Network-Coded Cooperation Protocol for Multiple Source – Multiple Relay Topologies in CellularNetworks

Mihai P. STEF, Zsolt A. POLGAR, Vasile BOTA
Technical University of Cluj Napoca, Telecommunications Department
C. Daicoviciu, 15, Cluj Napoca, 400020, Romaina
Tel: +40264401906, Fax: +0264401906, Email: Mihai.Stef@com.utcluj.ro

Abstract: This paper proposes a network coded cooperation protocol for the uplink transmission of relay enhanced cellular networks, which is intended to multiple source-multiple relay cooperative clusters. Two variants of the network coded protocol are proposed, one employing XOR-based erasure codes, which operates on the cluster cooperation graph, and one employing Reed Solomon codes. The two protocol variants were analyzed in terms of outage behaviour of the network decoder. Their performances were also evaluated in terms of the block error rate ensured.

Keywords: cooperative transmission, coded cooperation, network coding, Reed Solomon codes, outage probability.

1. Introduction

Relaying and cooperation between terminals are considered as some of the most promising approaches for performance improvement in wireless networks [1]. The channel-coded cooperation included in schemes where a relay-node (RN) serves only one user terminal (UT) in its connection to the base station (BS) is one of the techniques proposed in literature to accomplish those improvements [2] [3] [4]. Though this approach is shown to bring performance improvements for the served UT in terms of bit error rate (BER), block error rate (BLER) and/or coverage, the additional resources required by the RN are used to serve only one UT leading to a loss of performance in terms of spectral efficiency. In order to decrease the effect of the RN additional resources upon the spectral efficiency of the UT-BS transmission, Network Coding (NC) techniques [5] were included in coded cooperation algorithms [6]. Since the NC techniques allow cooperative structures within which the RN serves more UTs, such an approach leads to a more efficient employment of the additional resources of the RN. But, making these techniques effective raises new questions that have to be addressed.

A simple XOR-based network coding was employed in cooperative wireless networks [7] in order to obtain better diversity gain. Graph representation of a cooperative network was employed in [8] to use a known class of codes, codes on graph like the Low Density Parity Check (LDPC) codes, to cooperate in multi-source multi-relay topologies.

The paper describes a graph representation of the cooperative cellular network called cooperation graph. It also presents and discusses two cooperation strategies which employ network coding techniques based on simple XOR-coding or on Reed Solomon (RS) coding. Using an extension of the cooperation graph, the paper analyzes the performance provided by the proposed NC algorithms for some particular cooperation cluster topologies.

The paper is organized as follows: Section 2 introduces a graph representation of a cooperative cellular network as a bipartite graph. Section 3 describes the general principles of network coded cooperation protocols in a multiple-source cooperation cluster. Section 4 and Section 5 describe particular examples of NC based cooperation protocols employing XOR-based erasure codes respectively RS codes. Section 6 presents a performance analysis of these NC cooperation protocols, while Section 7 concludes the paper.

2. Network on Graph

This section describes a cooperative cellular network by means of a bipartite graph and defines the structures needed for such a graph representation. This representation of the network elements is more convenient to point out the processing involved and to analyze the cooperative network's performances.

Figure 1 (a) presents an example of a cooperative cellular network. Graph representation can be used to describe the cooperation in a cell or in a group of cell members (UTs and RNs), called cooperation cluster (CC).

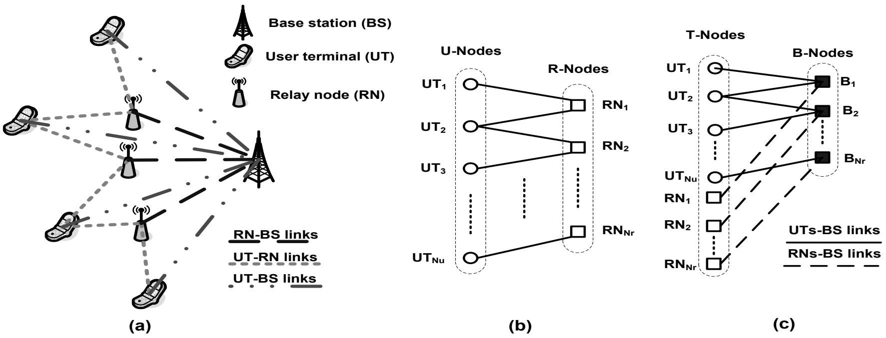


Figure 1. (a) Cooperative cellular network; (b) Cooperation graph; (c) Extended cooperation graph

Let us consider a cooperation cluster with *Nu* user terminals and *Nr* relay nodes. Figure 1 (b) presents the cooperation graph attached to this cluster. The cooperation graph shows the RNs with which an UT cooperates, also indicating the corresponding UT-RN links.

