Course 5

The theory of electric circuits – Second part of the course

Cap. 1. Basic notions

The electric circuits can be [passive (no sources) / active

Lumped circuit conditions : $a << \delta = \sqrt{\dfrac{2}{\sigma \cdot \mu \cdot \omega}}$
(Circuite cu elemente concentrate)

a → largest transversal dimention of the circuit

δ → Penetration depth (adâncime de pătrundere)

σ → conductivity (conductivitate)

μ → magnetic permeability

ω → frequency

l → length of circuit

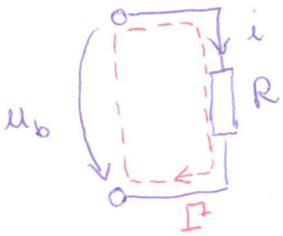
λ → wavelength

$l << \lambda = \dfrac{c}{f}$ (c → light speed, f → frequency)

$(f = 50\ Hz \Rightarrow \lambda = 6000\,km \Rightarrow l \leq 60\ km)$

1.1. Ideal circuit elements (Elmente de circuit ideale)

1) Ideal resistor



Rezistență ideală $\Rightarrow R \neq 0, L = 0, C = 0$

L – inductivity (inductivitate); C – capacity (capacitate)

Faraday law :

$e_\Gamma = -\dfrac{d\phi_{S_\Gamma}}{dt}$; $\phi_{S_\Gamma} = L \cdot i = 0 \Rightarrow e_\Gamma = 0$

Ohm's law : $e_\Gamma = u_f - u_b$

$u_f = R \cdot i$ (tensiunea de-a lungul firului conductor)

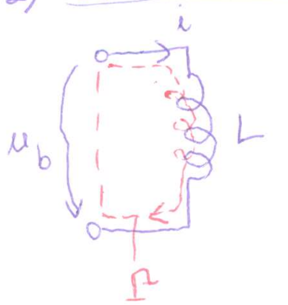
u_b – tensiunea la borne

$\Rightarrow \boxed{u_b = R \cdot i}$; $p = u_b \cdot i = R \cdot i^2 > 0$! (Joule law)

p – power (putere)

2) Ideal inductor

Simbol bobină (coil) : L or L



$L \neq 0, R = 0, C = 0$

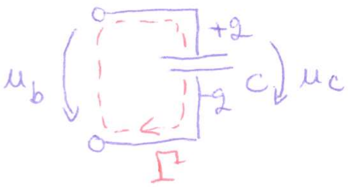
$e_\Gamma = -\dfrac{d\phi_{S_\Gamma}}{dt}$, $\phi_{S_\Gamma} = L \cdot i \Rightarrow e_\Gamma = -L \cdot \dfrac{di}{dt}$

$\left.\begin{array}{l} e_\Gamma = u_f - u_b \\ u_f = R \cdot i = 0 \end{array}\right\} \Rightarrow -L \cdot \dfrac{di}{dt} = -u_b \Rightarrow \boxed{u_b = L \cdot \dfrac{di}{dt}}$

$p = \dfrac{dW_{mg}}{dt} = \dfrac{d}{dt}\left(\dfrac{L \cdot i^2}{2}\right) \geq 0$ (W_{mg} → mg. energy)

W_{mg} – energie magnetică (crush)

3. Ideal capacitor



$$e_\Gamma = -\frac{d\phi_{S_\Gamma}}{dt};\quad \phi_{S_\Gamma} = L\cdot i = 0 \Rightarrow e_\Gamma = 0$$

$$e_\Gamma = u_f + u_c - u_b \Rightarrow u_c = u_b$$

$e_\Gamma = 0$; $u_f = R\cdot i = 0$

$C \neq 0,\ R = 0,\ L = 0$

$$C = \frac{q}{u_c} \Rightarrow u_c = \frac{q}{C}$$

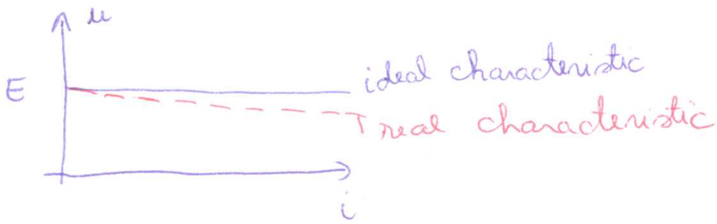
$$i = \frac{dq}{dt} \Rightarrow q(t) = \int i(t)\cdot dt$$

