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Mihai P. Stef, L. Boita, A. Botos, Zs. Polgar
Communications Department, Technical University of Cluj-Napoca

Mihai P. Stef
Communications Department
Faculty of Electronics and Telecommunications,
Technical University of Cluj-Napoca
26-28 G. Baritiu st
400027 Cluj-Napoca
ROMANIA
Phone: + 40-264 40 19 06
Fax: + 40-264 59 16 89
Email: Mihai.Stef@com.utcluj.ro

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Mihai P. Stef, Ph.D. student, L. Boita, B.Sc. student, A. Botos, Ph.D. student, Zs. Polgar, Ph.D.
Communications Department, Technical University of Cluj-Napoca

Abstract: Distributed channel coding and network coding techniques are means employed to improve the performance of cooperative transmissions. This paper studies the BLER performance provided by a cooperative scheme that combines distributed channel coding and network coding for multiple source-multiple relay architectures, within which the number of available relays is smaller than the number of sources. It includes a cooperation strategy which employs XOR-based network coding for such architectures and proposes a model to analyze the overall performance provided. The performance is analyzed for various values of the ratio between the source and relay numbers.

1. Introduction

Relaying and cooperation between terminals are considered as some of the most promising approaches for the performance improvement of the wireless networks [1]. The channel-coded cooperation included in schemes where a relay-node (RN) serves only one mobile or fix user terminal (UT) in its transmission to the base station (BS) is one of the techniques proposed in literature to accomplish those improvements [2] [3]. Though this approach is shown to bring performance improvements for the served UT in terms of bit error rate (BER), block error rate (BLER) and/or coverage, the additional time-frequency resources (TFR) required by the RN are used to serve only one UT leading to a loss of performances in terms of spectral efficiency. In order to decrease the effect of the additional TFR upon the spectral efficiency of the UT-BS transmission, Network Coding (NC) techniques [4] were included in coded cooperation algorithms. Since the NC techniques allow cooperation structures within which the RN serves more UTs, such an approach leads to a more efficient employment of the additional TFR of the RN. But, making these techniques effective raises new questions that have to be addressed.

A simple XOR-based network coding was employed in cooperative wireless networks [5] in order to obtain better diversity gain. Graph representation of a cooperative network was employed in [6] to use a known class of codes, codes on graph, like low density parity check (LDPC) codes, to cooperate in multi-sources multi-relays architectures.

The paper describes a graph representation of the cooperative cellular network called cooperation graph. It also presents and discusses a cooperation strategy which employs a XOR-based network coding technique and a network decoding algorithm which operates on the cooperation graph. Using an extension of the cooperation graph the paper analyzes the performance provided by the network decoding algorithm and presents the performance obtained for some cooperation architectures.

The paper is organized as follows: Section 2 introduces the representation of a cooperative cellular network as a bipartite graph. Section 3 discuses the network-coded cooperation strategy employed. Section 4 analyzes the network decoding algorithm. Section 5 presents the BLER performance obtained by network coded cooperation schemes. Finally, Section 6 concludes the paper.

2. Network on graph

This section describes a cooperative cellular network as a bipartite graph and defines the structures needed for a graph representation. This is a more convenient representation of the network elements to point out the processing involved and to analyze the cooperative network performance.

