LABORATORY 1

SPECTRUM AND EQUIPOTENTIAL SURFACES DETERMINATION FOR THE ELECTRICAL FIELD WITH AN ELECTRO KINETIC MODEL

1. Theoretical facts:

In electrostatic regime two cylindrical electrodes, concentrically set, of r_1 and r_2 radius charge with uniform charge distribution $\rho_l = \frac{q}{l}$ on their entire length, creates an electrical field having the expression:

$$\overline{E} = \frac{\rho_l}{2\pi\varepsilon}\frac{\overline{r}}{r^2} \tag{1}$$

being a plane-parallel field, with radial distribution and cylindrical symmetry (the equipotent surfaces are concentrically cylinders with this two electrodes).

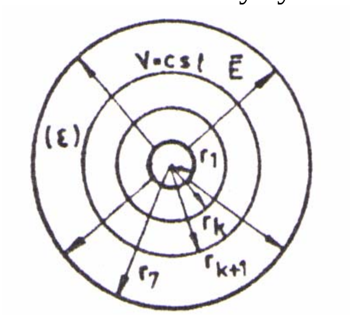

Figure 1

If the midmost electrode's potential is V_1, then the potential of one point in the field at the radius r_k will be:

$$V_k = V_1 - \int_{r_1}^{r_k} \overline{E}d\overline{r} = V_1 - \frac{\rho_l}{2\pi\varepsilon}\ln\frac{r_k}{r_1} \tag{2}$$

If the distance (r_7-r_1) is divided in six parts drawing concentrically circles, then the voltage between two adjacent circles is:

$$U_{k,k+1} = \frac{\rho_l}{2\pi\varepsilon}\ln\frac{r_{k+1}}{r_k} = \frac{U_{17}}{6} = \frac{1}{6}\frac{\rho_l}{2\pi\varepsilon}\ln\frac{r_7}{r_1} \tag{3}$$

respective:

$$r_{k+1} = r_k\left(\frac{r_7}{r_1}\right)^{\frac{1}{6}}, k \in [1,6] \tag{4}$$

The radius of the five equipotent surface drown between the two electrodes can be calculated using the relation (4).

If the central electrode of radius r_1 is placed with the d eccentrically inside of the cylinder of radius $r_{7,}$ then the electric field that appears between the two electrodes (Fig.2) can be determinate by replacing the two cylinders with their electrical axes placed in A and B points who's position is determined with the electrical imagines method is given by:

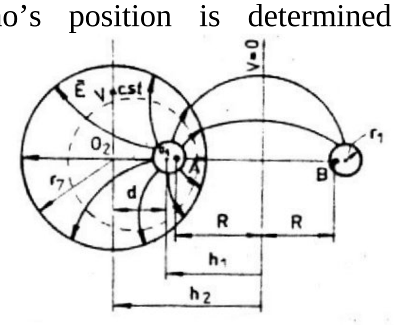

Figure 2

$$h_1 = \frac{r_7^2 - r_1^2 - d^2}{2d}$$

$$h_2 = h_1 + d \tag{5}$$

$$R = \sqrt{h_1^2 - r_1^2}$$

The equipotent surfaces are a family of circles orthogonally with the field lines $\overline{E}$.

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If the two electrodes placed concentrically (or eccentrically) are not in electrostatic regime but in electro kinetic regime (activated with a alternating-current (a.c.) voltage at a low fervency -50Hz- or direct-current (d.c.)voltage) the form of $\overline{E}$ lines, respectively of the current lines $\overline{J} = \sigma\overline{E}$ and the form of the equipotent lines (V=cst) are the same like in the electrostatic regime with the condition that the electrodes resistivity is lower that the resistivity of the separation electrode (when they are passed by a current their surfaces remain equipotent like in case of the electrostatic fields).

