Network-Coded Cooperation Algorithm for Multiple Source – Multiple Relay Topologies in Cellular Networks

Mihai P. Stef, Zsolt A. Polgar, Vasile Bota, Anghel Botos

Communications Department, Technical University of Cluj Napoca
Memorandumului, 28, 400114, Cluj-Napoca, Romania

Abstract— **This paper analyzes the performance provided on the uplink connection by a cooperative cluster, composed of user terminals and relays, which employ a XOR-based network coding algorithm. The paper presents the construction of the graphs that describe the operation of the algorithm within the selected topology. It then provides a theoretical analysis of the performance ensured by this algorithm in terms of the outage probability. The Block Error Rate (BLER) performance of the user terminals involved in cooperation are also evaluated for two cooperation topologies characterized by different cooperation graphs.**

Keywords: cooperative transmission, coded cooperation, network coding, outage probability

I. INTRODUCTION

Relaying and cooperation between terminals are considered as some of the most promising approaches for the performance improvement of wireless networks [1]. The distributed channel-coding cooperation included in schemes where a relay-node (RN) serves only one user terminal (UT) in its transmission to the base station (BS) is one of the techniques proposed in literature to accomplish those improvements [2]. Though this approach is shown to bring performance improvements for the served UT in terms of block error rate (BLER) and/or coverage, the additional resources required by the RN are used to serve only one UT leading to a loss of performance in terms of spectral efficiency. In order to decrease the effect of the additional resources upon the spectral efficiency of the UT-BS transmission, Network Coding (NC) techniques [3] were included in coded cooperation algorithms [4]. Since the NC techniques allow cooperation structures within which the RN serves more UTs, such an approach leads to a more efficient employment of the additional resources of the RN. But, making these techniques effective raises new questions that have to be addressed.

A simple XOR-based network coding was employed in cooperative wireless networks [5] in order to increase the diversity gain. For the multiple-source multiple-relay (MSMR) topologies, the graph representation of the cooperative MSMR topology was employed in [6] in connection with a class of codes on graph, e.g. Low Density Parity Check (LDPC) codes.

This paper describes a graph representation of the cooperative-cell of a cellular network, called cooperation graph. It also presents and discusses a cooperation strategy which employs a XOR-based network coding technique and a decoding algorithm which operates on the cooperation graph. Using an extension of the cooperation graph, the paper analyzes the performance provided by the proposed network decoding algorithm and presents the performance obtained for some particular cooperation topologies.

The paper is organized as follows: Section 2 introduces the representation of a cooperative cellular network as a bipartite graph. Section 3 discuses the network-coded cooperation strategy employed. Section 4 analyzes the network decoding algorithm, while Section 5 presents the BLER performance obtained by network coded cooperation schemes. Finally, Section 6 concludes the paper.

II. NETWORK ON GRAPH

This section describes a cooperative cell as a bipartite graph and defines the structures needed for a graph representation. This approach ensures a more convenient way to point out the processing involved and analyze the cooperative network performance.

Figure 1 (left) presents an example of a cooperative cell. The network elements of such a cell are: the BSs, the RN, the UTs and all their inter-connection links, i.e. the UT-RN links, RN-BS links and UT-BS links. The first two links are required by cooperation. The employment of the direct UT-BS links depends on the cooperation scheme employed. The number of UTs linked to a RN also depends on the cooperation strategy employed.

The cooperation of a group of cell's members, called cooperation cluster (CC), can be described by a graph representation, as shown in Figure 1 (left).

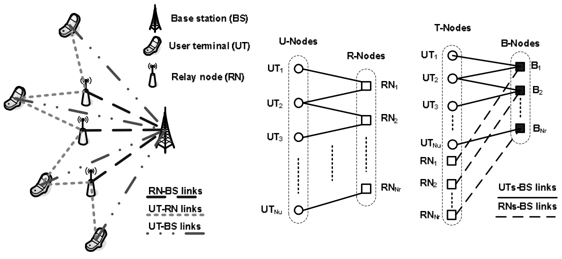

Figure 1. Cooperation cluster (left); Cooperation graph (center); Extended cooperation graph (right)

Let us consider a cooperation cluster with *Nu* user terminals and *Nr* relay nodes, whose cooperation graph is

---

This work was partially funded by the European Union within FP7-ICT-215477 CODIV Project

presented in Figure 1 (center). The cooperation graph shows all cooperating UT_i-RN_j pairs and also indicates the corresponding UT-RN links employed in the cooperation cluster. For the NC-based cooperation strategy employed in this paper the cooperation graph also indicates what data blocks should be NC-combined by each RN.

