as the potential origin must be adjacent to the branch containing that source.

The rule also applies to a circuit that contains several ideal voltage sources, placed on branches that have a common node (the reference node must be adjacent to a branch that contains an ideal voltage source).

The potential of node 1 can thus be calculated directly: $\underline{V}_1 = \underline{E}_1$ (The voltage drop on the ideal voltage source (V1=0) is equal to and opposite to the electromotive voltage).
The equation corresponding to node 1 in the method of node potentials becomes useless.
So we only write the equation for node 2:

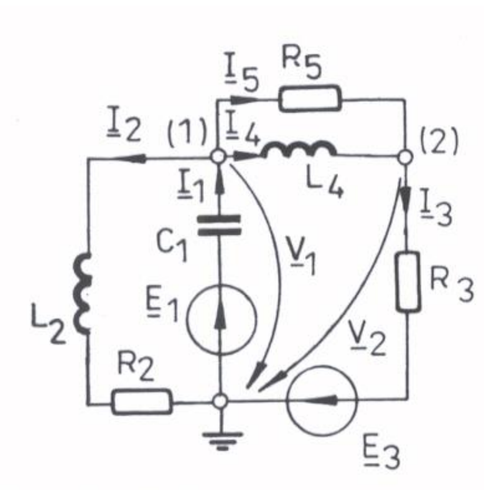
$$\begin{cases} \underline{V}_1 = \underline{E}_1 \\ \underline{V}_1 \cdot \underline{Y}_{21} + \underline{V}_2 \cdot \underline{Y}_{22} = \sum \underline{I}_{sc_2} \end{cases}$$

$$\underline{Y}_{21} = -\left(\frac{1}{R_1 + j\omega L_1} + j\omega C_2 \right); \ \underline{Y}_{22} = j\omega C_1 + j\omega C_2 + \frac{1}{R_1 + j\omega L_1};$$

$$\sum \underline{I}_{sc_2} = -\underline{E}_2 \cdot j\omega C_2 + \underline{I}_{01}.$$

Finally, the branch current must be computed.

3.



$$\begin{cases} \underline{Y}_{11}\underline{V}_1 + \underline{Y}_{12}\underline{V}_2 = \underline{I}_{SC1} \\ \underline{Y}_{21}\underline{V}_1 + \underline{Y}_{22}\underline{V}_2 = \underline{I}_{SC2} \end{cases}$$

$$\underline{Y}_{11} = j\omega C_1 + \frac{1}{R_2 + j\omega L_2} + \frac{1}{j\omega L_4} + \frac{1}{R_5}$$

$$\underline{Y}_{22} = \frac{1}{R_3} + \frac{1}{j\omega L_4} + \frac{1}{R_5}$$

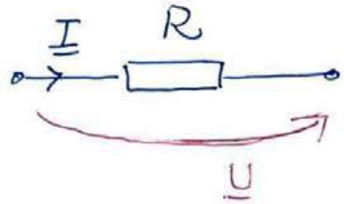
$$\underline{Y}_{12} = \underline{Y}_{21} = -\left(\frac{1}{j\omega L_4} + \frac{1}{R_5} \right)$$

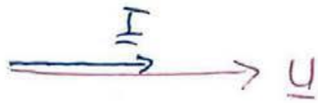
$$\underline{I}_{sc1} = \underline{E}_1 \, j\omega C \qquad \underline{I}_{sc2} = -\frac{\underline{E}_3}{R_3} \qquad \Rightarrow \underline{V}_1, \ \underline{V}_2$$

$$\underline{I}_1 = \frac{\underline{E}_1 - \underline{V}_1}{1/\,j\omega C_1}; \quad \underline{I}_2 = \frac{\underline{V}_1}{R_2 + j\omega L_2}; \quad \underline{I}_3 = \frac{\underline{E}_3 + \underline{V}_3}{R_3}$$

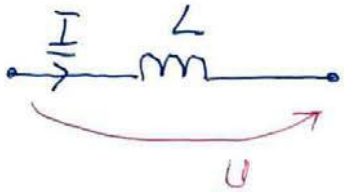
$$\underline{I}_4 = \frac{\underline{V}_1 - \underline{V}_2}{j\omega L_4}; \qquad \underline{I}_5 = \frac{\underline{V}_1 - \underline{V}_2}{R_5}$$

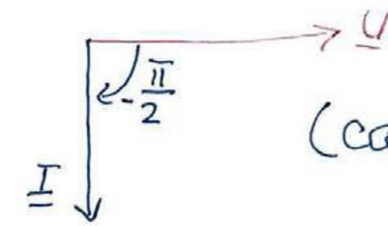
Exemple de reprezentare a circuitelor prin diagrame fazoriale:



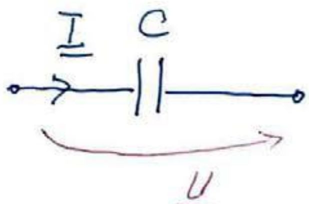


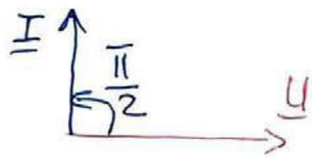
$\varphi = 0$ (rezonanță)





$\varphi > 0$ (caracter inductiv)





$\varphi < 0$ (caracter capacitiv)