COMMENTS ON SWITCHED RELUCTANCE MACHINE MATHEMATICAL MODEL

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ABSTRACT

In the paper the main analytical models of the switched reluctance (SR) machine are presented, based on geometry data, magnetic equivalent circuits and finite element (FEM) analysis results. In each case a representative example is given and finally the advantages and the weak points of each type of model are evinced.

INTRODUCTION

The switched reluctance machines (SRM) are receiving significant attention from the industries in the last decade thanks to its simple and robust construction, wide speed range capability, low rotor inertia, high starting torque and reduced costs [1]. Furthermore, since the SRM does not generate significant heat, it can be used with success in harsh operating environments, such as in automobiles. Nevertheless, the motion of the SRM is affected by the nonlinear characteristics of the magnetic materials, with high levels of saturation passing cyclically as the rotor continuously moves from unaligned to aligned positions with reference to energized stator phases. This obstacle requires first the calculation the flux linkages in the machine design phase, and secondly, mathematically representing them for analysis and control purposes. Therefore, the machine’s mathematical model is very important and it has to be accurate and easy to implement.

Obviously the numerical magnetic field calculation, usually via finite element method (FEM), gives very good results and allows a good description of a SRM operating process via look-up tables. Such a model, based on look-up tables, does not offer any possibility to analyze the machine behavior and, also it is quite difficult to implement it in an online control device requiring large amount of memory and quite long calculation time. Therefore, the analytical models, with different grades of complexity and accuracy, represent a viable alternative to the numerical models based on FEM results, and nowadays are widely used.

In the paper three main types of SRM analytical models are briefly presented and for each one a representative variant is discussed. Finally the advantages and the weak points of each model are evinced.

The SRM’s phase flux linkage variation function of current and rotor position represents the main part of all type of machine models. Many valuable works were published in the last years in this domain, as [2, 3, 4] which introduce analytical models based on geometry data, [5, 6] which develops models based on magnetic equivalent circuits, or [7, 8, 9] where the analytical model is created using FEM analysis results. Their final modeled function representations are quite accurate, the
The principal advantage of these models being that they allow an easier analysis of the machine, as they offer insight view into machine operating process and allow for control strategies formulating.

**MODEL OF SRM BASED ON GEOMETRY DATA**

The model proposed is a geometry-based simplified analytical model of SRM which has as inputs the machine geometry and the iron magnetic properties [2].

Usually the switched reluctance machines are modeled through functions of phase current and rotor position. To develop the geometry based simplified model, the flux linkage in aligned ($\lambda_a$) and unaligned ($\lambda_u$) position are first defined.

The aligned flux linkage is:

$$\lambda_a = \lambda_m (a_m i + b_m - \sqrt{b_m^2 - c_m i^2})$$

with

$$\lambda_m = (n_{ser} / n_{par}) \mu_0 (N_p / 2) l_{st} \cdot s_f \cdot (R_g / g) \cdot \beta_r$$

$$a_m = 1 + \frac{2g}{l_p}$$

$$b_m = \frac{n_{par} \cdot B_{sat}}{\mu N_p} \cdot [l_p + (\mu_r + 1)g]$$

$$c_m = \frac{2n_{par} \cdot B_{sat}}{\mu N_p} \cdot [l_p - (\mu_r - 1)g]$$

where: $N_p$ is the number of turns per pole coil, $n_{ser}$, $n_{par}$ are the winding number of series, respectively parallel paths, $l_{st}$ is the stack length, $s_f$ is the stacking factor, $R_g$ is the radius to rotor pole tips, $g$ is the air-gap length, $l_p$ is the total length of rotor and stator pole, $\beta_r$ is the rotor pole width, $B_{sat}$ is the flux density at saturation and $i$ is phase current.

The unaligned flux linkage is:

$$\lambda_u = L_u \cdot i$$

where $L_u$ is the unaligned phase inductance calculated from machine geometry [3].

The flux linkage in the intermediate position is stated as a function of $\lambda_a$, $\lambda_u$ and a position dependent function $f(\theta)$.

$$\lambda(i, \theta) = L_u i + (ai + b - c + \lambda_u^2 i^2) \cdot f(\theta)$$

where $a = \lambda_m a_m - L_u$, $b = \lambda_m b_m$, $c = \lambda_m^2 c_m$. 
where \( m \) is determined as in [2] and \( \theta \) is characterizing the rotor position.

The electromagnetic torque developed by one phase can be estimated from the flux linkage function as:

\[
T = \frac{\partial}{\partial \theta} \left( \int_0^\theta \lambda_i \, di \right)
\]

**MODELS BASED ON MAGNETIC FIELD ANALYSIS RESULTS**

The analytical model proposed is a model based on a Fourier series approximation of the flux linkage current characteristics [9]. The model is obtained by using three flux linkage current characteristics calculated via a two dimension finite element method (2D-FEM).

