On the Linear Stepper Motor Control Basics

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Abstract: **The linear stepper motor has many advantages such as a high speed, an accurate positioning capability and high servo stiffness. The dynamic performances and the positioning capabilities are improved by operating under a closed-loop control. In the paper, based on a theoretical approach, the control basics are developed. Some aspects of the control strategy to follow in order to obtain an optimum behavior are discussed and an intelligent controller is suggested.**

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ABSTRACT

The linear stepper motor has many advantages such as a high speed, an accurate positioning capability and a high servo stiffness. The dynamic performances and the positioning capabilities are improved by operating under a closed-loop control. In the paper, based on a theoretical approach, the control basics are developed. Some aspects of the control strategy to follow in order to obtain an optimum behavior are discussed and an intelligent controller is suggested.

1. INTRODUCTION

The linear stepper motor (LSM) operates under combined principles of the permanent magnet excitation, the variable air-gap reluctance and the step by step supplying of the control coils. The basic construction of the LSM is very simple. The LSM can operate in open-loop or in closed loop drive mode. In the open-loop drive mode the supplying control sequence is executed at a given frequency, existing a peril of losing the synchronism [5,6]. The control system must maintain, in certain limits, the prescribed motor speed with no dependence of load variation. The operating frequency is variable and depends on the LSM capability to achive an imposed displacement under given conditions as load and input source limits.

In the paper, under certain simplifying assumptions, the total tangential force is expressed in different cases of the supplying sequence. The simplifying assumptions, which do not affect basically the results, being usual, are done in order to make possible the analytical computations. The theoretical approach is developed on two LSM configurations, with two and respectively with four control coils. Also some different types of control voltage sequence are considered. The induced e.m.f. are computed in the control coils, excited or not. These e.m.f. prove to be useful in the closed-loop system, avoiding this way the use of other sensors. Based on the control strategy which follows from the theoretical approach an intelligent controller is suggested. It consist of a microcontroller and a data acquisition card.

2. LINEAR STEPPER MOTORS (LSM)

The LSM, shown in Fig.1 and Fig.2 in two slightly different variants, is in fact, a permanent-magnet excited, variable-reluctance synchronous motor.