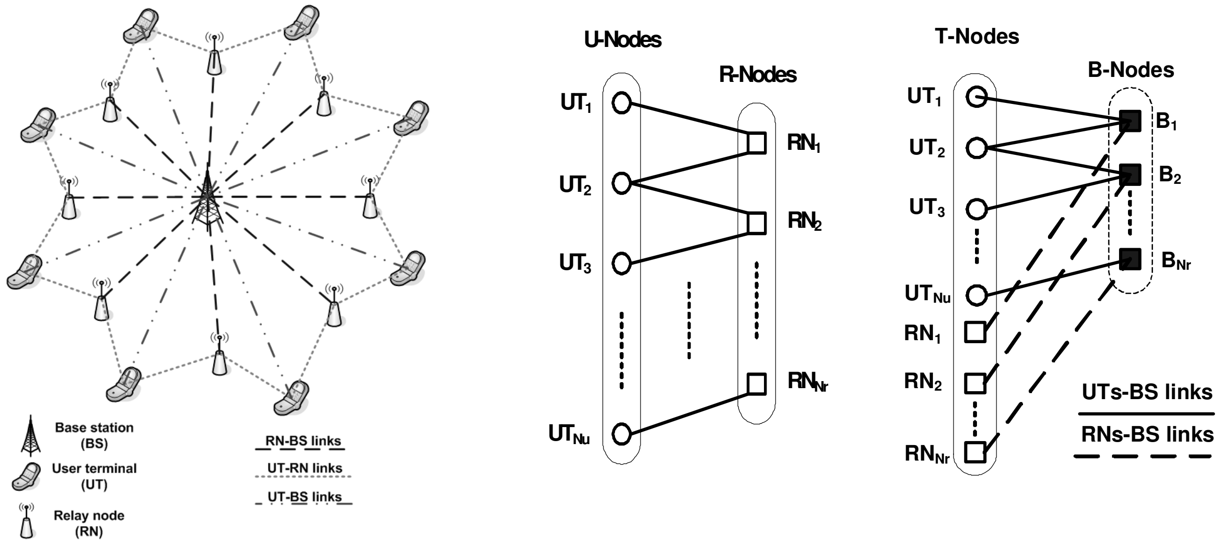

Figure 1. Network as a graph: left – Cooperative cellular network; center – Cooperation graph; right – Extended cooperation graph

Figure 1 (left) presents an example of a cooperative cellular network. The network elements of a cooperative cellular network are: base station (BS), relay nodes (RNs) and user terminals (UTs) and all the employed links are UT-RN links, RN-BS links and UT-BS links. The UT-RN and RN-BS links are mandatory in a cooperative network. Without these links the cooperation can not take place. The direct links (UT-BS) use depends on the cooperation scheme employed. Also the number of UT-RN links to a relay node depends on the cooperation strategy employed.

The paper considers a network coded (NC) cooperation strategy for the uplink direction, so for our example the relay nodes serve more than one user terminal (more than one UT-RN link for one RN) and direct links (UT-BS) data is used by the decoding process.

The graph representation can be used to describe the cooperation in a cell or for a group of cell members (UTs and RNs) which cooperate, called a cooperation cluster (CC).

Let us consider a cooperation cluster with *Nu* user terminals and *Nr* relay nodes. Figure 1 (center) presents the cooperation graph obtained. The cooperation graph shows for each user terminal with which RN it cooperates, also indicating the UT-RN links employed in the cooperation cluster. For the NC cooperation strategy employed in this paper the cooperation graph indicates for each RN what data blocks to combine.

To represent the cooperation graph two sets are defined, one for the UTs, U-nodes set, $U = \{UT_1, \ldots, UT_{Nu}\}$ and one for the RNs, R-nodes set, $R = \{RN_1, \ldots, RN_{Nr}\}$. Also, the matrix representation of the bipartite graphs was used to indicate the edges between U-nodes and R-nodes, $A[Nr \: x \: Nu]$, where:

$$A[\,j,i\,] = \begin{cases} 1, \ \ if \ \ RN_j \ \ and \ \ UT_i \ \ are \ \ connected \\ \\ 0, \ \ if \ \ RN_j \ \ and \ \ UT_i \ \ are \ \ not \ \ connected \end{cases} \tag{1}$$

The cooperation graph shows "*who cooperates with whom*", meaning that for each RN from the R-nodes, we define a set of UT neighbors from the U-nodes set, which represents the UTs which will be processed by the j^{th} RN:

$$nR_j = \left\{ UT_i \mid A[\,j,i\,] = 1, i = \overline{1,Nu} \right\} \tag{2}$$

Also, for each UT, we define the set of RN neighbors:

$$nU_i = \{RN_j \mid A[\,j,i\,] = 1, j = \overline{1,Nr}\} \tag{3}$$

Using the cooperation graph we can model the processing on the UT-RN links. To model the UT-BS and RN-BS processing we need to use the extended cooperation graph, figure 1 (right).

