Pattern recognition systems – Lab 5

Statistical Data Analysis

1. Objectives

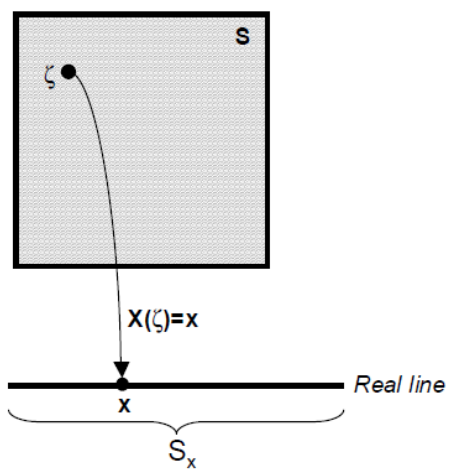
The purpose of this lab is to explore methods of analyzing statistical data used for classification and recognition. We will study the mean, standard deviation and covariance and the Gaussian probability density function. Our experiments will be done on a set of images containing faces. Using the covariance matrix we will study the correlations among different pixels.

2. Theoretical Background

2.1 Definitions

A *random variable* X is a function that assigns a real number X(ζ) to each outcome ζ in the sample space of a random experiment (see figure below). This function X(ζ) is performing a mapping from all the possible elements in the sample space onto the real line (real numbers). Random variables can be:

- Discrete: the resulting number after rolling a dice;
- Continuous: the weight of a sampled individual.



A *random variable vector* X is a function that assigns a vector of real numbers to each outcome X(ζ) in the sample space S. The notion of a random vector is an extension to that of a random variable:

$$X = [X_1 \, X_2 \ldots X_N]^T$$

2.2 Statistical Characterization of Random variables

A random variable can be partially characterized by:

1. Expectation: represents the center of mass of a density.

$$\mathbf{E[X]} = \boldsymbol{\mu} = \int_{-\infty}^{+\infty} \mathbf{x f_X(x) dx}$$

2. Variance: represents the spread about the mean

$$\mathbf{VAR[X]} = \mathbf{E[(X - E[X])^2]} = \int_{-\infty}^{+\infty} \mathbf{(x - \mu)^2 f_X(x) dx}$$

3. Standard deviation: The square root of the variance. It has the same units as the random variable

$$\mathbf{STD[X]} = \mathbf{VAR[X]^{1/2}}$$

2.3 Statistical Characterization of Random Vectors

We can (partially) describe a random vector with the following measures:

1. Mean vector:

$$E[X] = [E[X_1]E[X_2]\ldots E[X_N]]^T = [\mu_1 \mu_2 \ldots \mu_N] = \mu$$

2. Covariance matrix:

$$\begin{aligned} COV[X] &= \Sigma = E[(X-\mu)(X-\mu)^T] \\ &= \begin{bmatrix} E[(x_1-\mu_1)(x_1-\mu_1)] & \ldots & E[(x_1-\mu_1)(x_N-\mu_N)] \\ \ldots & \ldots & \\ E[(x_N-\mu_N)(x_1-\mu_1)] & \ldots & E[(x_N-\mu_N)(x_N-\mu_N)] \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \ldots & c_{1N} \\ \ldots & \ldots & \\ c_{1N} & \ldots & \sigma_N^2 \end{bmatrix} \end{aligned}$$

The covariance matrix indicates the tendency of each pair of features (dimensions in a random vector) to vary together, i.e., to co-vary. The covariance has several important properties:

- If $\mathbf{x_i}$ and $\mathbf{x_k}$ tend to increase together, then $\mathbf{c_{ik}}>0$
- If $\mathbf{x_i}$ tends to decrease when $\mathbf{x_k}$ increases, then $\mathbf{c_{ik}}<0$
- If $\mathbf{x_i}$ and $\mathbf{x_k}$ are **uncorrelated**, then $\mathbf{c_{ik}}=0$
- $|\mathbf{c_{ik}}| \leq \sigma_i \sigma_k$ where σ_i is the standard deviation of x_i
- $\mathbf{c_{ii}} = VAR(x_i)$
-

