Transformer Transient Behavior Simulation by a Coupled Circuit-Field Model

Ioan-Adrian VIOREL – Károly BIRO – Loránd SZABÓ
Technical University of Cluj-Napoca

Abstract: The design and optimization of electrical machines is one of the most important research areas in electrical engineering. Transient behavior is sometimes a critical part of designing and limits setting. The effects of saturation and secondary armature reaction are considered in the transformer circuit model by changing suitable the mutual inductance. The proposed circuit field model follows the idea that if the main path flux linkages is calculated by solving a field problem one can avoid the mutual inductance computation. The proposed circuit field model is conceived to be solved by means of computer and takes fully into account the nonlinearity. Computer results via the usual circuit model and circuit-field model, compared with given tests, stand by to prove the proposed model usefulness.

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REFERENCES

TRANSFORMER TRANSIENT BEHAVIOR SIMULATION BY A COUPLED CIRCUIT-FIELD MODEL

MODELE COUPLE CHAMP-CIRCUIT POUR L'ETUDE EN REGIME TRANSITOIRE DES TRANSFORMATEURS

VIOREL I.A., BIRO K., SZABO L.
TECHNICAL UNIVERSITY OF CLUJ
ROMANIA

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Abstract
The design and optimization of electrical machines is one of the most important research areas in electrical engineering. Transient behavior is sometimes a critical part of designing and limits setting. The effects of saturation and secondary armature reaction are considered in the transformer circuit model by changing suitable the mutual inductance. The proposed circuit-field model follows the idea that if the main path flux linkages is calculated by solving a field problem one can avoid the mutual inductance computation. The proposed circuit-field model is conceived to be solved by means of computer and takes fully into account the nonlinearity. Computer results via the usual circuit model and circuit-field model, compared with given tests, stand by to prove the proposed model usefulness.

Introduction
Electrical machines design and optimization constitutes one of the most important research areas in electrical engineering. Transient behavior is quite a critical part of designing and limit setting. Therefore any improvement made in describing the transient behavior of an electrical machine which conducts to better computational results compared with tests, may be of real help.

With any change in one or several of the quantities determining transformer operation, namely, voltage, frequency, load etc., a transient process occurs from one steady state to another. Usually this transient process lasts only a very short time, but it may be accompanied by considerable effects, dangerous for transformer operation. As a result of an over-current transient process can appear very large mechanical stresses between the windings or their parts and excessive overheating of the winding. These effects are of particular importance in modern high-voltage large-power transformers and so are the investigation of these phenomena.

Over-current transient processes arise when an unloaded transformer is connected to the power circuit, or when a short circuit occurs. The mathematical models suitable to describe the over-current and respectively the over-voltage transient processes are totally different. Because the ferro-magnetic circuit saturation affects basically the over-current transients this particular model will be discussed.

Usually the transformer circuit model consists of three
voltage equations, one on primary, one on secondary and one on the load. Because of ferromagnetic core saturation and secondary armature reaction the transformer circuit model is containing a procedure through which the mutual inductances are changed suitable. In fact, the circuit model accuracy is strongly dependent to the manner the mutual inductances are related to winding currents. The mutual inductances values and their variation caused by winding currents can be determined usually by no-load test with variable input voltage. The mutual inductance can be calculated by solving the field problem and computing, this way, the main path flux linkages. From this very point may arise a question. Is there any need to compute the mutual inductance if the main path flux linkages was determined? The circuit-field model proposed in this paper is based on the answer no on the previous question.

The combined circuit-field model is conceived to be solved by computer, the computational process consisting of a simultaneous iterative calculation of slightly modified circuit type equations and corresponding field problem. Computed results via the usual circuit model and circuit field model are compared with given tests, and some pertinent conclusions can be put in evidence.