Two sets should be defined in order to build up a cooperation graph: the U-nodes set, $U = \{UT_1, ..., UT_{Nu}\}$ and the R-nodes set, $R = \{RN_1, ..., RN_{Nr}\}$. A matrix representation of the bipartite graph is used to indicate the edges between U-nodes and R-nodes contained, i.e. $A[N_r\ x\ N_u]$, where:

$$A[j,i]=\begin{cases}1, \ if\ RN_j\ and\ UT_i\ are\ connected\\ 0, \ if\ RN_j\ and\ UT_i\ are\ \mathrm{not}\ connected\end{cases} \tag{1}$$

The cooperation graph shows "*who cooperates with who*", meaning that for each RN from the R-node set, we define a set of UT neighbors from the U-nodes set, which indicates the UTs which will be processed by the j^{th} RN, nR_j. Also, for each UT, we define the set of RN neighbors, nU_i.

$$nR_j = \{UT_i \mid A[j,i]=1, i=1,2,...,Nu\}; \qquad nU_i = \{RN_j \mid A[j,i]=1, j=1,2,...,Nr\} \tag{2}$$

Using the cooperation graph we can model the processing on the UT-RN links. To model the UT-BS and RN-BS processing we need to use the extended cooperation graph, see Figure 1 (c). The T-nodes set, $T=\{T_1, ... , T_i, ... , T_{Nu+Nr}\}$, of the extended cooperation graph is obtained from U-nodes set and R-nodes set of the cooperation graph, as follows: subset $TU=\{T_1, ... ,T_{Nu}\}$ represents the U-nodes and subset $TR=\{T_{Nu+1}, ..., T_{Nu+Nr}\}$ represents the R-nodes.

The B-nodes, $B = \{B_1, ..., B_{Nr}\}$, of the extended cooperation graph indicate the processing which has to be performed by the cluster (network) decoder, hosted in the BS.

As opposed to usual network-on-graph representations, where a node represents a check equation, a B-node could represent a system of equations involving the information blocks generated by the UTs (one block/UT/cooperation period) and the check blocks generated by the cluster's relay or relays. On this extended cooperation graph we have a representation of the UT-BS and RN-BS links. The extended graph's matrix, *A'*, becomes, *A'*[*A* | *I*], where *I* is [N_r *x* N_r] identity matrix.

Likewise, to the cooperation graph, we define the neighbor sets as follows:

$$nB_j = \{T_i \mid A'[j,i]=1, i=1,...,Nu+Nr\}; \qquad nT_i = \{B_j \mid A'[j,i]=1, j=1,...,Nr\} \tag{3}$$

3. Network Coded Cooperation Protocol

3.1 – Protocol description

The principle of the coded cooperation protocols for Multiple Source (MS) cooperation clusters is the following: the RNs generate and send to the BS additional check blocks that would be used for the recovery of the erred data blocks received by the BS from the UTs over their direct links. The coded cooperation scheme proposed for MS cooperation topologies employs two levels of coding:

- The first level of coding is represented by the channel code employed on each link of the cooperation cluster. The channel code employed by the UTs will have to ensure (at least theoretically) error-free UT-RN links.
- The second level of coding is represented by the network code (cluster code). The network code is applied only to the UTs' information bits. The lengths of the UTs' information blocks need to be equal at the input of the cluster encoder. The network code will be designed for the maximum block length, smaller-size blocks being padded with zeros.

