Study of the RLC serial circuit and the voltage resonance

1. Theoretical considerations

In the figure 1 is illustrated a serial circuit composed by: an ideal resistor with the resistance R, an ideal coil with the self inductance L and an ideal capacitor with the capacitance C. This circuit is supplied with sinusoidal voltage:

$$u=\sqrt{2}\cdot U\cdot\sin(\omega t+\gamma) \qquad (1)$$

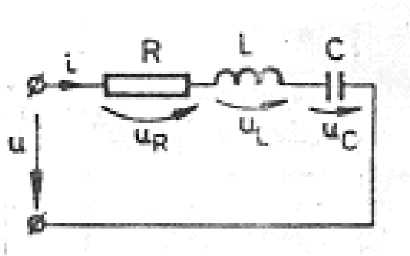

Figure 1

Applying the Ohm's law for the entire circuit we obtain the following equation:

$$u=u_R+u_L+u_C \qquad (2)$$

Where $$u_R=R\cdot i;\ u_L=L\frac{di}{dt};\ u_C=\frac{1}{C}\int i\cdot dt \qquad (3)$$

Therefore, equation (2) becomes: $R\cdot i+L\cdot\frac{di}{dt}+\frac{1}{C}\int i\cdot dt=\sqrt{2}U\sin(\omega t+\gamma)$ (4)

We rewrite the equation (4) in complex:

$$\underline{U}=R\cdot\underline{I}+j\omega L\cdot\underline{I}+\frac{1}{j\omega C}\cdot\underline{I}=\left[R+j(\omega L-\frac{1}{\omega C})\right]\cdot\underline{I}=(R+jX)\cdot\underline{I} \qquad (5)$$

Where X= $\omega L-\frac{1}{\omega C}$ is the equivalent reactance of the circuit.

The resonance conditions for a RLC serial circuit are:

- the current i must be in phase with the voltage u (φ=0),
- the reactive power absorbed by the circuit is zero (Q=UI$\cdot\sin\varphi$=0),
- the equivalent reactance of the circuit, X, is zero:

$$X=\omega L-\frac{1}{\omega C}=0 \qquad (6)$$

From this equation, one can observe that the resonance condition can be achieved by varying the frequency ($f=\omega/2\pi$) or the parameters L and C (the inductance L and the capacitance C).

- The resonance in a serial circuit is named the voltage resonance because at the resonance, the sum of the voltages through the coil and capacitor is zero:

$$\underline{U}_L+\underline{U}_C=j(\omega L-\frac{1}{\omega C})\cdot\underline{I}=0 \qquad (7)$$

and their effective values can be greater than the effective value U of the voltage u.

- the current I has the maximum value at the resonance:

$$I_0 = \frac{U}{R} \tag{8}$$

- the impedance of the circuit is minimum:

$$Z = \sqrt{R^2 + X^2} \tag{9}$$

- the active power, P, is maximum.

We consider that the coil is a real coil, with the inductance L and the resistance R_L is 3,80Ω.

2. The Arrangement

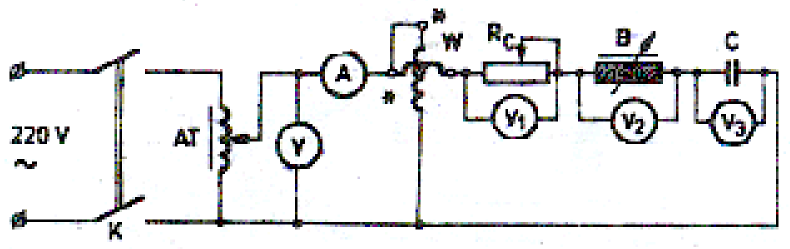

Figure 2

K - Bipolar switch
AT – Autotransformer
V, V_1, V_2, V_3 – AC current voltmeters
A – AC current ammeter
W – Wattmeter
R_C – Rheostat with cursor
B – Coil with variable inductivity
C – Capacitor

Complete table 1 with measured dates:

I [A]							
U_{RC} [V]							
U_B [V]							
U_C [V]							
P [div]							
P [W]							

3. The experimental results

$R_C = \frac{U_{R_C}}{I}$ - Resistance of the rheostat

$R = R_C + R_L$ - Equivalent resistance of the circuit

Total power in the circuit: $P = R \cdot I^2$

$R_L = \frac{P}{I^2} - R_C$ - Resistance of the coil

$Z_L = \frac{U_B}{I}$ - Impedance of the coil

$X_L = \sqrt{Z_L^2 - R_L^2}$ - Inductive reactance

$L = \frac{X_L}{\omega}$ - Inductance

$U_L = \sqrt{{U_B}^2 - (R_L \cdot I)^2}$ - Inductive voltage drop

The reactance and the capacitance of the capacitor are:

$$X_C = \frac{1}{\omega C} = \frac{U_C}{I}; \quad C = \frac{10^6}{\omega X_C}$$

The power factor of the circuit is: $\cos\varphi = \frac{P}{UI} = \frac{R_C + R_L}{\sqrt{(R_C + R_L)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

1. Complete table 2 with the experimental results:

R_C [Ω]							
R_L [Ω]							
R [Ω]							
Z_L [Ω]							
X_L [Ω]							
L [H]							
U_L [V]							
cosφ							
X_C [Ω]							
C [μF]							

2. The phase diagrams:

- before the resonance (figure 3(a))
- at the resonance (figure 3(b))
- after the resonance (figure 3(c))

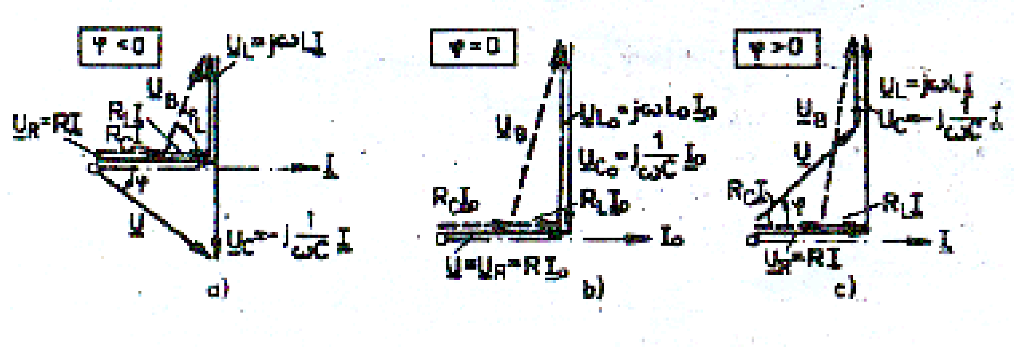

Figure 3

3. Represents the graphs for: I, P, cosφ=f (X_L)

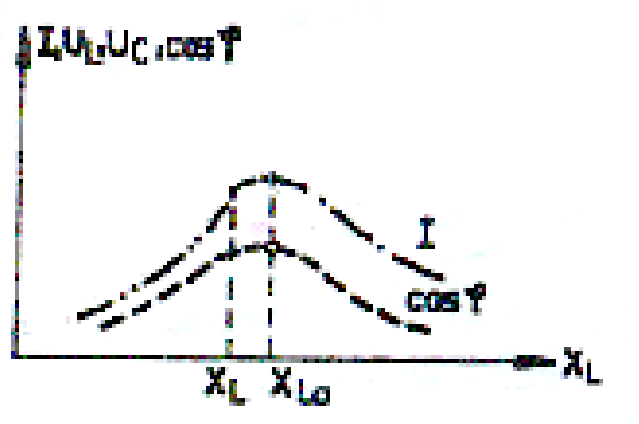

Figure 4