Mathematical Model

The transformer is the simplest practical illustration of a two-coil electromagnetic device since in its basic form it consists of two adjacent co-axial coils. One coil, the primary, coil 1 say, is connected to the supply system, and the induced voltage in the secondary, coil 2, is, usually, supplying power, obviously at a different voltage, to a circuit electrically isolated from the primary circuit. The primary winding has \( N_1 \) turns and the secondary \( N_2 \), but usually the transformer equation are written with an equivalent secondary which has also \( N_1 \) turns. It is referred to primary transformer [1]. In such a case, with all secondary values referred to primary, the transformer equations are:

\[
\begin{align*}
v_1 &= R_1 i_1 + \frac{d\lambda_1}{dt} \\
v_2 &= R_2 i_2 + \frac{d\lambda_2}{dt} \\
v_2 &= R_e i_2 + L_e \frac{di_2}{dt} \\
\lambda_1 &= L_1 i_1 + \lambda_m \\
\lambda_2 &= L_2 i_2 + \lambda_m \\
\lambda_m &= M i_m \\
i_m &= i_1 + i_2
\end{align*}
\]

where \( v_1, i_1, R_1, \lambda_1 \) are primary voltage, current, leakage inductance and total flux linkages, respectively; \( v_2, i_2, R_2, \lambda_2 \) are the referred to primary secondary voltage, current, leakage inductance and total flux linkages, respectively; \( \lambda_m \) is the main path flux linkages, or magnetizing flux, \( i_m \) the magnetizing current, \( M \) the mutual or magnetizing inductance, \( R_e \) and \( L_e \) the referred to primary load resistance and inductance, respectively.

Circuit Model

In order to obtain the usual circuit model equations in the
system (1) to (7) the equations (4) and (5) will be introduced in the equations (1) and (2), taking eq. (6) into account. After some computation the resulting system is:

\[
v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + \left(M + i_m \frac{dM}{di_m}\right) \frac{di_m}{dt} \tag{8}
\]

\[
v_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + \left(M + i_m \frac{dM}{di_m}\right) \frac{di_m}{dt} \tag{9}
\]

where the leakage inductances \(L_1\) and \(L_2\) were considered constants, which means that they are not affected by the saturation.

If there is no saturation effect \(dM/di_m = 0\), and we get the usual equations:

\[
v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_m}{dt} \tag{10}
\]

\[
v_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_m}{dt} \tag{11}
\]

It is obviously that the equation (8) and (9) or (10) and (11) together with equations (3) and (7) are describing the transformer operation with or without consideration of ferromagnetic core saturation effects. If the saturation effects has to be considered one needs to know the variation of the magnetizing inductance function of the magnetizing current,

\[M = f(i_m)\tag{12}\]

The mutual, or magnetizing inductance values function of the magnetizing current can be determined by a no-load test with variable input voltage, the plot looking like the magnetization characteristic, but with different scales.

The mutual inductance can be calculated by solving the field problem and by computing the main path flux linkages based on nonlinear equivalent magnetic circuit for example.

**Circuit-Field Model**

By considering the saturation effects the circuit model becomes nonlinear and the computer solving procedure will be adequately made. It means that at a certain time moment the new currents are computed with the previous time moment calculated values of mutual inductance and its variation versus magnetizing current. Therefore it will be more convenient to use the main path flux linkages, which has to be computed any way, and to avoid the mutual inductance calculation. The transformer leakage inductances are not too affected by saturation and this inductances can be calculated or determined by tests with quite a good accuracy. Based on this the currents \(i_1\) and \(i_2\) can be expressed from equations number (4) and (5), and introduced into equation (1), (2), obtaining:

\[
\frac{d\lambda_1}{dt} + \frac{R_1}{L_1} \lambda_1 = v_1 - \frac{R_1}{L_1} \lambda_m \tag{13}
\]

\[
\frac{d\lambda_2}{dt} + \frac{R_2}{L_2} \lambda_2 = v_2 - \frac{R_2}{L_2} \lambda_m \tag{14}
\]

where the currents are:

\[
i_1 = \frac{\lambda_1 - \lambda_m}{L_1} \tag{15}
\]

\[
i_2 = \frac{\lambda_2 - \lambda_m}{L_2} \tag{16}
\]

The equations (13) to (16) constitutes the circuit part of the circuit.
circuit-field model. The field part of this model is based on the main path flux linkages, \( \lambda \), computation, and can be made via nonlinear equivalent magnetic circuit solving, or via a numerical method, such as finite differences or finite elements, applied to the given field domain.