Figure 2 (a) shows the operating principle of the coded cooperation protocol. In the first phase of the cooperation each UT channel encodes its N_i information bits using a rate R_i code. The Nc_i-long coded blocks are broadcasted to the (BS) over the $\mathrm{UT_i}$-BS links and to the RNs over the $\mathrm{UT_i}$-$\mathrm{RN_j}$ links. In the second phase of cooperation, each RN encodes the information blocks generated by the $\mathrm{UT_i}$-$\mathrm{RN_j}$ links channel decoders using the network code, thus generating one or several check blocks. The smaller-length information blocks applied to the cluster encoder will be zero-padded to the maximum length block, the number of zero padding bits employed being:

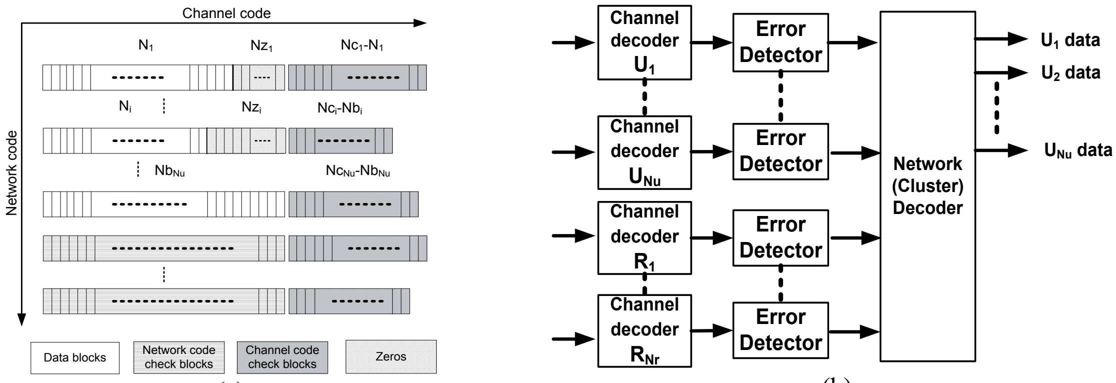
$$Nz_i = \max_k (N_k) - N_i \tag{4}$$

The check blocks computed by each RN are obtained by encoding one symbol from all user data blocks available in that symbol period. These check blocks are channel-encoded and sent over the $\mathrm{RN_j}$-BS links.

3.2 – Performance analysis

The performance metrics employed to evaluate the studied protocols are the outage probability and the recovery probability, i.e the probability to recover all data blocks.

The BS performs independent channel decoding of the blocks received from the UTs and the RNs and checks the correctness of each block. If there are erred blocks on the direct $\mathrm{UT_i}$-BS links, the cluster decoder tries to recover them using the NC-coded blocks received on the $\mathrm{RN_j}$-BS links. The NC code employed can correct errors or erasures, the later case being analyzed in this paper. Figure 2 (b) illustrates the processing performed by the BS.



(a) (b)

Figure 2 – (a) NC-based cooperation protocol for RN-enhanced cellular network; (b) Decoder structure

The *worst case* for the cluster decoder is when it can not recover any of the lost blocks; we call this situation the *outage state* of the cluster decoder. The *outage event* is the event for which the cluster decoder reaches the *outage state,* meaning that it is not able to recover any of the lost blocks. The outage probability is defined as the occurrence probability of an *outage event.* In contrast to the *outage event*, the most desired case for the cluster decoder is when all data blocks are recovered. We call this event the "*all blocks recovery" event* and its occurrence probability is called "*all blocks recovery" probability,* $P_{recover}$.

The channel experienced by the NC code can be modeled as N_u+N_r parallel block erasure channels (BLEC). The erasure probability of each BLEC is given by the block error rate (BLER) ensured by the channel codes employed over that link.