The T-nodes set, $T=\{T_1, \ldots, T_i, \ldots, T_{Nu+Nr}\}$, of the extended cooperation graph is obtained from the cooperation graphs U-nodes set and R-nodes set as follows: subset $TU=\{T_1, \ldots, T_{Nu}\}$ represents the U-nodes and subset $TR=\{T_{Nu+1}, \ldots, T_{Nu+Nr}\}$ represents the R-nodes.

B-nodes, $B = \{B_1, \ldots, B_{Nr}\}$, indicates the equations which have to be solved by the network decoder. On this extended cooperation graph we have a representation of the UT-BS and RN-BS links. The graph matrix, *A'*, for the extended cooperation graph becomes, $A' = [\,A \mid I\,]$, where I is $[Nr \times Nr]$ identity matrix.

Likewise, for the cooperation graph, we define the neighbors sets as follows:

$$\begin{aligned} nB_j &= \{T_i \mid A'[\,j,i\,] = 1, i = \overline{1,Nu+Nr}\} \\ nT_i &= \{B_j \mid A[\,j,i\,] = 1, j = \overline{1,Nr}\} \end{aligned} \tag{4}$$

To analyze the cooperation clusters performance we need to know the quality of each link involved in cooperation. For this purpose we define the link quality set $TL=\{TL_1, \ldots, TL_i, \ldots, TL_{Nu+Nr}\}$ which is constructed as follows: subset $\{\mathrm{TL}_1, \ldots, \mathrm{TL}_{\mathrm{Nu}}\}$ represents the UTs-BS links quality and subset $\{\mathrm{TL}_{\mathrm{Nu+1}}, \ldots, \mathrm{TL}_{\mathrm{Nu+Nr}}\}$ represents the RNs-BS links quality.

The performance of the cooperation scheme is expressed in terms of block error rate, to be able to do this, a mapping rule from link quality metric to block error rate, *BLER(link),* is used.

3. Network-coded cooperation strategy

The considered cooperation strategy is one that employs network coding on the RN-BS links. The motivation for using network coding is that one relay node can serve more than one user terminal using the same physical resources.

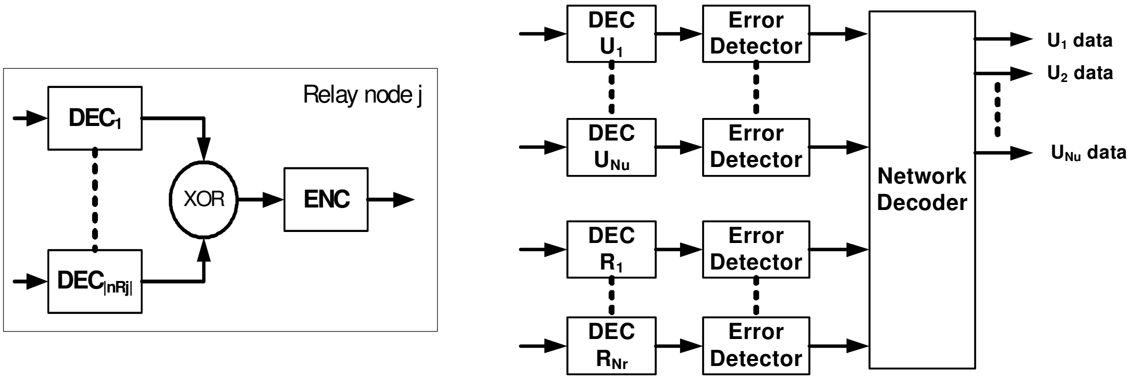

Figure 2. Network encoder (left) and network decoder (right)

According to decode and forward cooperation strategy, each user terminal will channel encode their N_i information bits, using a R_{UT} rate channel code, obtaining an N_c bits length coded block. To obtain the global rate we must take into account the additional information provided by the relay nodes. For each user terminal, using the cooperation graph, the global rate can be computed as follows:

$$R_g^{UT_i} = \frac{N_i}{N_c \cdot \left(1+\sum_{R_j\in nUi}\frac{1}{|nRj|}\right)} = \frac{N_i}{N_c}\cdot\frac{1}{1+\sum_{R_j\in nUi}\frac{1}{|nRj|}} = R_{UT}\cdot\frac{1}{1+\sum_{R_j\in nUi}\frac{1}{|nRj|}} \tag{5}$$

Each relay node, RN_j, will process the coded blocks received from all its neighbors $U_i \in nRj$. As it is shown in figure 2 left, RN will decode each received block and the network encoded block is computed using the XOR operation. The XOR-ed block is encoded using the same channel code with the same rate as the one employed by the UTs. These restrictions are not mandatory; they are imposed to simplify the analysis of the cooperation scheme.