In this paper, the field problem has been solved by a nonlinear equivalent magnetic circuit, shown in Figure 1, because this solution asks for shorter computer time, and because the main purpose of the paper was just to introduce the circuit-field model.

![Figure 1. The nonlinear simplified equivalent magnetic circuit](image)

In Figure 1, \( R_{m_1} \) and \( R_{m_2} \) are the leakage magnetic resistances,

\[ F_{m_1} = N_1 i_1 \]  
\[ F_{m_2} = N_2 i_2 \]

are the amperturns for primary, respectively, secondary, referred to primary, windings. The total magnetic resistance of the transformer iron-core, \( R_{m_\text{Fe}} \) in Figure 1, is resulting as a sum of the magnetic resistance calculated on the constant section iron core portions. The saturation is fully considered by taking the magnetic permeability variable in accordance with the magnetization characteristic.

The main computational steps in a circuit-field model manner solved problem are:

(i) At a given time value with the input voltage \( v \), computed and the magnetizing flux \( \lambda \) calculated in the previous moment the total flux linkages \( \lambda_1 \) and \( \lambda_2 \) are obtained by solving equations (13) and (14).

(ii) The new currents \( i_1 \) and \( i_2 \) are given by relations (15) and (16).

(iii) With this new amperturns \( F_{m_1} \) and \( F_{m_2} \) the equivalent magnetic circuit is solved, via an iterative process which allows saturation taking into account, and the new value of the magnetizing flux \( \lambda \) is determined.

(iv) New calculated total flux linkages are compared with the values obtained at step (i). If the differences are greater than the given error, the procedure is run up from the beginning at the same time moment with the new under-relaxated magnetizing flux. If the differences are smaller than the given error the procedure is run up from the first step at increased time with an under-relaxed magnetizing flux value and the corresponding new input voltage.

In the case of circuit model the computational procedure is based on a determined by test relation (12), analytically expressed via a polynomial approximation, and follows quite the usual steps.

In order to consider the iron core losses a functions winding was introduced. This short-circuited winding has \( N_1 \) turns, the same leakage inductance as the primary and its resistance and current are function of iron core losses [4].
Results and Conclusions

A 1 kVA, one phase, 220V/24V transformer was considered with the main data: \( R_L = 1.234 \Omega \), \( L_L = 7.46 \text{mH} \), \( R_T = 1.342 \Omega \), \( L_T = 11 \text{mH} \), \( N_L = 350 \text{ turns} \), \( N_T = 40 \text{ turns} \).

On this transformer the usual tests were performed to determine the leakage inductances and the variation of the magnetizing inductances function of magnetizing current. Unfortunately the design data were not available, therefore the equivalent magnetic circuit was build up quite approximately. The no-load transformer connection to the power circuit and short circuit transient processes were studied. The tests conducted on transformer are given in figure 2. a and b at rated input voltage and frequency. The steady-state no-load, respectively short circuit currents are: \( I_u = 0.7 \text{A} \), \( I_s = 38 \text{A} \). The no-load and short circuit transient primary currents and flux linkages versus time, calculated via usual circuit model are given in figure 3. a and b, respectively. 4. a and b, at the same zero initial input voltage phase in p.u. In figure 5. a and b the primary current, respectively the flux linkages and the magnetizing flux linkages are given versus time, for no-load connecting transient at the zero initial input voltage phase, in the case of circuit-field model.

The results confirm the worth of the circuit-field model, even the obtained results are quite spoiled by the equivalent magnetic circuit approximate definition.

References

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