$$\Rightarrow \boxed{u_b = \frac{q}{C} = \frac{1}{C}\int i(t)\cdot dt}$$

$$\Rightarrow p = \frac{dW_{el}}{dt} = \frac{d}{dt}\left(\frac{C\cdot u^2}{2}\right) \geq 0$$

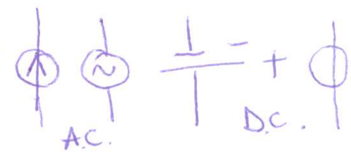
W_{el} – electric energy (curs 3)

4. Ideal voltage source (sursă de tensiune)



E – constant, indiferent de valoarea curentului

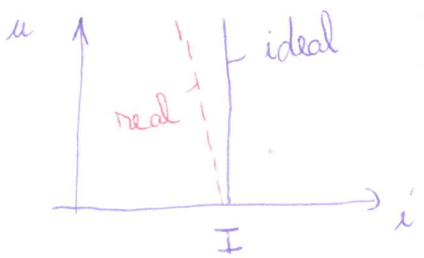
Simboluri:



Se cunoaște: valoarea tensiunii furnizate de sursă

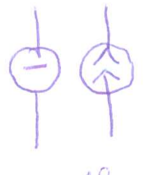
Necunoscut: curentul ce circulă prin sursă (depinde de configurația circuitului din care face parte sursa)

5. Ideal current source (sursă de curent)



I – constant, indiferent de valoarea tensiunii

Simboluri:



We know: the value of the current given by the source

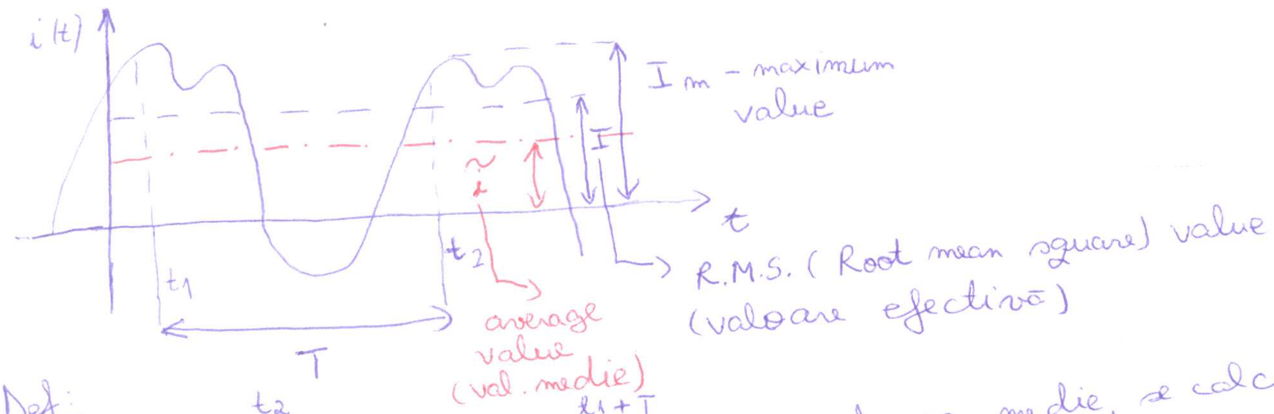
Unknown: the voltage drop on the current source (depends on the circuit's configuration)

Cap 2. Sinusoidal quantities. Definitions. Characteristic values.