At the BS (figure 2, right) all the received coded blocks, from the UTs on UT-BS links and from the RNs over the RN-BS links, are channel decoded and the obtained data blocks are checked for errors (by syndrome or CRC). The network decoder takes as input the result of the error detection blocks and all the decoded blocks. The extended cooperation graph is supposed to be known at the BS.

The network decoder stores its input data in two types of buffers: one status buffer, SbuffT and *Nu+Nr* data buffers. The status buffer is a *Nu+Nr* bits length buffer which stores the error detection result for each data block, 1 for no errors and 0 for detected errors. Data buffers are *Ni* bits length and store the decoded data blocks by the channel decoders.

The basic idea of the network decoding algorithm is to search for each erroneous block if it can be recovered from its neighbors' data. The *separate network decoding* algorithm is trying to solve each equation separately. Then pseudocode of this decoding process and some explanations are presented in the following:

```
Start:
For each $T_i \in TU$
If SbuffT[i]=0 then
  For each $B_j \in nT_i$
    e=0;
    For each $T_k \in nB_j \setminus T_i$
      If SbuffT[k]=1 then
        e=e+1
      End If
    End For
    If e=|nBj|-1 then
      SbuffT[i]=1
      Dbuff[i] = XOR(nBj\Ti)
      Goto Start
    End If
  End For
End If
End For
```

- the algorithm verifies for each UT if the received data block is correct, using the status buffer. If an erroneous data block is found, it searches all the B-nodes neighbors of the T-node owner of the erroneous data block.
- for each of the B-nodes found, the algorithm verifies if for all its T-nodes neighbors the received data blocks are correct, except the data block which the algorithm tries to recover.
- if a B-node neighbor, for which all the T-node neighbors data blocks are correctly received, is found, then the erroneous block can be recovered using the bitwise XOR operator. The recovered data block is stored in its data buffer, marking it as a correct data block and the algorithm is restarted.
- if the erroneous data block can not be recovered the algorithm will search for the next erroneous block and will try to recover it.

4. Network decoding algorithm analysis

From the network decoder point of view, all the links are modeled as block erasure channels, with the erasure probability equal to *BLER(link)*. Knowing the *BLER* provided by the channel code of each link, network decoder performances can be evaluated. The *worst case* for the network decoder is when it can not recover any of the lost blocks; we call this situation the *outage state* of the network decoder. The paper will analyze this situation.

The *outage event* is the event for which the network decoder reaches the *outage state,* meaning it is not able to recover any of the lost blocks. The outage event appears when all the erroneous blocks were transmitted by T nodes forming an *outage set*. Similar to the stopping set definition [7], the *outage set*, S, is defined as a subset of the T-nodes set, T, such that all the B-nodes neighbors of the T-nodes in S are connected to S at least twice.

According to the outage set definition, a subset, S, of the T-nodes set, T, is an outage set if the next condition if fulfilled:

$$card\left\{ B_j \in B \mid \sum_{T_i \in S} A'[\,j,i\,] = 1 \right\} = 0 \tag{6}$$

Outage probability is defined as the probability to have *k* erroneous blocks which form an *outage set*. This can be written as follows:

$$P_{outage} = \sum_{k}^{N_u+N_r} P_{OS}^{k} \cdot P_{k-err} \tag{7}$$

where P_{OS}^{k} is the probability to have k-length outage sets according to the extended cooperation graph and $P_{k\text{-}err}$ is the probability to have k erroneous blocks.

The probability to have a k-length outage set is not the same for all the k-length outage sets due to the different links quality. For simplification we consider that the probability of k-length outage sets are equal and can be computed as follows:

$$P_{OS}^{k} = \frac{N_{OS}^{k}}{C(|\,T\,|,k)} \tag{8}$$

where $C(|\,T\,|,k)$ represents the number of all the k-subsets of the T-node set and N_{OS}^{k} represents the number of the k-length outage sets.