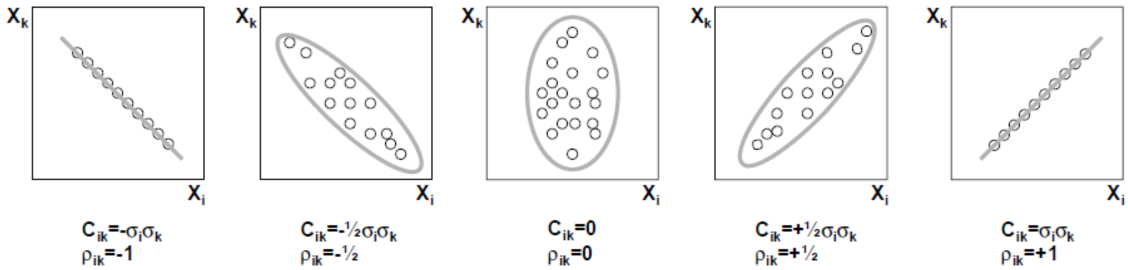
The covariance terms can be expressed as:

$$c_{ik} = E[(x_k - \mu_k)(x_i - \mu_i)]$$

$$c_{ii} = \sigma_i^2 \text{ and } c_{ik} = \rho_{ik}\sigma_i\sigma_k$$

where ρ_{ik} is called the **correlation coefficient.**

The next figures represent the correlation charts between two features, x_i and x_k.

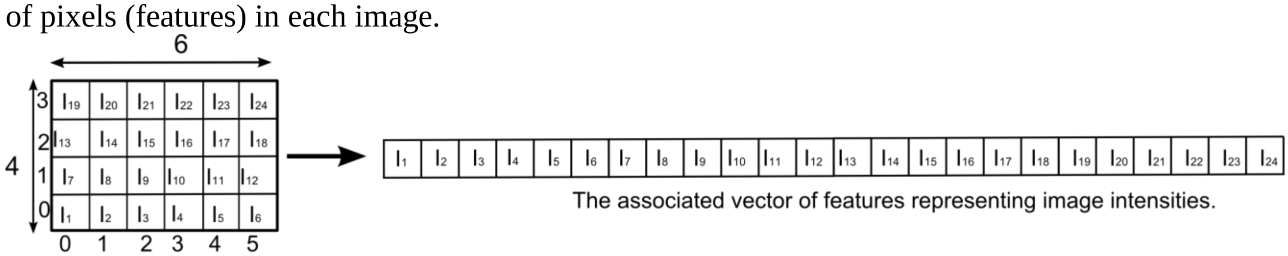


3. Practical Issues

In this lab session you are required to study the correlation between pixels belonging to human faces. You are given *p=400* images that contain human faces. The figure below shows a montage of all the input images:

The vector $X = (X_1, \ldots, X_p)$, where X_k represents an image.

Each image is characterized by a set of features representing pixel intensities, hence $X_k = (I_{1k}, \ldots, I_{Nk})$, where $N = image\_width \times image\_height$, i.e. N represents the total number of pixels (features) in each image.



The image

Each image in the set has the dimension of 19x19 pixels.

Your task will be to compute the covariance matrix of the given set of images and study how different features vary with respect to each other.

The mean value of a feature, I_i (position i in the image, along the whole set) is:

$$\mu_i = \frac{1}{p}\sum_{k=1}^{p} I_{ik}$$

Where I_{ik} represents the value of feature *i* in image *k*.
The standard deviation of a feature, I_i is:

$$\sigma_i = \sqrt{\frac{1}{p}\sum_{k=1}^{p}(I_{ik}-\mu_i)^2}$$

The elements of the covariance matrix, c_{ij} can be computed by:

$$c_{ij} = \frac{1}{p}\sum_{k=1}^{p}(I_{ik}-\mu_i)(I_{jk}-\mu_j)$$

The correlation coefficient is:

$$\rho_{ij} = \frac{c_{ij}}{\sigma_i \sigma_j}$$

4. Practical Work

1. Load the 400 images and store the intensity values as columns in the feature matrix *I*. The code that loads several images from a folder is:

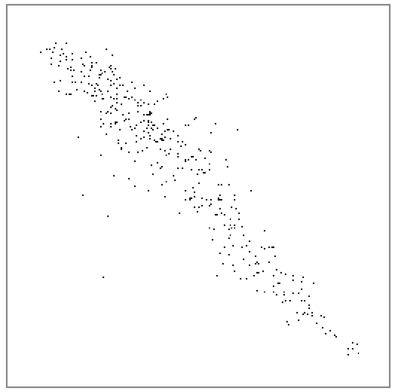
```
char folder[256] = "faces";
char fname[256];
for(int i=1; i<=400; i++){
      sprintf(fname,"%s/face%05d.bmp", folder, i);
      Mat img = imread(fname,0);
}
```

2. Compute mean values for each feature and save to a csv (comma separated values) text file. Csv files are viewable in Microsoft Excel as tables.

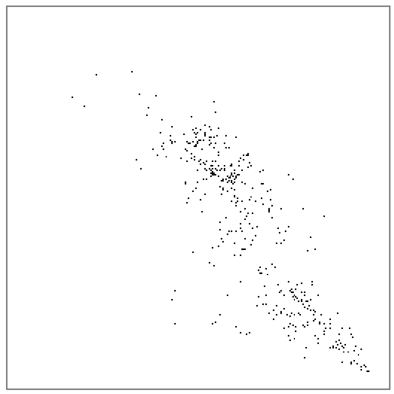
3. Compute the covariance matrix and save it to a csv text file.

4. Compute the correlation coefficients matrix and save it to a csv text file.

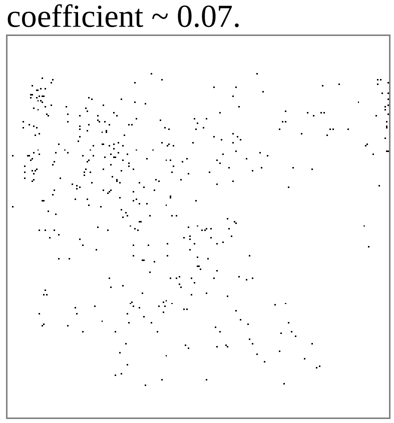
5. Compute the correlation coefficient and display the correlation chart between the intensity feature pairs. The correlation chart between features I_i and I_j is a 256x256 image on which we display the values of the two features for each of the p images, and we take I_i intensity on the horizontal axis and I_j intensity on the vertical axis. Use the following coordinate pairs (row, column):
    a. (5,4) and (5,14). These points correspond to pixels belonging to left eye and right eye. Your result should resemble the one in figure below having the correlation coefficient ~ 0.94. The positions need to be linearized (transformed to a single value) to find the correct row index from *I*.



b. (10,3) and (9, 15). These points correspond to pixels belonging to left cheek and right cheek. Your result should resemble the one in figure below having the correlation coefficient ~ 0.84.



c. (5,4) and (18,0). These points correspond to pixels belonging to left eye and the left bottom corner of the face images (notice these points are not highly correlated). Your result should resemble the one in figure below having the correlation coefficient ~ 0.07.



6. Plot the probability density function for a selected feature having the form of a one dimensional Gaussian probability density function:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

where μ is the mean and σ is the standard deviation for the selected feature. Normalize the density values so that the peak reaches the height of the image.

7. Optionally, plot the 2d probability density function as a grayscale image for two selected features using the 2d Gaussian probability density function:

$$p(x_i, x_j) = \frac{1}{2\pi\sqrt{\det(C_{ij})}} exp\left(-0.5\left(\left[x_i - \mu_i, x_j - \mu_j\right] C_{ij}^{-1} \begin{bmatrix} x_i - \mu_i \\ x_j - \mu_j \end{bmatrix}\right)\right)$$

where μ_i is the mean for feature i and C_{ij} is the covariance matrix between features i and j. Normalize the density values to fit inside the range 0:255.

5. References

R.Gutierez-Osuna – Introduction to Pattern Recognition, Wright State University http://www.engr.sjsu.edu/~knapp/HCIRODPR/PR_home.htm