Let $i(t)$ be a periodic function: $i(t) = i(t+T)$

T – perioada semnalului; $f = \frac{1}{T}$ – frequency (frecvență)

$\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow \omega T = 2\pi$ $\quad\omega$ – angular frequency (pulsație)



Def:

$$\begin{cases} \tilde{i} = \dfrac{1}{t_2 - t_1} \displaystyle\int_{t_1}^{t_2} i(t)\cdot dt = \dfrac{1}{T}\int_{t_1}^{t_1+T} i(t)\cdot dt \\ I = \sqrt{\dfrac{1}{T}\displaystyle\int_{t_1}^{t_1+T} i^2(t)\cdot dt} \end{cases}$$

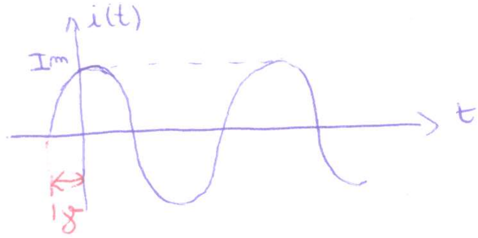
– valoarea medie se calculează pentru o perioadă (T) a semnalului

– RMS value (valoare efectivă) !!! VERY IMPORTANT

Now, let $i(t)$ be a sinusoidal function: $i(t) = I_m \cdot \sin(\omega t + \gamma)$

Def: $\omega t + \gamma$ = the phase of the quantity (current)

γ – epoch angle or initial phase (faza inițială a semnalului)



For a sinusoidal quantity:

$$\tilde{i} = \frac{1}{T}\int_{t_1}^{t_1+T} i(t)\cdot dt = \frac{1}{T}\int_{t_1}^{t_1+T} I_m \sin(\omega t + \gamma)\cdot dt =$$

$$= -\frac{I_m}{\omega T}\cdot \cos(\omega t + \gamma)\Big|_{t_1}^{t_1+T} = \frac{I_m}{\omega t}\Big[\underbrace{-\cos(\omega t_1 + \overset{2\pi}{\omega T} + \gamma)}_{\cos(\omega t_1 + \gamma)} +$$

$$+ \cos(\omega t_1 + \gamma)\Big] = 0$$

(valoarea medie a unei mărimi sinusoidale e nulă pe o perioadă)

$$I^2 = \frac{1}{T}\int_{t_1}^{t_1+T} I_m^2 \sin^2(\omega t + \gamma)\, dt = \frac{1}{T}\cdot I_m^2 \cdot \int_{t_1}^{t_1+T} \frac{[1 - \cos(2\omega t + 2\gamma)]}{2}\cdot dt =$$

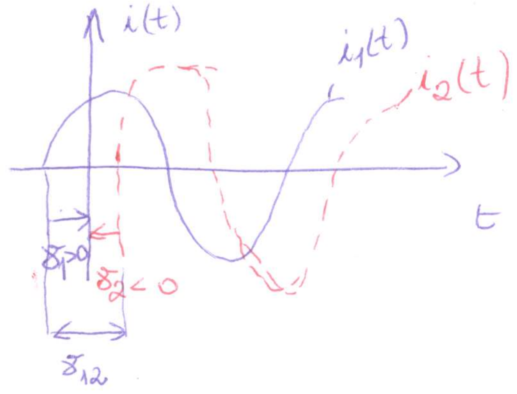
$$= \frac{I_m^2}{2T}\cdot\Big[\int_{t_1}^{t_1+T} dt + \underbrace{\int_{t_1}^{t_1+T} \cos(2\omega t + 2\gamma)\cdot dt}_{0}\Big] = \frac{I_m^2}{2}$$

val med pe o perioadă = 0

$$\Rightarrow I = \frac{I_m}{\sqrt{2}} \quad \text{or} \quad \boxed{I_m = \sqrt{2}\cdot I}$$

$$\Rightarrow \boxed{i(t) = \sqrt{2}\cdot I \cdot \sin(\omega t + \gamma)}$$

This form will be used from now on!