Taking into account the quality of all the links, the probability to have k erroneous blocks is computed as follows:

$$P_{k-er} = \sum_{T' \in T^{(k)}} \left(\prod_{t_i \in T'} BLER(TL_i) \cdot \prod_{t_i \in T \setminus T'} \left(1 - BLER(TL_i)\right) \right) \tag{9}$$

where $T^{(k)}$ represents the family of all k-subsets of the T-node set.

Figure 3 (right) compares the outage probability obtained from the simulations to the one obtained by theoretical evaluation using relations (7) – (9) for two 4 sources – 2 relays architectures, figure 3 (left). The results presented in figure 3 shows that the theoretical estimation of the outage probability is very close to the simulated one. Small differences appear due to the fact that the relay nodes have better link quality (with 6 dB greater than the user terminals links) and the outage set probability is not the same for all outage sets. The same figure also shows that for the second architecture, figure 3 down, better outage probability is obtained than the one obtained for the first architecture, figure 3 up.

The obtained results indicates that the outage probability can be used as an overall performance indicator and outage sets characterize the worst case situation for the network decoding algorithm discussed above.

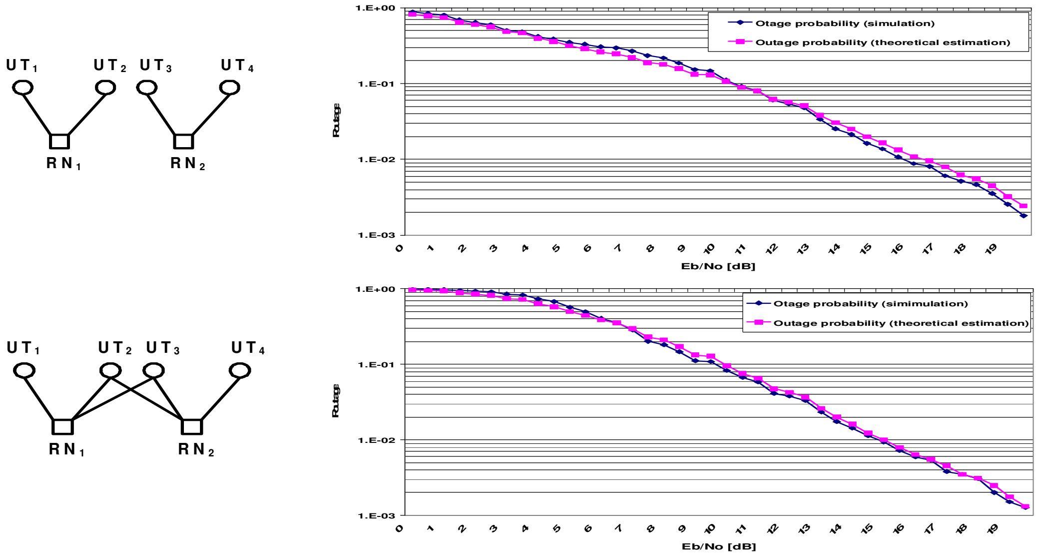

Figure 3. left: Cooperation graphs for the two considered architectures; right: Outage probability for the two considered architectures

5. Performance of the network coded cooperation strategy

The performance of the network coded cooperation strategy was evaluated for four architectures with four user terminals and two or three relay nodes. Figure 4 presents the cooperation graphs for the four architectures. All user terminals have identical UT-BS channels, the same E_b/N_0 on all channels, and the relay nodes have better RN-BS channels, E_b/N_0 greater with 6dB than the E_b/N_0 of the direct channels. The UT-RN channels are considered to be quasi error free. The employed channel code is a turbo code having the parameters shown in table 1.