The capacity formed by the concentrically electrodes on the h length are:

$$C = \frac{2\pi\varepsilon h}{\ln\frac{r_7}{r_1}} \tag{6}$$

By replacing the ε dielectric between the electrodes with the resistivity ρ, the electrical resistance between the two electrodes will be:

$$R = \left(\frac{1}{C}\right)_{\varepsilon=\frac{1}{\rho}} = \frac{\rho}{2\pi h}\ln\frac{r_7}{r_1} \tag{7}$$

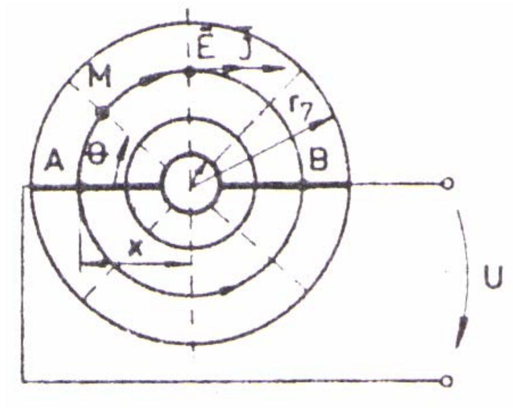

Figure 3.

The second electro kinetic model has surfaces of r_1 and r_7 isolated radius, and on the radial direction are displaced two electrodes in shape of two rectangular plates of (r_7-r_1) length and h height like in Fig.3, activated with U voltage. On a ⊥ plan on axis, the electro kinetic's field lines $\overline{E}$ and the current lines $\overline{J} = \frac{1}{\rho}\overline{E}$ would be concentrically circles whit the circle of r_1 radius, and the equipotent lines would be radials.

The M point's potential from the tank, in the field line of x radius would be:

$$U_{AM} = \int_A^M \overline{E}d\overline{l} = \rho\int_A^M \overline{J}d\overline{l} = \rho\int_A^M Jxd\theta = \rho Jx\theta \tag{8}$$

admitting that $\overline{E}$ and $\overline{J}$ have the constant module by the length of the field line of x radius.

But: $$U = \int_A^B \overline{E}d\overline{l} = \int_A^B \rho Jxd\theta \;\Rightarrow\; J = \frac{U}{\pi\rho x} \;\Rightarrow\; U_{AM} = \frac{\theta}{\pi}U \tag{9}$$

the voltage between an electrode and a point in the tank is proportional with θ, the equipotent lines being radial.

The conductor environment resistance between the two electrodes is:

$$R = \frac{1}{G};\; G = 2\int_{Aelectrod}\frac{ds}{\rho l} = 2\int_{r_1}^{r_7}\frac{1}{\rho}\frac{hdr}{\pi r} = \frac{2h}{\pi\rho}\ln\frac{r_7}{r_1} \tag{10}$$

being formed from the two half cylinders parallel resistances.

If the interior cylinder (by r_1 radius) is placed with d eccentricity inside the tank Fig. 4, the field lines $\overline{E}$ and the current lines $\overline{J}$ that are establish between the electrodes activated with

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U would be the Apollonius circles and the equipotent lines (V=cst) orthogonal with the field lines.

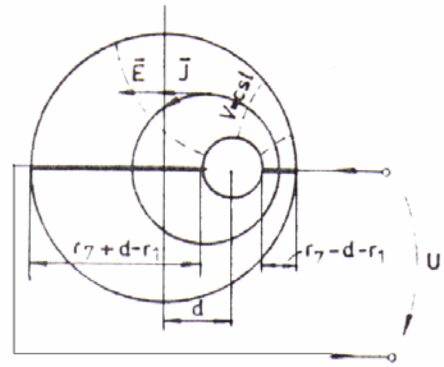

Figure 4.

So, the field lines of the electro kinetic model show in Fig.1 and 2 are the same with the equipotent lines of the Fig.3 and 4 and reciprocally, the equipotent lines from the first models, coincidence with the field lines from the last models.

At each model we can emphases only the equipotent lines, the field lines will be emphases like the equipotent lines of the dual model.

2. Work objectives

- For the concentric (and eccentric) position of the two electrodes show in Fig.1 and 2 we will trace five equipotent lines which will divide the voltage U between the electrodes in six equal parts. The surfaces (curves) positions experimental traced will be compared with those deducted from the relations (4).

- For the dual model from Fig.3 and 4 we'll trace the equipotent curves and we'll verify their coincidence with the field lines $\overline{E}$ from the first models (the orthogonally of the two curves families).

