

Introduction to „frequency response” of LTI systems

1 Aims of the laboratory work

The main interest is on refreshing the knowledges regarding the Linear, Time – Invariant systems, especially on their time response to standard inputs (unity step signal, ramp signal, sinewave, etc.). Complementary, the superposition principle must be reviewed.

The Matlab companion as a “computer assisted” tool in making the simulations stage is also covered by this work.

Classical notions about “reading” oscilloscope captures are also described

2 Theoretical considerations

Processes like analog circuit in figure 1 can be mathematically treated as “systems” described by a model. The two classical approaches in mathematical modelling consists in deriving the “transfer function” (figure 2) or “the state space” model.

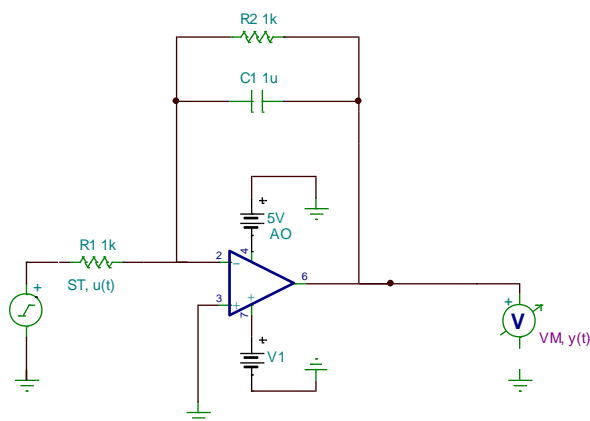


Figure 1 Analog circuit based on inverting OpAmp

AO –
operational
amplifier

R1- resistor
(1kΩ)

R2- resistor
(1kΩ)

C1- capacitor
(1μF)

ST- voltage
source

VM-voltmeter

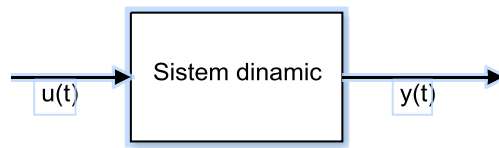


Figure 2 Transfer function model – $H(s)$

The transfer function can be derived based on the proprieties of operational amplifier in inverting connection. The immediate result of the impedances ratio can be applied:

$$H(s) = -\frac{R_2 || C_1}{R_1} = -\frac{\frac{R_2 \cdot \frac{1}{sC_1}}{R_2 + \frac{1}{sC_1}}}{R_1} = -\frac{R_2}{R_1} \frac{1}{R_2 C_1 s + 1} = \frac{k}{Ts + 1} \quad (1)$$

In relation (1), it should be recognized the transfer function of the first order element with its two parameters: k - the proportional gain and T - the time constant.

The steady state response to a sinewave $u(t) = \sin(\omega t)$ changes in amplitude and phase, i.e. $y_{ss}(t) = A_{ss} \sin(\omega t + \varphi_{ss})$. The second important result is that the bandwidth of a FOE is directly connected to its parameters in the next relation:

$$BW \in \left(0, \frac{k}{T}\right) \quad (2)$$

FOE behaves like a low pass filter, having the cutting frequency $\omega_c = \frac{k}{T} \left[\frac{rad}{sec} \right]$ or $f_t = \frac{k}{T} \frac{1}{2\pi} [Hz]$.