Feedback generator polynomial	13_8
Feedforward generator polynomial	15_8
"Mother" code rate	0.50
UT coding rate	0.75
No. of iterations of the turbodecoders	8

Table 1. Channel code parameters

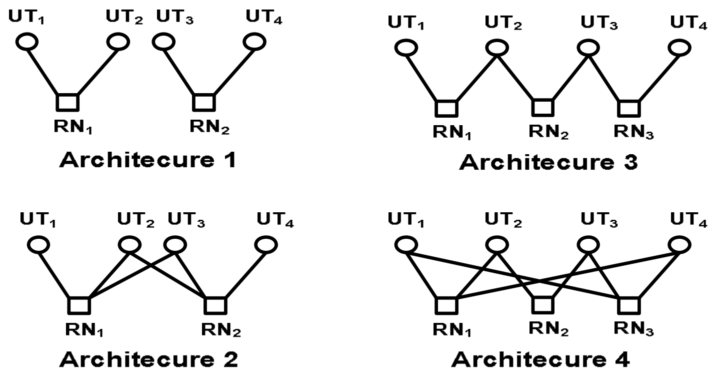

Fig. 4. Cooperation graphs of the considered architectures

The global performance is evaluated in terms of global BLER which is computed as the ratio between the number of all incorrect data blocks at the BS after network decoding and the number of all data blocks send by all user terminals.

Figure 5 (left) shows the global BLER performance of the considered architectures and figure 5 (right) presents the outage probability obtained from simulations. The first observation is that the outage probability gives an indication of the overall performance of different architectures.

For architectures 1 and 2, two relay nodes serve four user terminals, but using different cooperation graphs. Figure 5 (left) shows that the architecture 2 obtains better overall performance than the architecture 1, only by the modification of the cooperation graph. As it is showed in figure 5, increasing the number of relay nodes will not always provide gain in overall performance, as this depends on the cooperation graph. Architecture 4 provides similar overall performance with architecture 2, but architecture 2 uses two relay nodes and architecture 4 uses three relay nodes.

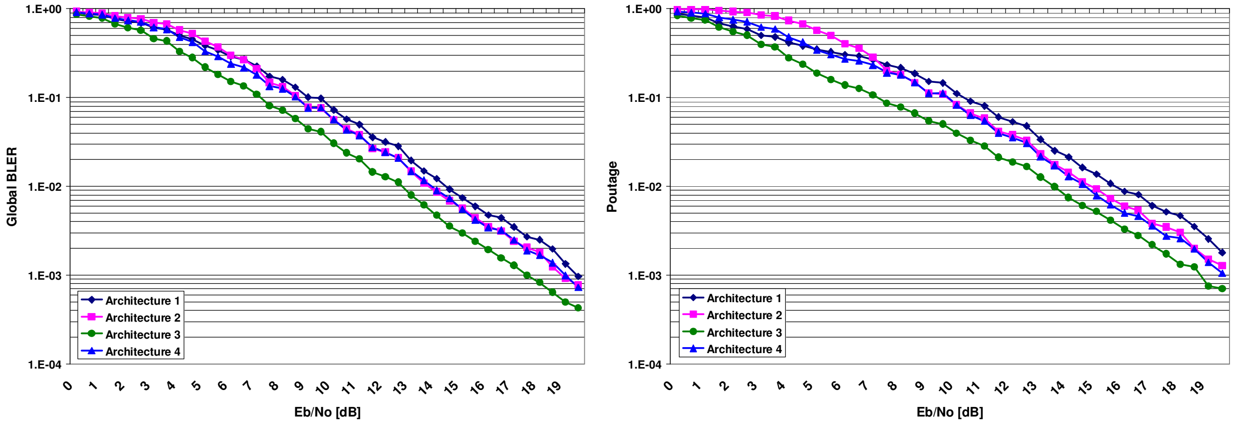

Figure 5. Overall performance for the considered architectures: left – global BLER; right – Outage probability

Figure 6 presents the individual BLER performance for each user terminal. Except for architecture 1 the cooperation does not provide the same performance for each user terminal. The differences between user terminals BLER performance can be explained by affiliation of each user terminal and each relay node to a different outage set, but this requires further investigations. For example, architecture 3 provides better BLER performance for UT2 and UT3 than for UT1 and UT4. This can be explained by the fact that UT1 and UT4 form length 2 outage sets with their RNs and UT2 and UT4 do not form any length 2 outage sets. The minimum length outage sets have greater influence on the performance because these outage sets are more probable to appear.