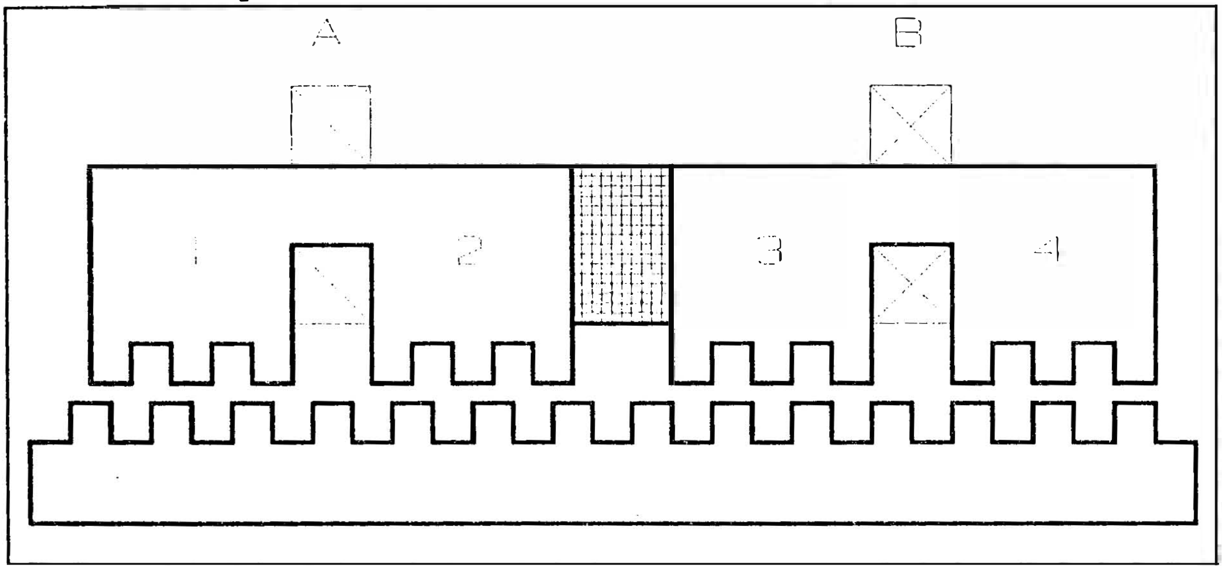

Figure 1

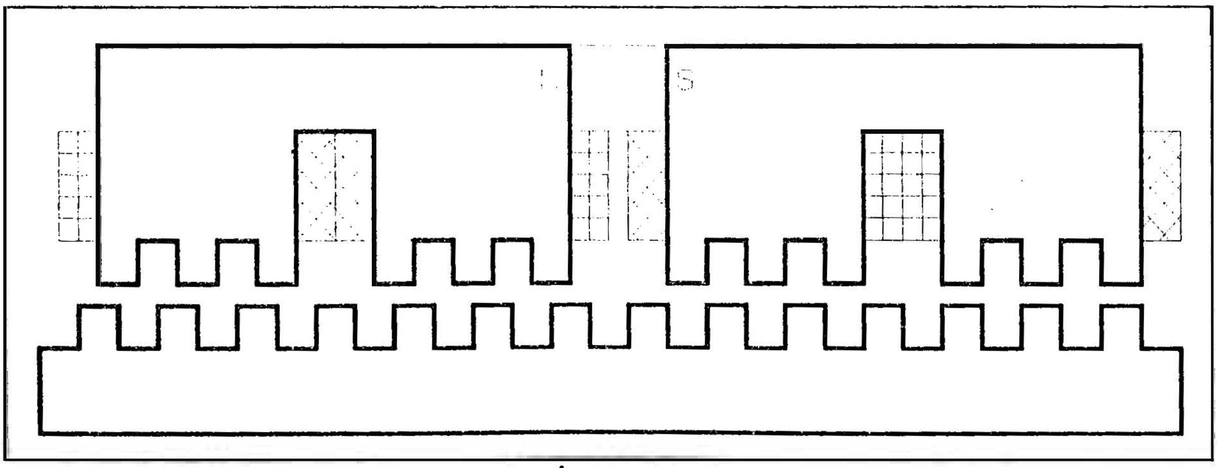

Figure 2

It has, basically, a movable armature (the mover) placed over a fixed one (the platen). The platen is an equidistantly toothed bar of any length fabricated from high-permeability cold-rolled steel. The mover consists of two electromagnets having command coils (two or four) and a permanent magnet between them, which is the excitation source. Each electromagnet has two poles; all poles having the same number of teeth. The tooth pitch is the same on both armatures (mover and platen) Each of the two poles of one electromagnet are displaced with respect to the platen slotting by half of a tooth-pitch. The right side electromagnet (see Figs. 1 or 2) is placed by multiple of a quarter of tooth-pitch with respect to the left side electromagnet.

The permanent-magnet flux passes through the mover iron core, air-gap and platen. By commuting the permanent-magnet flux in a way to concentrate it into a single pole of one electromagnet is resulting in a tangential force. It will tend to align the teeth of that mover's pole with the platen teeth in a manner as to minimize the air-gap magnetic reluctance. In order to displace the mover one step to the right from the initial position shown in Figs. 1 and 2 the command coils must be supplied in a way to assure the flux concentration into pole number four. It brings the flux under this pole to a maximum while under the other pole of the right side electromagnet (pole number three) the flux is reduced almost to zero. On the next step in the mover's displacement to the right the flux under the pole number two must be raised at its maximum value.

The above presented LSM construction is not the only one possible, but is the simplest one. The mathematical model and the theoretical approach are developed for this motor's variant and all the results will come along with this one also.

3. THEORETICAL APPROACH

The complex toothed configuration, the magnetic saturation of the iron core parts and the permanent magnet operating point change due to air-gap variable reluctance and control amperturns can be covered accurately only by a coupled circuit-field model. The block diagram of the circuit-field model, composed of three main submodels, is given in Fig.3.