The cooperation graph is based on two sets of nodes: the U-nodes set, $U = \{UT_1, \ldots, UT_{Nu}\}$ and the R-nodes set, $R = \{RN_1, \ldots, RN_{Nr}\}$. A matrix representation $A[N_r \times N_u]$ of the bipartite graph is used to indicate the edges (employed links) between the U-nodes and R-nodes, where:

$$A[j,i] = \begin{cases} 1, \; if\ RN_j\ and\ UT_i\ are\ connected \\ 0, \; if\ RN_j\ and\ UT_i\ are\ \text{not}\ connected \end{cases} \qquad (1)$$

The cooperation graph shows "*who cooperates with who*", meaning that for each RN from the R-node set, we define a set of UT neighbors from the U-node set, which contains the UTs which will be processed by the j^{th} RN:

$$nR_j = \left\{ UT_i \mid A[j,i] = 1, i = \overline{1,Nu} \right\} \qquad (2)$$

Also, for each UT, we define the set of RN neighbors:

$$nU_i = \left\{ RN_j \mid A[j,i] = 1, j = \overline{1,Nr} \right\} \qquad (3)$$

Using the cooperation graph we can model the processing on the UT-RN links. To model the UT-BS and RN-BS processing we have to use the extended cooperation graph, (Figure 1 - right).

The T-node set of the extended cooperation graph, $T=\{T_1, \ldots, T_i, \ldots, T_{Nu+Nr}\}$, is obtained from the cooperation graph's U-node and R-node sets as follows: subset $TU=\{T_1, \ldots, T_{Nu}\}$ represents the U-nodes and subset $TR=\{T_{Nu+1}, \ldots, T_{Nu+Nr}\}$ represents the R-nodes.

The B-node set, $B = \{B_1, \ldots, B_{Nr}\}$, indicates the processing which has to be performed by the network decoder. Opposite to usual networks on graph representations [6], where a node represents a check equation, a B-node represents a system of equations involving the information blocks generated by the UTs (one block/UT/cooperation period) and the check blocks generated by the cluster's RN which serves those UTs. The extended cooperation graph represents both the UT-BS and RN-BS links. The graph matrix, of the extended cooperation graph becomes $A' = [A \mid I]$, where I is $[N_r \times N_r]$ identity matrix.

Likewise, for the cooperation graph, we define the neighbor sets as follows:

$$\begin{aligned} nB_j &= \left\{ T_i \mid A'[j,i] = 1, i = \overline{1, Nu+Nr} \right\} \\ nT_i &= \left\{ B_j \mid A'[j,i] = 1, j = \overline{1,Nr} \right\} \end{aligned} \qquad (4)$$

III. NETWORK-CODED COOPERATION STRATEGY

The cooperation strategy considered employs network coding on the RN_j-BS links. The motivation for using network coding is that one RN can serve more than one UTs while employing the same physical resources.

According to the employed decode-and-forward (DF) cooperation strategy, each UT FEC-encodes its N_i information bits, using a R_{UT} rate channel-code, generating an N_c bit-long coded block and each RN_j, processes the coded blocks received from all its neighbors $UT_i \in nR_j$. As shown in figure 2 (left), the RN decodes each received block and computes the network-encoded block by using bit-level XOR operations. The XOR-generated block is then FEC-encoded using the same channel-code with the same rate as the one employed by the UTs. This restriction is not mandatory, but it is imposed only to simplify the analysis of the cooperation scheme.