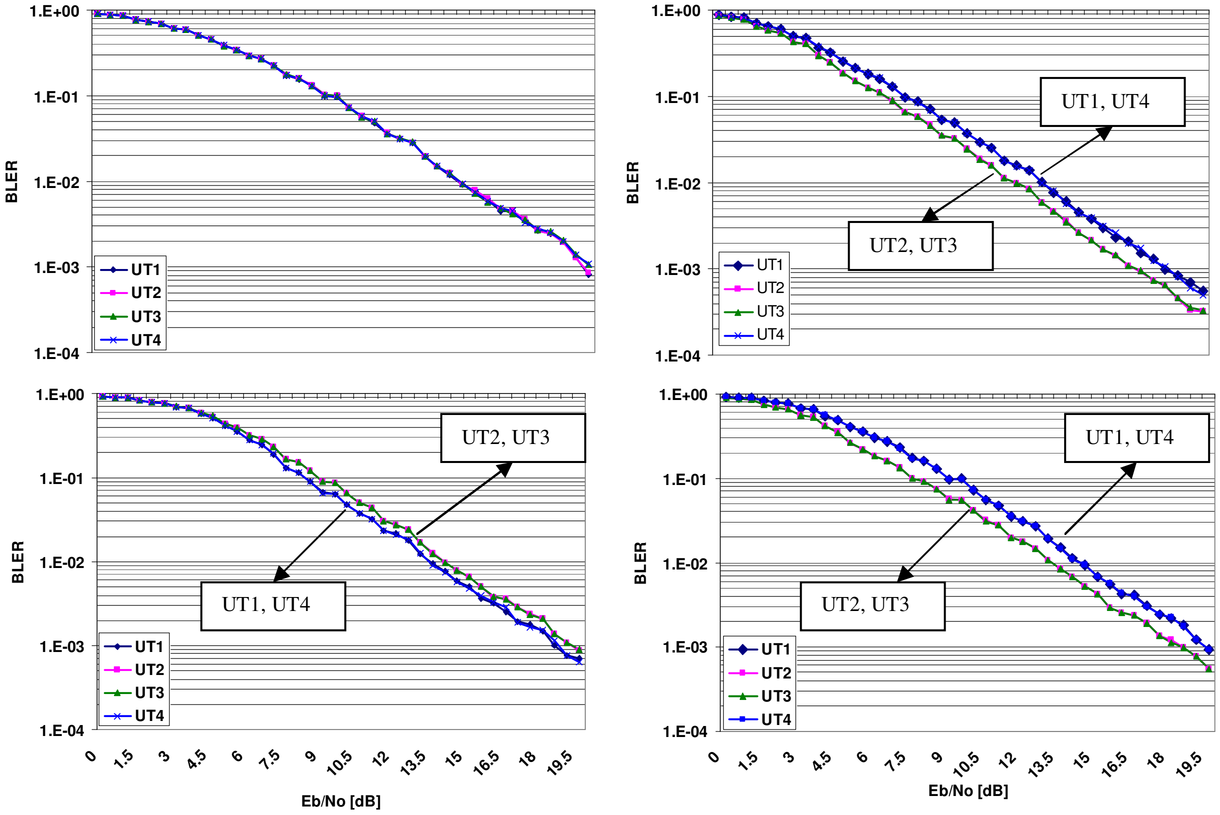

Fig. 6. Individual BLER performance of the considered architectures: left, up – architecture 1; left, down – architecture 2; right, up – architecture 3; right, down – architecture 4

6. Conclusions

The paper presents an analysis of a network coding based cooperation technique for multiple sources multiple relays cellular cooperative architectures and proposes a belief propagation based method for XOR based NC decoding. The proposed performance analysis is based on the graph representation of the cooperative cellular network, defining also the construction of a bipartite graph which describes the network coding operations performed.

The paper defines the outage event in the NC decoding process and computes the probability of such an event using the stopping set definition employed in the analysis of LDPC codes over binary erasure channels. Preliminary results presented show that the outage probability is a reliable overall performance indicator for cooperative architectures. Using the outage probability one can compare the global BLER performance of different cooperative architectures.

We also showed that one can improve the overall performance of a cooperation cluster by carefully constructing the cooperation graph.

Preliminary studies show also that the individual BLER performances can be estimated using the defined outage sets.

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