- The position of the electrical axes will be determined from the Fig.2 and 4 models with relations (5) and will be experimentally verified if the A and B points are convergence points for the drawn equipotent lines.

- The resistance between the electrodes will be experimental measured and will be compared with those calculated with relations (7) and (10) admitting $\rho \approx 10^4\,\Omega$m for water or the value of ρ from relation (7) will be used in (10).

3. The circuit and required equipment

The circuit arrangement will be executed from Fig.5-a, b each one in concentrically setting and eccentrically one.

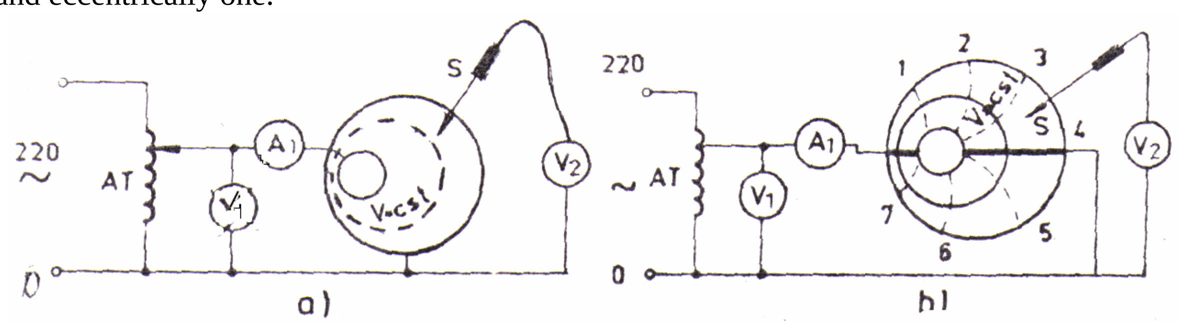

Figure 5.

The used equipment will be:

AT – autotransformer ATR-8; 0-250 V, 8 A

V_1 – voltmeter a.c. (0-250) V

V_2 – voltmeter DU- 20 (on 100 scales V)

S – check rod for searching the equipotent points

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A_1 – ammeter (1A)

4. Work directions

- The electrodes won't be activated with direct-current voltage to avoid the electrolysis phenomenon but with alternating-current voltage of 50 Hz fervency.
- The autotransformer sliding plug is set in a way that the exit voltage be 50V.
- For the model show in Fig.5-a, the equipotent lines will be drown from 10V in 10V, united all the points that have the same voltage (10, 20…) V related to the exterior electrode. These points are repeated with S check rod. Watching the V_2 voltmeter the equipotent lines of 10V, 20…V will be drawn. The points position will be placed immediately on a math piece of paper and in the same time will be drown the equipotent lines and then immediately return on the determination of a point that is not aligning to the curve.
- For the model from fig.5-b activated at U=40V, starting from the 7 points marked on the tank the equipotent lines will be drawn from 5V in 5V which for the concentrically cylinder will be radial lines from 45 in 45° or arcs eccentrically placed.

5. Experimental result and data processing

- The equipotent lines and the field lines of each model are drown and it will be checked if the field lines of a model are the same with the equipotent lines of the dual model and reciprocally.
- For the concentrically electrodes the position of the equipotent lines will be calculated with (4) and (8) and will be compared with the experimentally ones.
- For the eccentrically electrodes the position of the electrical axes will be determined with relation (5) and it will be checked if the extended (equipotent) field lines are concurrent in the electrical axe A.
- The electrical resistance between the electrodes is determined on the base of V_1 and A_1 indications and is compared with the calculated results using the (7) and (10) relations.

For eccentrically electrodes, in the dual model (Fig 5-b) the resistance between the places will be calculated with the relation:

$$R = \frac{\pi\rho}{2b}\frac{1}{\ln\frac{r_7 - d}{r_7 + d}\frac{r_7}{r_1}}$$

and compared with the experimentally measured one.

For both models will be used the constructive data:

r_1=20mm; r_2=195mm; h=97mm; d=30mm.

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