To obtain the global rate of the NC-coded transmission we must take into account the additional information inserted by the RNs., The global coding-rate can e computed for each UT_i by using the cooperation graph. It is expressed by:

$$R_g^{UT_i} = \frac{N_i}{N_c \cdot \left(1 + \sum\limits_{RN_j \in nU_i} \frac{1}{|nR_j|} \right)} = \frac{R_{UT}}{1 + \sum\limits_{RN_j \in nU_i} \frac{1}{|nR_j|}} \qquad (5)$$

At the BS (figure 2, right) all the received coded blocks, from the UTs on the UT-BS links and from the RNs over the RN-BS links, are channel-decoded and the decoded data-blocks are checked for errors (by FEC-syndrome or CRC). The network decoder takes as input the information provided by the error-detection operation (each block is correct or not) and all the decoded blocks. The extended cooperation graph is supposed to be known at the BS.

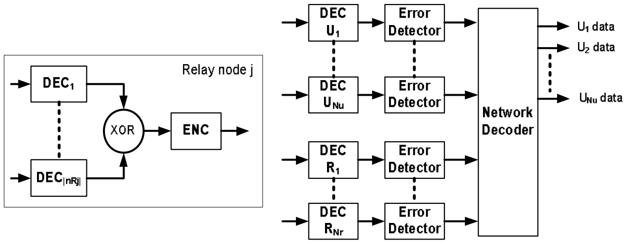

Figure 2. Network encoder (left) and network decoder (right)

The basic idea of the network decoding algorithm is to check if an erroneous block can be "recovered" using the blocks received from the UT's neighbors. This approach is applied to blocks received from all UTs in the cluster. If the recovery is possible, the *separate network decoding* algorithm is trying to solve each equation independently. Using the principles stated above, the network decoding process can be summarized by the following steps:

- the algorithm checks if the data block received, from each UT_i is correct. If an erroneous data block is found, it searches all the B-node neighbors of the T-node that generated the erroneous received data block.
- for each of the B-nodes found, the algorithm verifies if for all its T-node neighbors the received data blocks are correct, except the data block which the algorithm tries to recover.
- if a B-node neighbor, for which all the T-node neighbors' data blocks are correctly received, is found, then the erroneous block can be recovered using the bitwise XOR operator. The recovered data-block is stored and the algorithm is restarted.

- if the erroneous data block can not be recovered, the algorithm will search for the next erroneous block and will try to recover it.

IV. NETWORK DECODING ALGORITHM ANALYSIS

From the network decoder point of view, all links are modeled as block erasure channels, with the erasure probability equal to *BLER(link)*. Then, the network decoder's performances can be evaluated by knowing the *BLER* provided by the channel-code of each link. The *worst case* for the network decoder is when it can not recover any of the lost blocks; we call this situation the *outage state* of the network decoder. This "worst case" situation is analyzed below.

The *outage event* is the event for which the network decoder reaches the *outage state,* meaning it is not able to recover any of the lost blocks. The outage event appears when all erroneous blocks were transmitted by T nodes that form an *outage set*. Similar to the definition of a stopping set [7], the *outage set*, S is defined as a subset of the T-nodes set T, so that all the B-node neighbors of the T-nodes from S are connected to S at least twice.

According to the definition of the outage set, a subset S, from the T-node set T, is an outage set if the following condition is fulfilled:

$$\left| \left\{ B_j \in B \mid \sum_{T_i \in S} A'[j,i] = 1 \right\} \right| = 0 \tag{6}$$

The outage probability is defined as the probability to have *k* erroneous blocks which form an *outage set*. This can be written as follows:

$$P_{outage} = \sum_{k}^{N_u+N_r} P_{OS}^{k} \cdot P_{k-err} \tag{7}$$

where P_{OS}^{k} is the probability to have k-long outage sets according to the extended cooperation graph and P_{k-err} is the probability to have k erroneous blocks out of the (N_u+N_r) received blocks.

The probability to have a k-long outage set is not the same for all the k-long outage sets due to the different qualities of the links employed. To simplify the analysis this paper considers that the probabilities of all k-long outage sets are equal and can be computed by:

$$P_{OS}^{k} = \frac{N_{OS}^{k}}{C(|T|,k)} \tag{8}$$

C(|T|,k) denotes the number of all the k-subsets of the T-node set and N_{OS}^{k} represents the number of the k-long outage sets.