The performance of the protocol depends on the network code employed, but the upper bound of the $P_{recover}$ probability can be computed according to the Singleton bound [9] and the erasure correction bound. The Singleton bound states that the minimum Hamming distance of a linear block code is smaller than or equal to the number of check blocks plus one. If each RN generates one check block, the Singleton bound can be written as:

$$d_{min} \le N_r + 1 \tag{5}$$

The erasures correction bound states that the number of erasures which can be corrected is lower than d_{min}. So, the upper bound of "*all blocks recovery"* probability of the jth cluster's decoder can be computed as the probability to have less than $N^j_{check}+1$ erroneous blocks.

$$P_{recover} \le 1 - \sum_{k=N_r+1}^{N_u+N_r} p_{k-err} \tag{6}$$

where p_{k-err} is the probability to have exactly k erred blocks, taking into account the BLECs of all cluster members, and is computed as it follows:,

$$p_{k-err} = \sum_{T'\in T^{(k)}} \left(\prod_{t_i\in T'} BLER_{t_i-BS} \cdot \prod_{t_i\in T\setminus T'} \left(1 - BLER_{t_i-BS}\right) \right) \tag{7}$$

where $T^{(k)}$ represents the family of all k-subsets of the T-node set.

Codes that achieve this limit are Maximum Distance Separable (MDS) codes. Also this bound is achievable only when the UTs-RNs links are error-free.

If the UT-RN links are not error-free the cooperation protocol has two possibilities: the first consists in discarding the erred data blocks in the RNs, and not use them in the check blocks computation; the second one is to exclude from cooperation the RNs that receive erred blocks. This paper considers error-free UT-RN links.

4. Network Coded Cooperation With XOR-based Erasure Codes

A possible method to implement an NC-based cluster cooperation protocol is to use an XOR-based erasure codes at the RNs. In this case each RN computes its check block by simply XOR-ing the information bits of its neighbor UTs.

$$RN_j \ check\ block = \underset{UT\ \in nR_j}{XOR}\left(UT\ data\ block\right) \tag{8}$$

At the BS, the decoding algorithm tries to recover each erred block by employing the correct blocks received from the neighbour nodes. The decoding process should take the following steps:

- the algorithm checks if the data blocks received from each UT_i are correct. If an erred data block is found, it searches all the B-node neighbours of the T-node that generated the incorrectly received data block.
- for each of the B-nodes found, the algorithm verifies if for all its T-node neighbours the received data blocks are correct, except the data block which the algorithm tries to recover.
- if a B-node neighbour, for which all the data blocks of its T-node neighbours are correctly received, is found, then the erred block can be recovered using the bit-wise XOR operator. The recovered data-block is stored and the algorithm is restarted.
- if the erred data block can not be recovered, the algorithm will search for the next erred block and will try to recover it.

Similar to the stopping set definition [10], the *outage set* S, is defined as a subset of the T-nodes set, such that all the B-nodes neighbours of the T-nodes from S are connected to S at least twice. According to the outage set definition, a subset S, of the T-nodes set is an *outage set* if the following condition is fulfilled:

$$\left|\left\{B_j \in B \mid \sum_{T_i \in S} A'[j,i] = 1\right\}\right| = 0 \tag{9}$$

The outage probability is computed as the probability to have *k* erroneous blocks which form an *outage set*. This can be written as follows:

$$P_{outage} = \sum_{k=2}^{N_u+N_r} P_{OS}^k \cdot p_{k-err} \tag{10}$$

where P_{OS}^k is the probability to have k-long outage sets according to the extended cooperation graph and $p_{k\text{-}err}$ is the probability to have k erroneous blocks computed in (7).

The probability to have a k-long *outage set* is not the same for all the k-length *outage sets* due to the different link's qualities. For simplification we consider that the probability of k-long *outage sets* are equal and can be computed as follows:

$$P_{OS}^k = \frac{N_{OS}^k}{C\left(|T|,k\right)} \tag{11}$$

where $C(|T|,k)$ represents the number of all the k-subsets of the T-node set and N_{OS}^k represents the number of the k-long *outage sets*.

5. Network Coded Cooperation With Reed Solomon Codes

A well known class of MDS codes are the Reed Solomon (RS) codes. In this section the RS code, acting as erasure correcting code, is used as a network code. This type of code can be used only for “complete cooperation clusters”, meaning that all UTs are connected to all RNs. We considered only the situation in which each RN computes one check block.

The RS code employed has to be selected according to the numbers of the UTs and RNs in the cooperation cluster The parameters of the RS(n, k) code employed in cooperation cluster *j* has to satisfy the inequalities:

$$\begin{cases} k \geq Nu \\ n - k \geq Nr \end{cases} \tag{12}$$

Due to the fact that RS codes are MDS codes, the outage probability of the cluster decoder can be computed as follows:

$$P_{out} = \sum_{k=N_r+1}^{N_u+N_r} p_{k-err} \tag{13}$$

where $p_{k\text{-}err}$ is computed as in (7).