$\delta_{12} = \delta_1 - \delta_2$ phase shift (defazaj între i_1 și i_2)

i_1 – lead (defazat înainte)

i_2 – lags (defazat în urmă)

$i_1(t) = I_1 \sqrt{2} \cdot \sin(\omega t + \delta_1)$

$i_2(t) = I_2 \cdot \sqrt{2} \cdot \sin(\omega t + \delta_2)$

δ_{12} – the two currents are in phase (sunt în fază)

$\delta_{12} = \pm \pi$ – the two are in opposite phase (sunt în antifază)

$\delta_{12} = \frac{\pi}{2}$ – quadrature (cuadratură)

$\sin(\omega t + \delta_1) = 0$

$\Rightarrow \omega t + \delta_1 = 0$

$\Rightarrow \delta_1 = -\omega t$

$\Rightarrow$ for $t < 0 \Rightarrow \delta_1 > 0$!

2.1 Mathematical operations with sinusoidal quantities

a) Multiplication by a scalar (înmulțire cu un scalar)

$$a \cdot i(t) = a \cdot I \cdot \sqrt{2} \cdot \sin(\omega t + \delta)$$

b) Addition (adunare)

$$\left.\begin{aligned} i_1(t) &= I_1 \sqrt{2} \sin(\omega t + \delta_1) \\ i_2(t) &= I_2 \sqrt{2} \sin(\omega t + \delta_2) \end{aligned}\right\} \Rightarrow i(t) = i_1 + i_2 = I \cdot \sqrt{2} \sin(\omega t + \delta)$$

with
$$\begin{cases} I = \sqrt{I_1^2 + I_2^2 + 2 I_1 I_2 \cos(\delta_1 - \delta_2)} \\ tg\, \delta = \dfrac{I_1 \cdot \sin \delta_1 + I_2 \cdot \sin \delta_2}{I_1 \cdot \cos \delta_1 + I_2 \cdot \cos \delta_2} \end{cases}$$

c) Derivation (derivare)

$$\frac{di}{dt} = \frac{d}{dt}\left[I \sqrt{2} \sin(\omega t + \delta)\right] = \omega I \cdot \sqrt{2} \cdot \cos(\omega t + \delta) = \sqrt{2} \cdot \omega \cdot I \cdot \sin\left(\omega t + \delta + \frac{\pi}{2}\right)$$

↳ leads the initial value by $\frac{\pi}{2}$ (această mărime e defazată cu $\frac{\pi}{2}$ înaintea curentului inițial)

↳ the amplitude is ω times larger

d) Integration (integrare)

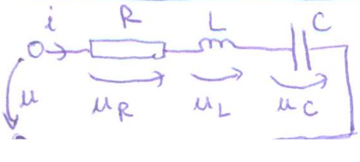
$$\int i(t) \cdot dt = \int I \sqrt{2} \sin(\omega t + \delta) \cdot dt = \frac{-I \cdot \sqrt{2}}{\omega} \cdot \cos(\omega t + \delta) = \frac{I \sqrt{2}}{\omega} \cdot \sin\left(\omega t + \delta - \frac{\pi}{2}\right)$$

$$\boxed{-\cos \alpha = \sin\left(\alpha - \frac{\pi}{2}\right)}$$

↳ the amplitude is ω times smaller

↳ lags behind the initial value by $\frac{\pi}{2}$ (defazat în urmă cu $\frac{\pi}{2}$)

RLC series circuit (circuit RLC serie)



$u(t) = u_R + u_L + u_C$ — Ohm's law

$$\begin{cases} u(t) = U \sqrt{2} \sin(\omega t + \delta_u) \\ i(t) = I \sqrt{2} \sin(\omega t + \delta_i) \end{cases}$$

$$u(t) = u_R + u_L + u_C = R \cdot i + L \cdot \frac{di}{dt} + \frac{1}{C}\int i(t) \cdot dt$$

<u>Cases</u>: 1) If $\gamma_u = 0$ phase horigin (origine de fază) $\begin{cases} u(t) = \sqrt{2}U \cdot \sin(\omega t) \\ i(t) = \sqrt{2} \cdot I \sin(\omega t - \varphi) \end{cases}$