The phase flux linkage approximation is:

\[
\lambda(i, \theta) = iL(i, \theta)
\]

\[
\lambda(i, \theta) = \lambda_0 + \lambda_1 \cos(Q_r \theta) + \lambda_2 \cos(2Q_r \theta)
\]

where \( Q_r \) is the number of rotor poles and \( \lambda_0, \lambda_1, \lambda_2 \) are coefficients that can be derived as functions of the aligned \( \lambda_a \), averaged \( \lambda_{av} \), and unaligned \( \lambda_u \) positions of the rotor.

\[
\lambda_0 = \frac{1}{2} \left[ \frac{1}{2} (\lambda_a + \lambda_u) + \lambda_{av} \right]
\]

\[
\lambda_1 = \frac{1}{2} (\lambda_a - \lambda_u)
\]

\[
\lambda_2 = \frac{1}{2} \left[ \frac{1}{2} (\lambda_a + \lambda_u) - \lambda_{av} \right]
\]

The equation that describes the flux-linkage characteristic function of the current values is estimated via a curve fitting procedure by employing a ratio of polynomials:

\[
\lambda = \frac{i}{ai^2 + bi + c}
\]

where the coefficients \( a, b \) and \( c \) are specifically calculated via curve fitting procedure in the case of the three flux linkage characteristics considered.

The flux-linkage \( \lambda_u \) for the unaligned position can be described too as:

\[
\lambda_u = L_u i
\]
where $L_u$ is a constant and represents the phase inductance in unaligned position.

The SRM’s electromagnetic torque will be calculated using the following equation:

$$T_e = \int_0^i [-Q_u \lambda_1 \sin(Q, \theta) - 2Q_u \lambda_2 \sin(2Q, \theta)] di$$  \hspace{1cm} (13)

The model developed by Roux [7] is simple and very useful for fast sizing and initial valuations in computer-aided designs. The magnetization curves are approximated by three equations. One can be used to approximate the magnetization curve for the unaligned position, and the other two for the aligned position.

The unaligned position curve is approximated by a straight line as (12).

The curve for the aligned position is split in two parts. When the phase current $I < i_s$ the flux can be determined as:

$$\lambda_a(i) = L_a i$$  \hspace{1cm} (14)

where $L_a$ is the phase inductance in aligned position and $i_s$ is the saturated current.

For the saturated region ($I > i_s$) the flux curve is stated using the equation

$$\lambda_a(i) = \lambda_{s0} + \sqrt{4a(i - i_{s0})}$$  \hspace{1cm} (15)

where $a, \lambda_{s0}, i_{s0}$ are constants to be determined as in [8].

The final equation of the flux can be expressed as a function of $\lambda_{u}, \lambda_a$ and $\theta$ as:

$$\lambda(i, \theta) = \frac{1}{2} [\lambda_u(i) - \lambda_a(i)] \cdot [\cos(Q, \theta) + 1] + \lambda_a(i)$$  \hspace{1cm} (16)

The instantaneous torque developed by one phase can be determined as usual by using equation (6).

**MODELS USING MAGNETIC EQUIVALENT CIRCUIT ANALYSIS**

Lately there have been several attempts to produce accurate magnetic equivalent circuit models for SRM.

The model discussed here [1], is a simplified magnetic equivalent circuit (MEC) which take into account only the equivalent air-gap reluctance for one stator pole.

The flux produced by the coil mmf, $F_A$, through pole $A$ is:

$$\lambda_{A(\theta)} = \frac{F_A}{R_g + R_{gcl}}$$  \hspace{1cm} (17)
\[ R_{gd} = \frac{1}{P_o + P_1 \cdot \cos \gamma} \]  

(18)

where:

\[ R_{gcd} = \frac{1}{Q_s P_o - (P_o + P_1 \cdot \cos \gamma)} \]  

(19)

where \( P_o, P_1 \) - air-gap permeance coefficients and \( Q_s \) – the number of stator slots.

The flux produced by the coil \( K \) mmf, \( F_K \), through pole \( A \) is:

\[ \lambda_{A(K)} = \frac{F_K}{R_{gd}} \left[ 1 - \frac{R_{gK}}{R_{gd} + R_{gck}} \right] \]  

(20)

\[ R_{gK} = \frac{1}{P_o + P_1 \cdot \cos[\gamma - (K-1)2\beta]} \]  

(21)

where:

\[ R_{gck} = \frac{1}{Q_s P_o - (P_o + P_1 \cdot \cos[\gamma - (K-1)2\beta])} \]  

(22)

with the notations

\[ \gamma = Q_r \cdot \theta, \ \beta = \frac{2\pi}{Q_s} \]  

(23)

The magnetizing flux through pole \( K \) is computed in the same way as the magnetizing flux through pole \( A \).

The torque developed by the motor when the coils \( A \) and \( K \) are being fed is:

\[ T_{A(K)} = \frac{1}{2} \left( F_A \frac{\partial \lambda_A}{\partial \theta} + F_K \frac{\partial \lambda_K}{\partial \theta} \right) \]  

(24)

There are some other more sophisticated models, as the ones given in \([5, 6]\) but this one is very simple and, based on variable equivalent air-gap permeance concept \([1]\), offers good first evaluation of the machine behavior.

**CONCLUSIONS**

The current paper presents several flux linkage models for the SRM, each of them having errors, but accurate enough for the proposed task.

The analytical model based on geometry data is a simple one and it is easy to be implemented. The model can be used for dynamic simulation of the drive system and to verify the machine’s design.

Analytical models of SRM based on magnetic field analysis results are the most accurate one, but are simple and don’t require too any preliminary measurements on the motor. The models can be employed in the online control systems and their results are useful for sizing motors and converter components.
The results provided by models using magnetic equivalent circuit analysis have a level of accuracy dependent of the MEC complexity, but usually they give preliminary estimation of the SRM behavior.

REFERENCES