The block error rate ensured for each link by the network code can be computed as the probability to have more than N_r erred blocks and one of the erred blocks comes from the node for which the BLER is computed.

$$BLER_{(UT)} = \sum_{k=N_r+1}^{Nu+Nr} \left(\sum_{\substack{T' \in T^{(k)} \\ UT \in T'}} \left(\prod_{t_i \in T'} BLER_{t_i-BS} \cdot \prod_{t_i \in T \setminus T'} \left(1 - BLER_{t_i-BS}\right) \right) \right) \tag{14}$$

The RS code is designed according to the maximum number of UTs and RNs of the cooperation cluster. Different network code rates can be obtained by using puncturing and shortening.

A drawback of such a realization of the network coded protocol is that it requires a “complete cooperation graph”, meaning that all UTs have to be able cu communicate with all RNs on quasi error-free channels. If this condition can not be fulfilled some UTs or RNs have to be removed from the cooperation cluster

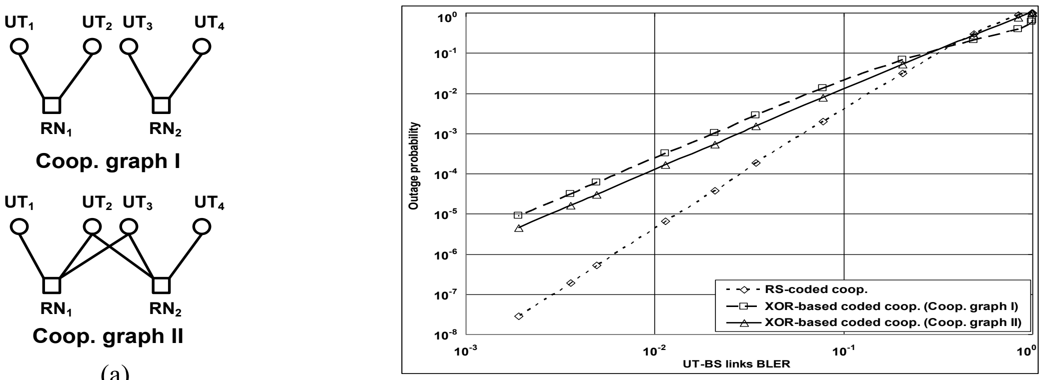
6. Performances of the Network Coded Cooperation Protocols

In order to evaluate the performances of the two cooperation protocols proposed in the previous sections, we considered a cluster of four UTs and two RNs. For the XOR-based coded cooperation protocol, two cooperation topologies defined by the cooperation graphs presented in figure 3 (a) were considered. For the RS code based cooperation protocol, the topology considers four UTs, all being connected to each of the two RNs.

All UTs and RNs use as channel code a rate 0.5 convolutional code and BPSK signalling. The UT-BS links are identical and the RN-BS links are considered to have an SNR greater with 2 dB than the one of the UT-BS links. All links are affected by additive white Gaussian noise (AWGN). A (15, 11) RS code is used for the RS code based cluster.

The performance analysis of the two cooperation protocols proposed can be performed by using the outage probability as main performance metric. The outage probability obtained for the two cooperation protocols considered is plotted in figure 3 (b), according to the direct link’s BLER. Such a probability vs. probability representation has the advantage of being independent of the channel codes employed on the cooperative cluster’s links, and thus characterizing only the performances of the cluster NC code.

Figure 3 (b) shows that for low and medium BLER of the direct links, the RS-based network code has lower outage probability than the XOR-based network codes. The outage probabilities ensured by the XOR-based network codes are different since they are depending on the structures of the cooperation graphs, which have different *outage sets* distributions.