2.1 Matlab simulations

If the interest is in analyzing the behavior of the circuit for a sinewave of 1kHz, the next Matlab commands can be used:

```
R1=1e3;R2=R1;C1=1e-6;% the circuit parameters
f=1e3;% the frequency in Hz for the input sinewave
T=1/f;% the period of the input sinewave in sec.
w=2*pi*f;% the frequency in [rad/sec];
t=0:T/50:12*T;% simulation time basis STRONGLY! connected to T
H=tf(-R2/R1,[R2*C1 1]);% the transfer function
u=sin(w*t);% the input sinewave, or generally the input signal
lsim(H,u,t);% linear simulation of the response
%% in case of modifying the plot title and legend
y=lsim(H,u,t);% memorize the output signal in y variable
plot(t,u,'r',t,y,'b');
legend('Input signal','Output signal');
title('Sinewave response of FOE');
grid;shg
```

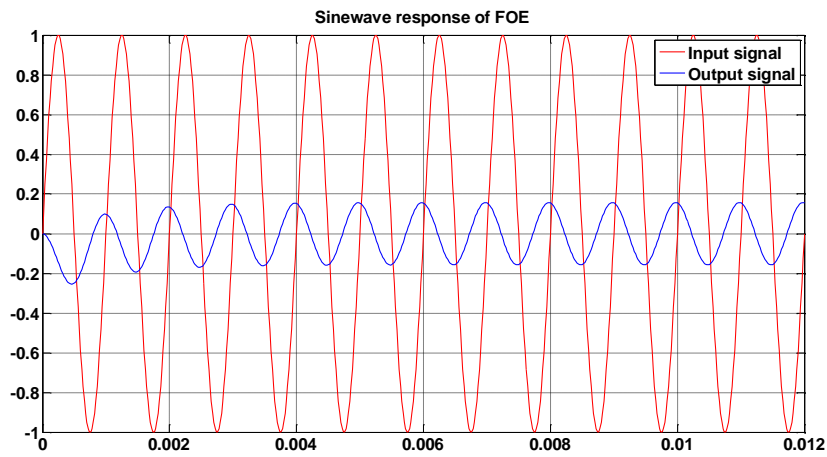


Figure 3 graphical result of the second section in the script

2.2 Practical measurements

In figure 4 and 5 there are two oscilloscope captures for the circuit in figure 1.