$\gamma_{12} = \boxed{\varphi = \gamma_u - \gamma_i}$ (always $\gamma_u - \gamma_i$) $\Rightarrow$

$\Rightarrow \varphi = -\gamma_i$

2) If $\gamma_i = 0$ $\Rightarrow \begin{cases} u(t) = \sqrt{2}U \sin(\omega t + \varphi) \\ i(t) = \sqrt{2} \cdot I \cdot \sin \omega t \end{cases}$

$\gamma_{12} = \varphi = \gamma_u - \gamma_i = \gamma_u$

We select as reference quantity the one that appears more often in the function equation of the circuit.

For series connection, the current is the reference $\to$ the same current crosses every circuit element.

$$\Rightarrow R \cdot i + L \cdot \frac{di}{dt} + \frac{1}{C}\int i(t) \cdot dt = u(t)$$

$$\Rightarrow R \cdot \sqrt{2} \cdot I \cdot \sin(\omega t) + \omega L \cdot I\sqrt{2} \cdot \cos \omega t - \frac{\sqrt{2} \cdot I}{C \cdot \omega} \cdot \cos \omega t = \sqrt{2}U \sin \omega t \cdot \cos\varphi + \sqrt{2}U \cos\omega t \sin\varphi$$

We identify the coefficients of $\boxed{\sin \omega t}$ from the left and right terms of the eq.:

$R \cdot \sqrt{2} \cdot I = \sqrt{2} \cdot U \cdot \cos\varphi$ (1)

Identificăm coeficienții lui $\boxed{\cos\omega t}$ din termenul stâng și cel drept al ecuației:

$I\sqrt{2}\left(\omega L - \frac{1}{\omega C}\right) = \sqrt{2} \cdot U \cdot \sin\varphi$ (2)

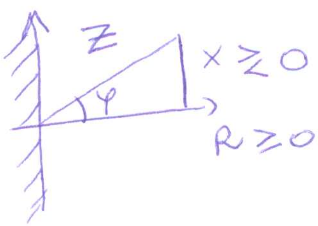
Din ecuațiile $(1)^2 + (2)^2 \Rightarrow$ $\boxed{I = \frac{U}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}$

From eq $\frac{(2)}{(1)} \Rightarrow$ $\boxed{tg\,\varphi = \frac{\omega L - \frac{1}{\omega C}}{R}}$

<u>Notations</u>: $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ IMPEDANCE (impedanță)

$X = \omega L - \frac{1}{\omega C}$ REACTANCE (reactanță)

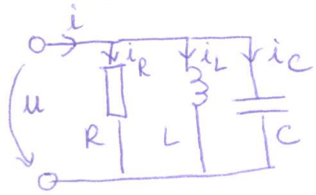
$\Rightarrow Z^2 = R^2 + X^2$



$Z \in$ cadran I sau IV

X poate fi pozitiv sau negativ, R pozitiv

RLC parallel circuit (circuit RLC paralel)



Origine de fază $\gamma_u = 0$ (Same voltage on all circuit elements)

$$\Rightarrow \begin{cases} u(t) = U\cdot\sqrt{2}\,\sin(\omega t) \\ i(t) = I\sqrt{2}\,\sin(\omega t - \varphi) \end{cases}$$

Kirchhoff current law (KCL) – Teorema I a lui Kirchhoff pt. curenți

$$\Rightarrow i = i_R + i_L + i_C \iff i = \frac{u}{R} + \frac{1}{L}\int u\cdot dt + C\cdot\frac{du}{dt}$$ $$\boxed{\frac{1}{R} = G \text{ - conductance}}$$

$$\Rightarrow I\sqrt{2}\sin\omega t\cdot\cos\varphi - I\sqrt{2}\sin\varphi\cdot\cos\omega t = U\sqrt{2}\cdot G\cdot\sin\omega t - \frac{1}{\omega L}\cdot U\sqrt{2}\cdot\cos\omega t + \omega C\cdot U\sqrt{2}\cdot\cos\omega t$$