(a)

(b)

Figure 3 – (a) Test cooperation cluster topologies. (b) Outage probability of the NC coded protocols

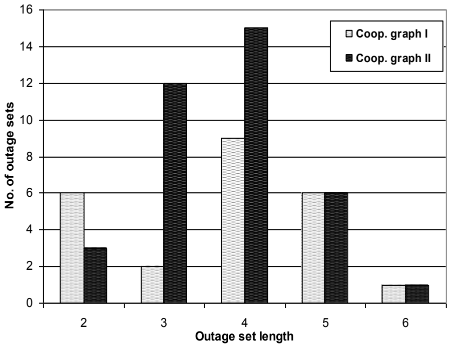


Figure 4 - Outage sets histograms

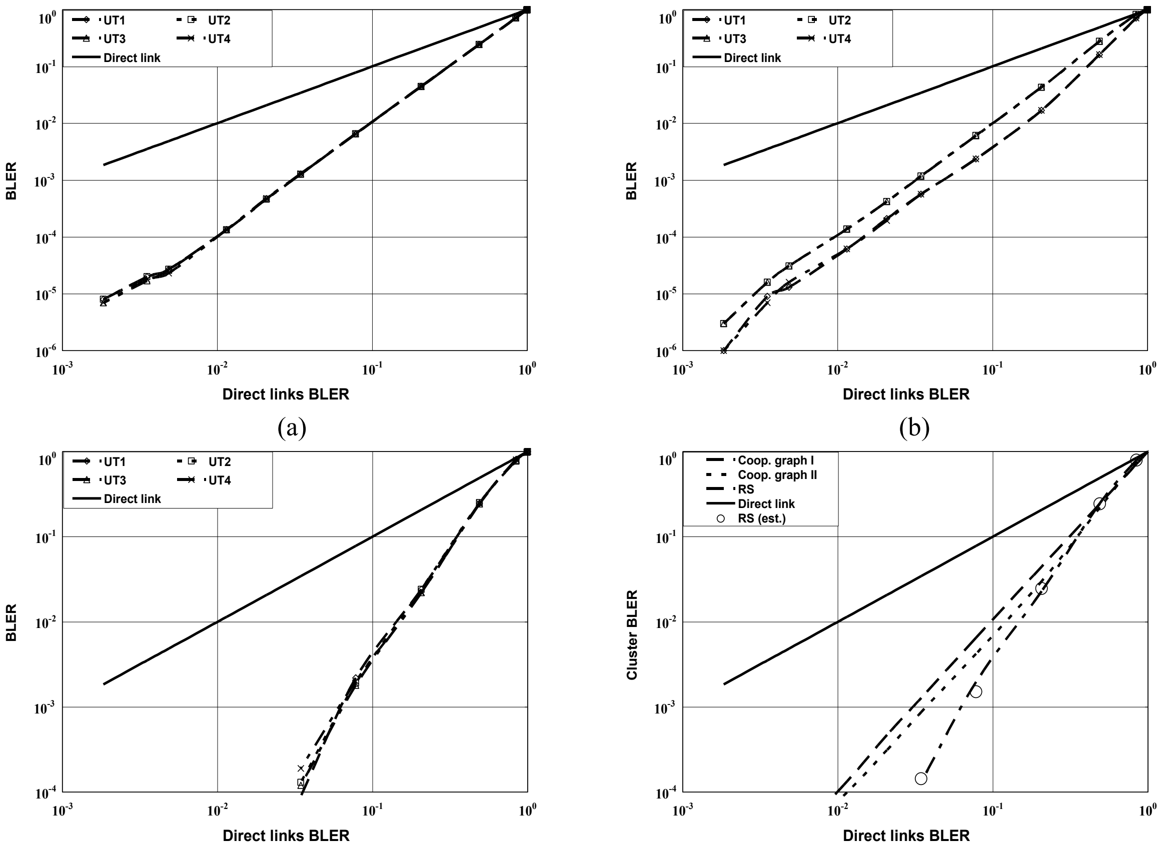
Figure 4 presents the distributions (histograms) of the *outage sets* for cooperation graphs I and II. Though the number of outage sets of cooperation graph II is higher than the one of cooperation graph I, the cooperation graph II provides better performances. This is explained by the lower number of outage sets of length two obtained for cooperation graph II. The two-long outage sets have greater occurrence probability than other outage sets.