Taking into account the quality of all links, the probability to have k erroneous blocks out of the (N_u+N_r) received blocks is expressed by (9), where $T^{(k)}$ denotes the family of all k-length subsets of the T-node set.

$$P_{k-er} = \sum_{T' \in T^{(k)}} \left(\prod_{t_i \in T'} BLER_{t_i-BS} \cdot \prod_{t_i \in T \setminus T'} \left(1 - BLER_{t_i-BS}\right) \right) \tag{9}$$

V. PERFORMANCE OF THE NETWORK CODED COOPERATION ALGORITHM

The performance of the network coded cooperation algorithm was evaluated for two cooperative topologies that include four UTs and two RNs. Figure 3 presents the cooperation graphs of the two topologies.

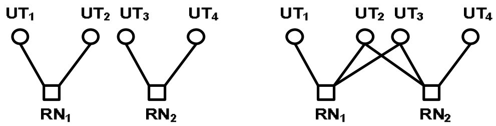

Figure 3. Cooperation graphs of the test topologies

It is assumed that all UTs have identical UT_i-BS direct channels, i.e. the same E_b/N_0, and the RNs have better RN_j-BS channels, i.e. an E_b/N_0 greater with 6dB than the E_b/N_0 of the direct channels. All direct channels are affected by additive Gaussian noise and Rayleigh block fading. The UT_i-RN_j channels are considered to be quasi error-free. The channel-code employed is a turbo code having the parameters shown in table 1.

Feedback generator polynomial	13_8
Feedforward generator polynomial	15_8
"Mother" code rate	0.50
UT coding rate	0.75
No. of iterations of the turbo decoders	8

Table 1. Channel-code parameters

Figure 4 compares the outage probabilities provided by computer simulations to the ones obtained by theoretical evaluation, using relations (7) – (9), for the two test topologies presented in figure 3.

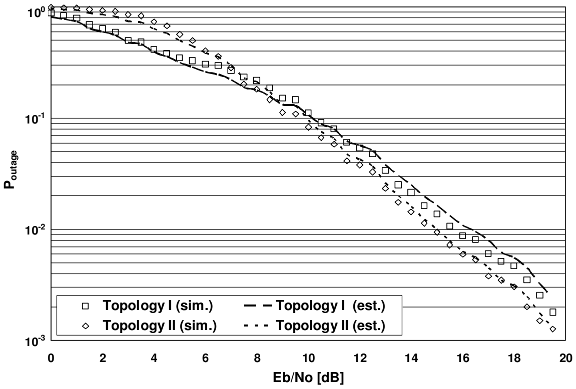

Figure 4. Outage probability of the two test topologies

The curves of figure 4 show that the theoretical estimated outage probabilities are very close to the ones provided by simulations. Small differences appear due to the fact that the RNs have better link-qualities, while the theoretical analysis considers identical links, and the outage set probability is not the same for all outage sets.

The smaller outage probability provided by the second test topology for medium and great E_b/N_0 values is explained by the smaller number of outage sets of length 2 for this topology. In the high E_b/N_0 domain, large outage sets have small influences upon the outage probability. In the low E_b/N_0 domain, the large outage sets have a greater impact upon the outage probability.

The performance provided by the cooperation cluster is also evaluated in terms of the global BLER, which is computed as the ratio between the number of all incorrect data blocks at the BS after network decoding and the number of all data blocks send by all UTs in the cluster.

Figure 5 shows the global BLER performance provided by the two cooperation topologies defined in figure 3.

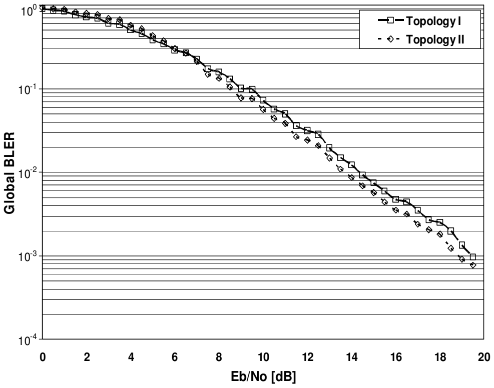

Figure 5. Global BLER performance for the test topologies

There should be noted that the second topology provides smaller BLER in about the same SNR domain where it provides a smaller outage probability; therefore, the outage probability could be employed as an indicator of the overall performance of different topologies. The overall performance also depends on the structure of the cooperation graph of that topology.