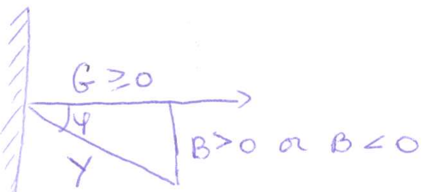
Prin identificare

$$\begin{cases} \cancel{\sqrt{2}} I\cdot\cos\varphi = \cancel{\sqrt{2}}\cdot U\cdot G & (3) \\ -\cancel{\sqrt{2}}\cdot I\cdot\sin\varphi = U\cdot\cancel{\sqrt{2}}\cdot\left(\omega C - \frac{1}{\omega L}\right) & (4) \end{cases} \Longrightarrow \begin{cases} (3)^2 + (4)^2 \Rightarrow I = U\sqrt{G^2 + \left(\frac{1}{\omega L} - \omega C\right)^2} \\ \frac{(4)}{(3)} \Rightarrow \operatorname{tg}\varphi = \dfrac{\frac{1}{\omega L} - \omega C}{G} \end{cases}$$

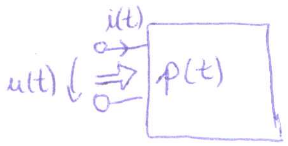
$Y = \sqrt{G^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}$ – ADMITANCE (admitanță)

$B = \frac{1}{\omega L} - \omega C$ – SUSCEPTANCE (susceptanță)

$$\Rightarrow \boxed{Y^2 = G^2 + B^2}$$



2.2. Power in sinusoidal regime



$u(t) = \sqrt{2}U\cdot\sin(\omega t + \varphi)$ $\quad \boxed{\gamma_i = 0} \Rightarrow \boxed{\gamma_u = \varphi}$

$i(t) = \sqrt{2}\cdot I\cdot\sin\omega t$

$p(t) = u(t)\cdot i(t)$ INSTANTANEOUS POWER (PUTERE INSTANTANEE)

$$p(t) = \sqrt{2}\cdot U\cdot\sin(\omega t + \varphi)\cdot\sqrt{2}\cdot I\cdot\sin\omega t = 2\cdot U I\cdot\sin\omega t\cdot\sin(\omega t + \varphi)$$

$$\Rightarrow p(t) = U\cdot I\cdot[\cos\varphi - \cos(2\omega t + \varphi)]$$ $\sin a\cdot\sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$

ACTIVE POWER (PUTERE ACTIVĂ):

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\cdot dt} \qquad P = \frac{1}{T}\left[\int_0^T U\cdot I\cdot\cos\varphi\, dt - \underbrace{\int_0^T \cos(2\omega t + \varphi)\cdot U\cdot I\cdot dt}_{0}\right]$$

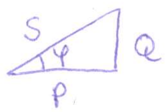
$\boxed{P = U\cdot I\cdot\cos\varphi}$ [W] – measurement unit
Watt – unitate de măsură

REACTIVE POWER (PUTERE REACTIVĂ): $\boxed{Q = U\cdot I\cdot\sin\varphi}$ [VAr]

VAr – Volt Ampère reactive

APARENT POWER : $\boxed{S = U \cdot I}$ [VA] Volt Ampère
(Putere aparentă)