The RS-coded cooperation protocol provides identical BLER performance for all UTs if the channel codes employed by the UTs ensure identical BLER on the UT-BS links, see Figure 5 (c), thus ensuring fairness between the UTs involved. The fairness ensured by the XOR-based cooperation protocol depends on the cooperation graph, see figure 5 (a) and 5 (b). Within cooperation graph I, the XOR-based cooperation protocol is a fair one, providing identical BLER performance for all UTs, see figure 5 (a), while within cooperation graph II the protocol does not ensure fairness, see figure 5 (b).

The overall performances of the cooperation cluster are evaluated in terms of cluster or global BLER, which is computed as the ratio between the number of erred NC decoded blocks and the total number of data blocks transmitted by the UTs of the cluster. The global BLER ensured by cooperation graph II is smaller than the one of cooperation graph I, see Figure 5 (d). The differences between the outage probability ensured by the XOR-based protocol in the two test topologies, see Figure 3 (b), can also be identified in the cluster (global) BLER, the lower cluster BLER provided in cooperation graph II, being related to the lower outage probability. Figure 5 (d) also shows that the RS-coded cooperation protocol outperforms the XOR-based cooperation protocol in terms of cluster BLER and that the BLER computation method, proposed in section 5, is accurate.

7. Conclusions

The paper proposes a cluster-based network coded cooperation protocol for the uplink transmission of relay enhanced cellular networks. The protocol described and analyzed is intended for multiple source – multiple relay cooperation clusters that have no connectivity between UTs. The paper analyzed two types of erasure codes, used as network codes, namely: an XOR-based erasure code and RS codes. These two variants of the cooperation protocol were analyzed in terms of outage probability, fairness and block error rate. An upper bound of the "*all blocks recovery*" probability of the network decoder was also derived for the proposed cooperation protocol. The simulations results show that the RS-coded cooperation protocol outperforms the XOR-based protocol, both in outage probability and BLER. As for



(a) (b)

(c) (d)

Figure 5 – Individual UT BLER performances: (a) XOR-based coding topology I; (b) XOR-based coding topology II; (c) RS coding; (d) Global BLER performances of the XOR-based coding and RS coding

fairness, it is shown that the RS-coded protocol is always a fair one, if the channel codes employed by the UTs ensure identical BLER on the UT-BS links, while the XOR-based protocol ensures fairness only for some particular cooperation graphs and link parameters.

Acknowledgment

This work was partially funded by the EU within FP7-ICT-215477 CODIV Project.

References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior,” IEEE Trans. Inform. Theory, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[2] T. E. Hunter and A. Nosratinia, “Cooperative Diversity through Coding,” Proc. IEEE ISIT, Laussane, Switzerland, July 2002.

[3] B. Zhao, M. C. Valenti, “Distributed Turbo Coded Diversity for Relay Channel”, Electronics Letters, vol. 39, May 2003.

[4] A. Chakrabarti, E. Erkip, A. Sabharwal, and B. Aazhang, “Code Designs for Cooperative Communication”, IEEE Signal Processing Magazine, Vol. 24, Issue 5 , pp. 16 – 26, Sept. 2007.

[5] R. Ahlswede, N. Cai, S.-Y. R. Li and R. W. Yeung, “Network Information Flow”, IEEE Transactions on Information Theory, 46(4):1204-1216, July 2000.

[6] C. Hausl, P. Dupraz, “Joint Network-Channel Coding for the Multiple-Access Relay Channel”, IEEE SECON, Vol. 3, pp. 817-822, 2006.

[7] Y. Chen, S. Kishore and J. Li, “Wireless Diversity through Network Coding”, IEEE Wireless Communications and Networking Conference, 2006.

[8] X. Bao and J. Li, “Matching Code-on-Graph with Network-on-Graph: Adaptive Network Coding for Wireless Relay Networks”, 43rd Annual Allerton Conference, 2005.

[9] R. C. Singleton, “Maximum Distance Q-nary Codes”, IEEE Trans. on Inf. Theory, Vol. 10, pp. 116-118, 1964.

[10] C. Di, D. Proietti, I. E. Telatar, T. J. Richardson and R. Urbanke, “Finite Length Analysis of Low-Density Parity-Check Codes on the Binary Erasure Channel,” IEEE Trans. Inform. Theory., vol. 48, no. 6, pp. 1570-1579, June 2002.