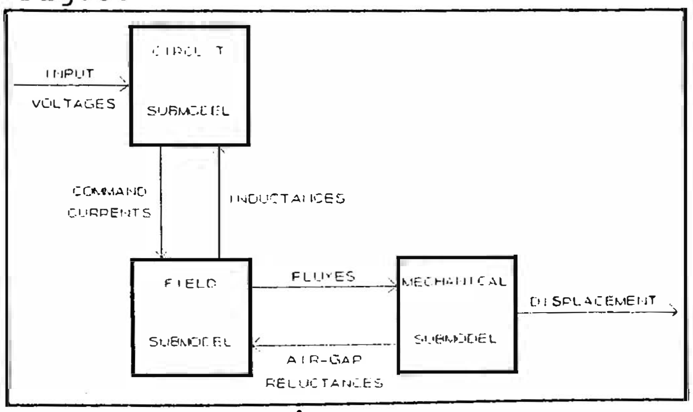

Figure 3

The circuit-field model has been introduced in previous papers [4,6] and it was not developed for an analytically solving purpose. This model, which takes fully into account all the nonlinearities, is based on an iterative procedure and can be solved only by means of a computer program. In order to obtain analytical results, which can offer important information for elaborating the control strategy all the calculus will lay basically on three assumptions:

- the air-gap reluctance are greater than all other reluctances, excepting of the permanent magnet;
- the permanent magnet reluctance is so great that no flux linkages produced by the currents which pass though the command coils will cross from one electromagnet to the other;

• the superposition principle can be applied because the core parts of the LSM are not affected by saturation and the permanent magnet operating point does not change.

The equivalent magnetic circuit of the LSM with four command coils (Fig.2) is given in Fig.4.

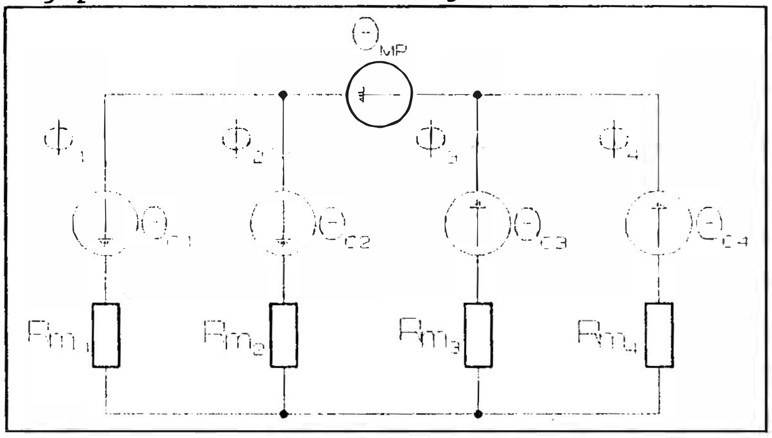

Figure 4

In fact there is no difference between the two variant of LSM given in Figs.1 and 2 as far as the magnetic circuit is concerned, excepting of the number of control amperturns.

The initial position of the mover, considered in the computational process, is that one given in Figs.1 and 2 and the displacement is to the right increasing thus the x-coordinate value. These conditions does not affect the results and offer the possibilities to explain more easier the motors's behavior and to simplify the mathematics.

As it was proved in a previous paper [5] the magnetic permeance of an electromagnet (one side of the magnetic circuit) is not dependent of the mover position,

$$P_{me} = \frac{\mu_o A}{Z\delta'}(2Z + \lambda - 1) \tag{1}$$

The equivalent variable air-gap under the j pole of the mover is, [5,6],

$$\delta_{ej(x)} = \frac{2Z\delta'}{(2Z + \lambda - 1)(1 + a\cos\alpha_j)} \tag{2}$$

with the constant a,

$$a = \frac{\lambda[1 + \lambda(2Z - 1)]}{2(2Z + \lambda - 1)} \tag{3}$$

and

$$\alpha = x\frac{2\pi}{t_d} \tag{4}$$

which means, for the booth variant of LSM,

$$\alpha_1 = \alpha,\ \alpha_2 = \alpha + \pi,\ \alpha_3 = \alpha + \frac{\pi}{2},\ \alpha_4 = \alpha - \frac{\pi}{2}\ . \tag{5}$$

The tangential force under one mover pole j is

$$F_{tj} = -\left(\frac{\delta W_{mj}}{\delta x}\right)|\Phi_j = ct. \quad . \tag{6}$$

which conducts to

$$F_{tj} = -\frac{\Phi_j^2}{2\mu_0 A}\frac{d}{dx}[\delta_{ej(x)}] \quad . \tag{7}$$