$S^2 = P^2 + Q^2$

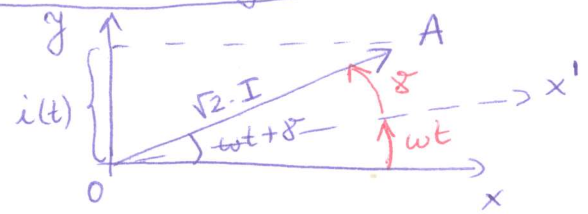


Power factor (factor de putere) $\boxed{k_p = \frac{P}{S} = \cos\varphi}$

2.3. Symbolic representation of sinusoidal quantities

$i(t) = I\sqrt{2}\sin(\omega t + \gamma)$

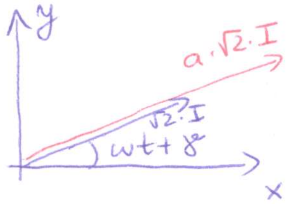
1) The vector form



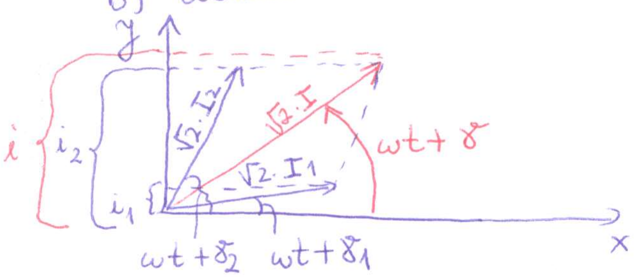
PHASOR (Fazor)

For every sinusoidal quantity we can define a Phasor of length = max value and angle between OA and Ox = phase.

a) multiplication

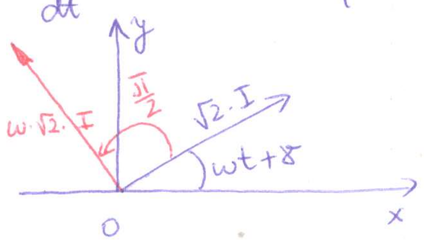


b) addition

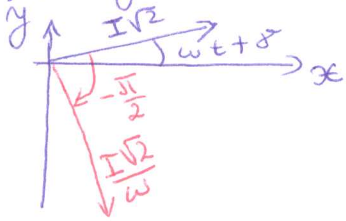


c) derivation

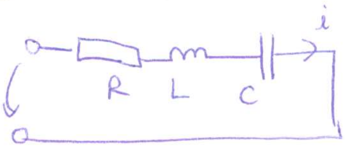
$\frac{di}{dt} = \omega \cdot I\sqrt{2}\cdot\sin\left(\omega t + \gamma + \frac{\pi}{2}\right)$



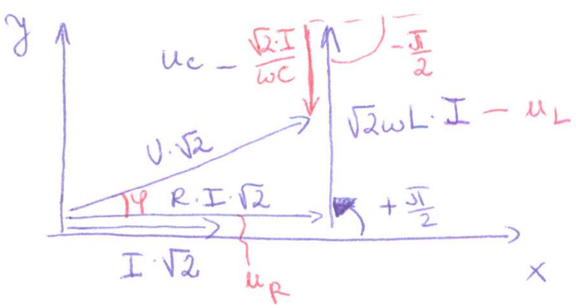
d) integration $\int i\,dt = \frac{\sqrt{2}\cdot I}{\omega}\sin\left(\omega t + \gamma - \frac{\pi}{2}\right)$



Example:



$u = \underbrace{R\cdot i}_{u_R} + \underbrace{L\cdot\frac{di}{dt}}_{u_L} + \underbrace{\frac{1}{C}\int i\,dt}_{u_C}$; $\begin{cases}\gamma_i = 0\\ \omega t + \gamma_i = 0\end{cases}$



$u = \sqrt{2}\cdot U\cdot\sin(\omega t + \gamma_u)$

$U^2 = (R\cdot I)^2 + \left(\omega L\cdot I - \frac{I}{\omega C}\right)^2$

$U^2 = I^2\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)$

2) The complex form

Se asociază un număr complex unei mărimi sinusoidale

$i(t) = I\cdot\sqrt{2}\sin(\omega t + \gamma) \Longleftrightarrow \underline{i(t)} = \sqrt{2}\cdot I\cdot e^{j(\omega t + \gamma)}$ (cu i se notează curentul)