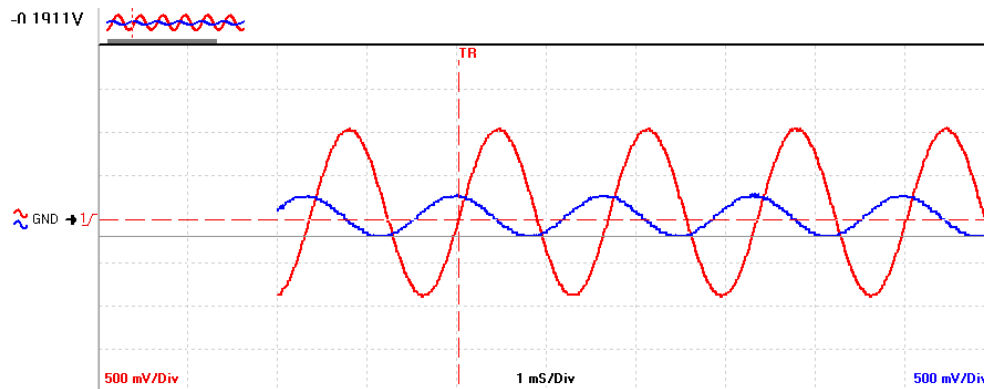


Figure 4 Oscilloscope capture with no measurements included

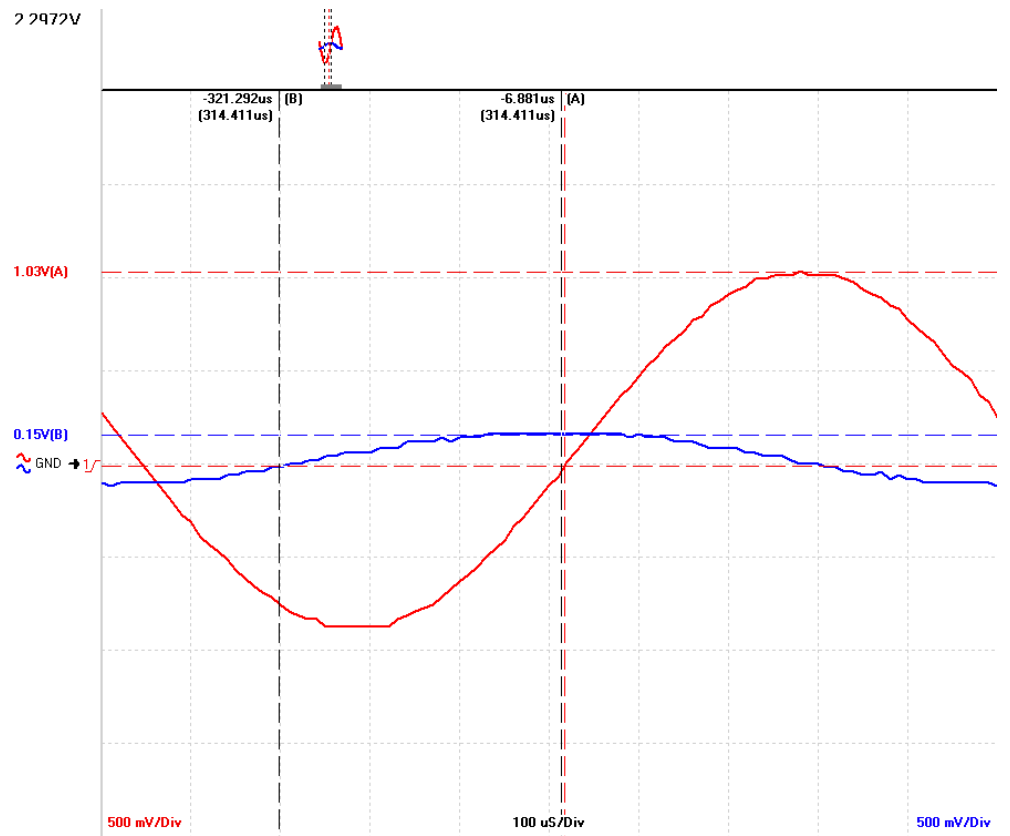


Figure 5 Oscilloscope capture with amplitude and phase shift included

References

- [1]. Bernstein, Dennis S. "Feedback control: an invisible thread in the history of technology." *Control Systems*, IEEE 22.2 (2002): 53-68.
- [2]. Black, Harold S. "Inventing the negative feedback amplifier." *IEEE spectrum* 14.12 (1977): 55-61.
- [3]. Callier, Frank M., and C. A. Desoer. "Linear System Theory (Springer Texts in Electrical Engineering)." (1991).
- [4]. Cartianu, Gh, M. Săvescu, D. Stanomir, I. Constantin. *Semnale, circuite și sisteme*. Editura Didactica și Pedagogică, București (1980).
- [5]. Chen, Chi-Tsong. *Linear system theory and design*. Oxford University Press, Inc., 1995.
- [6]. Colaneri, Patrizio, Jose C. Geromel, and Arturo Locatelli. *Control Theory and Design: An RH2 and RH Viewpoint*. Academic Press, 1997.
- [7]. Desoer, C. A. "In memoriam: Harold Stephen Black (1898-1983)." *Automatic Control*, IEEE Transactions on 29.8 (1984): 673-674.
- [8]. Dorf, Richard C. *Modern control systems*. Addison-Wesley Longman Publishing Co., Inc., 1995.

homework

- "a Matlab script that return the phase shift (in radians) between the input and the output of the circuit in this lab"

Glossar de Termen

- LTI system
- OpAmp circuit (inverting connection)
- Transfer function
- Response — transient response

- steady state response
- step response (unity step signal)
- frequency response (in sine wave response)
- frequency ($\frac{\text{rad}}{\text{s}}$) — ω
- Amplitude of the output
 - ↳ amplified signal
 - ↳ attenuated (out) signal
- Phase shift (negative/positive)
 - ↳ rising edge
 - ↳ falling edge
- Filters → LPF
 - ↳ bandwidth