Figures 6 and 7 present the individual BLER performance of each UT involved in respective topology.

For the first test topology, cooperation provides the same performance improvement for each UT, see figure 6. This behavior is explained by the perfect symmetry of the cooperation graph, see figure 3 (right).

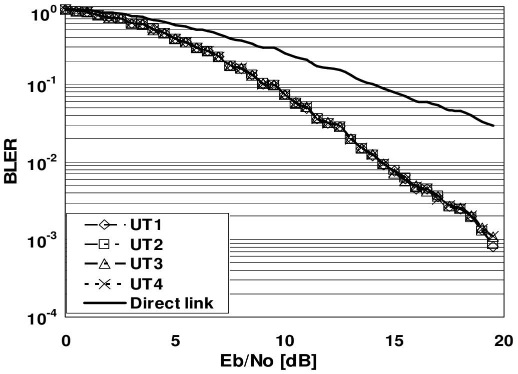

Figure 6. BLER performance for the first test topology

The differences between the BLER performance of the UTs ensured by the second test topology can be explained by the affiliation of each UT and each RN to different outage sets, due to the asymmetry of the cooperation graph. The influence of the topology's asymmetry upon the individual BLER of the UTs requires further investigations.

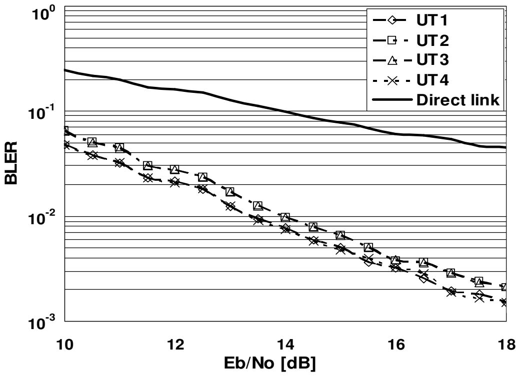

Figure 7. BLER performance for the second test topology

VI. CONCLUSIONS

This paper presents an analysis of a network coding-based cooperation technique for multiple-source multiple-relay cooperative clusters integrated in cellular networks and proposes a graph-based decoding method for the case when the network coding is performed by bitwise XOR operations. The performance analysis provided is based on the graph representation of the cooperative cluster, whose NC mechanism is described by a bipartite-graph, whose construction is also presented.

The paper defines the outage event in the network decoding process and computes the probability of such an event using the stopping set definition employed in the analysis of LDPC codes over binary erasure channels. Preliminary results show that the outage probability is a reliable indicator of the cluster's global performance, since it has the same trend as the global BLER performance of the cooperative topology employed.

The paper also shows that the overall and the individual BLER performance of a cooperation cluster depend on the structure of the cooperation graph.

REFERENCES

[1] J.N. Laneman, D.N.C. Tse, and G.W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," IEEE Trans. Inform. Theory, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[2] T. E. Hunter and A. Nosratinia, "Cooperative diversity through coding," Proc. IEEE ISIT, Laussane, Switzerland, July 2002.

[3] R. Ahlswede, N. Cai, S.-Y. R. Li and R. W. Yeung, "Network information flow", IEEE Transactions on Information Theory, 46(4):1204-1216, July 2000.

[4] C. Hausl, P. Dupraz, "Joint Network-Channel Coding for the Multiple-Access Relay Channel", IEEE SECON, Vol. 3, pp. 817-822, 2006

[5] Yingda Chen, Shalinee Kishore and Jing (Tiffany) Li, Wireless Diversity through Network Coding, IEEE Wireless Communications and Networking Conference, 2006.

[6] X. Bao and J. Li, "Matching code-on-graph with network-on-graph: Adaptive network coding for wireless relay networks," 43rd Annual Allerton Conference, 2005.

[7] C. Di, D. Proietti, I. E. Telatar, T. J. Richardson and R. Urbanke, "Finite length analysis of low-density parity-check codes on the binary erasure channel," *IEEE Trans. Inform. Theory.*, vol. 48, no. 6, pp. 1570-1579, June 2002