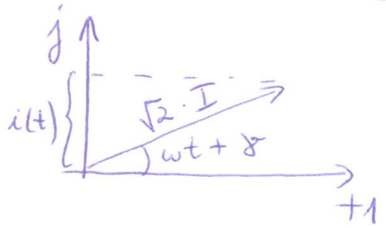
Mărime subliniată = mărime complexă ; $\boxed{j^2 = -1}$

$$\underline{i(t)} = \sqrt{2}\cdot I\cdot\cos(\omega t+\gamma) + j\cdot\underbrace{\sqrt{2}\cdot I\cdot\sin(\omega t+\gamma)}_{i(t)}$$

imaginary part

Euler: $\boxed{e^{j\cdot\alpha} = \cos\alpha + j\cdot\sin\alpha}$

$$\Rightarrow i(t) = \operatorname{Im}\{\underline{i(t)}\}$$



a) Multiplication by a scalar:

$$a\cdot i(t) \rightleftarrows a\cdot\underline{i(t)}$$

b) Addition

$$i_1(t) + i_2(t) \rightleftarrows \underline{i_1(t)} + \underline{i_2(t)}$$

c) Derivation: $\dfrac{di}{dt} = \omega\cdot I\cdot\sqrt{2}\sin\left(\omega t+\gamma+\dfrac{\pi}{2}\right) \rightleftarrows \omega\cdot\sqrt{2}\cdot I\cdot e^{j\left(\omega t+\gamma+\frac{\pi}{2}\right)} =$

$\boxed{e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j}$ $\quad = \omega\cdot\underbrace{\sqrt{2}\cdot I\cdot e^{j(\omega t+\gamma)}}_{\underline{i(t)}}\cdot\underbrace{e^{j\cdot\frac{\pi}{2}}}_{j} = j\cdot\omega\cdot\underline{i(t)}$

d) Integration: $\displaystyle\int i(t)\cdot dt = \frac{\sqrt{2}\cdot I}{\omega}\cdot\sin\left(\omega t+\gamma-\frac{\pi}{2}\right) \rightleftarrows \frac{\sqrt{2}\cdot I}{\omega}\cdot e^{j\left(\omega t+\gamma-\frac{\pi}{2}\right)} =$

$\boxed{-j = \frac{1}{j}}$ $\quad = \dfrac{1}{\omega}\cdot\underbrace{I\cdot\sqrt{2}\cdot e^{j(\omega t+\gamma)}}_{\underline{i(t)}}\cdot\underbrace{e^{j\left(-\frac{\pi}{2}\right)}}_{-j} = \dfrac{1}{j\cdot\omega}\cdot\underline{i(t)}$

Obs: $\sqrt{2}$ and ωt is present for all complex numbers. We will consider the simplified complex form:

$$\underline{i(t)} = I\sqrt{2}\cdot e^{j(\omega t+\gamma)} = \sqrt{2}\cdot e^{j\omega t}\cdot\underbrace{I\cdot e^{j\gamma}}_{\text{simplified complex form}} \rightarrow \boxed{\underline{I} = I\cdot e^{j\cdot\gamma}}$$

forma complexă simplificată!

Examples:

1) $i(t) = 5\cdot\sin\left(\omega t+\dfrac{\pi}{6}\right) \rightarrow \underline{I} = \dfrac{5}{\sqrt{2}}\cdot e^{j\frac{\pi}{6}} = \dfrac{5}{\sqrt{2}}\left(\cos\dfrac{\pi}{6} + j\sin\dfrac{\pi}{6}\right) = \dfrac{5}{\sqrt{2}}\cdot\left(\dfrac{\sqrt{3}}{2} + j\cdot\dfrac{1}{2}\right)$

2) $i(t) = 7\cdot\sqrt{2}\cdot\cos\omega t = 7\cdot\sqrt{2}\cdot\sin\left(\omega t+\dfrac{\pi}{2}\right) \rightarrow \underline{I} = 7\cdot e^{j\frac{\pi}{2}} = 7j$

3) $u(t) = 3\cdot\sqrt{2}\cdot\sin(\omega t+\pi) \rightarrow \underline{U} = 3\cdot e^{j\pi} = -3$