It means that the tangential force can be determined by knowing the flux through each pole of the mover. This flux is a sum of two fluxes, one produced by the permanent magnet and another produced by the command amperturns. The flux produced by the permanent magnet goes through all the poles and is, [4,5],

$$\Phi_{0j} = \frac{\Theta_{MP}P_{me}}{4}(1 + a\cdot\cos\alpha_j)\ ,\ j = 1\div 4\ . \tag{8}$$

The flux produced by command amperturns, supposing that not more than one command coil is excited once is, for the left side electromagnet, [5],

$$\Phi_{CA} = \frac{\Theta_{CA}P_{me}}{4}(1 + a\cdot\cos\alpha_1)(1 + a\cdot\cos\alpha_2)\ , \tag{9}$$

and for the right side electromagnet

$$\Phi_{CB} = \frac{\Theta_{CB}P_{me}}{4}(1 + a\cdot\cos\alpha_3)(1 + a\cdot\cos\alpha_4)\ , \tag{10}$$

where Θ_{CA} and Θ_{CB} are the command amperturns of one coil of the left side electromagnet (A) respectively of the right side (B).

Two cases will be considered, they being the most usual. In the first case the supplying control voltage will be sinusoidal and in the second case will be a square wave pulse sequence.

When the voltages are sinusoidal the frequencies are the same for all signals and the phases differ. It means that the command amperturn expressions are:

$$\begin{aligned}\Theta_{CA} &= \Theta_{CAM}\sin(\omega t + \gamma_A)\\ \Theta_{CB} &= \Theta_{CBM}\sin(\omega t + \gamma_B)\end{aligned} \tag{11}$$

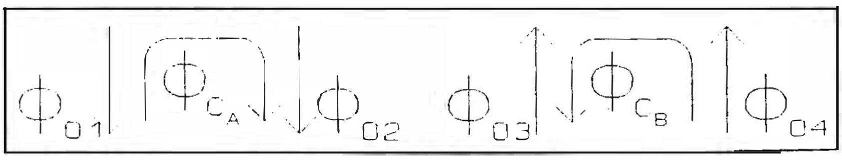

Figure 5

The resulting flux through the mover's poles is a sum of two fluxes, as in Fig.5, which conducts to:

$$\Phi_1=\Phi_{01}-\Phi_{CA}=\frac{\Theta_{MP}P_{me}}{4}(1+a\cdot\cos\alpha)[1-k_{iA}\sin(\omega t+\gamma_A)(1-a\cdot\cos\alpha)] \tag{12}$$

$$\Phi_2=\Phi_{02}+\Phi_{CA}=\frac{\Theta_{MP}P_{me}}{4}(1-a\cdot\cos\alpha)[1+k_{iA}\sin(\omega t+\gamma_A)(1+a\cdot\cos\alpha)] \tag{13}$$

$$\Phi_3=\Phi_{03}-\Phi_{CB}=\frac{\Theta_{MP}P_{me}}{4}(1-a\cdot\sin\alpha)[1-k_{iB}\sin(\omega t+\gamma_B)(1+a\cdot\sin\alpha)] \tag{14}$$

$$\Phi_4=\Phi_{04}+\Phi_{CB}=\frac{\Theta_{MP}P_{me}}{4}(1+a\cdot\sin\alpha)[1+k_{iB}\sin(\omega t+\gamma_B)(1-a\cdot\sin\alpha)] \tag{15}$$

where the m.m.f's factors k_{iA} and k_{iB} are:

$$k_{iA}=\frac{\Theta_{CAM}}{\Theta_{MP}}\;,\quad k_{iB}=\frac{\Theta_{CBM}}{\Theta_{MP}} \tag{16}$$

The tangential forces developed under the mover's poles have the expressions:

$$F_{t1}=-K_F\sin\alpha[1+k_{iA}\sin(\omega t+\gamma_A)(1-a\cdot\cos\alpha)]^2 \tag{17}$$

$$F_{t2}=K_F\sin\alpha[1-k_{iA}\sin(\omega t+\gamma_A)(1+a\cdot\cos\alpha)]^2 \tag{18}$$

$$F_{t3}=-K_F\cos\alpha[1-k_{iB}\sin(\omega t+\gamma_B)(1+a\cdot\sin\alpha)]^2 \tag{19}$$

$$F_{t4}=K_F\cos\alpha[1+k_{iB}\sin(\omega t+\gamma_B)(1-a\cdot\sin\alpha)]^2 \tag{20}$$

The resulting tangential force developed by the motor, which is given by the sum of the pole's tangential force, eq.(17) ÷ (20), is:

$$\begin{aligned}F_t=4K_F[&-k_{iA}\sin\alpha\sin(\omega t+\gamma_A)+k_{iB}\cos\alpha\sin(\omega t+\gamma_B)\\&-\frac{a}{2}\sin2\alpha[k_{iB}^2\sin^2(\omega t+\gamma_B)-k_{iA}^2\sin^2(\omega t+\gamma_A)]]\end{aligned} \tag{21}$$

with the tangential force coefficient K_F,

$$k_F = \left(\frac{\theta_{MP}}{4}\right)^2 P_{me} a \frac{2\pi}{t_d} \quad . \tag{22}$$

To come to the particular case presented in a previous paper, [6], where only coil B was supplied with a square wave pulse, the conditions are:

$$\sin(\omega t+\gamma_A) = 0 \ , \ \sin(\omega t+\gamma_B) = 1 \ , \tag{23}$$

and is resulting the same expression

$$F_t = 4K_F k_{iB}\cos\alpha\,(1 - a k_{iB}\sin\alpha) \tag{24}$$

With equation (21) the total tangential force can be computed only if, besides the command voltage frequency, phase and amplitude, the variation of the displacement α function of time is known. The ideal situation comes when sinα and sinωt has the same variation function of time. So if at the initial moment of time (t = 0), the A coil command voltage is zero as α and both, displacement and voltage are increasing, than the total unitary tangential force, f_t^*, is:

$$f_t^* = \frac{F_t^*}{4K_F} = k_{iB}\cos\alpha\sin(\alpha + \gamma_B) - k_{iA}\sin^2\alpha - \\ - \frac{a}{2}\sin 2\alpha\,[k_{iB}^2\sin^2(\alpha + \gamma_B) - k_{iA}^2\sin^2\alpha] \ . \tag{25}$$

Its variation function of α for three different values of γ_B (0; π/4; π/2) when $k_{iA} = k_{iB} = 1$ is given in Fig.6 in the case of the sample motor under consideration.

Let consider an usual case when

$$k_{iA} = k_{iB} = k_i \ , \ \gamma_B = \frac{\pi}{2} \ , \tag{26}$$

Now the total unitary tangential force is

$$f_{ti}^* = k_i\cos 2\alpha\,(1 - \frac{a}{2} k_i\sin 2\alpha) \tag{27}$$

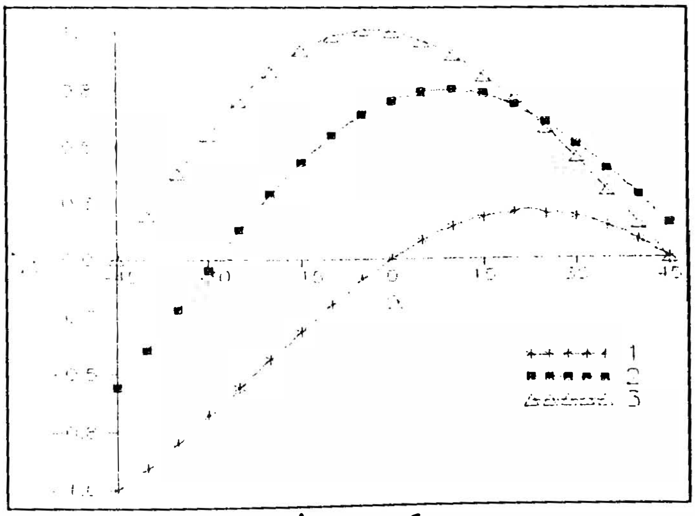

Figure 6

and its variation function of α for three different values of k_i, (1: 1.5: 2) is presented in Fig.7 (the same sample motor).

The unitary average tangential force f^*_{tim} is:

$$f^{*}_{tim} = \frac{2}{\pi} \int_{\alpha_0 - \frac{\pi}{4}}^{\alpha_0 + \frac{\pi}{4}} f^{*}_{ti} d\alpha \qquad (28)$$

which gives

$$f^{*}_{tim} = \frac{4}{\pi} k_i \cos 2\alpha_0 \qquad (29)$$

The maximum value of the unitary average tangential force is obtained at the optimum displacement,

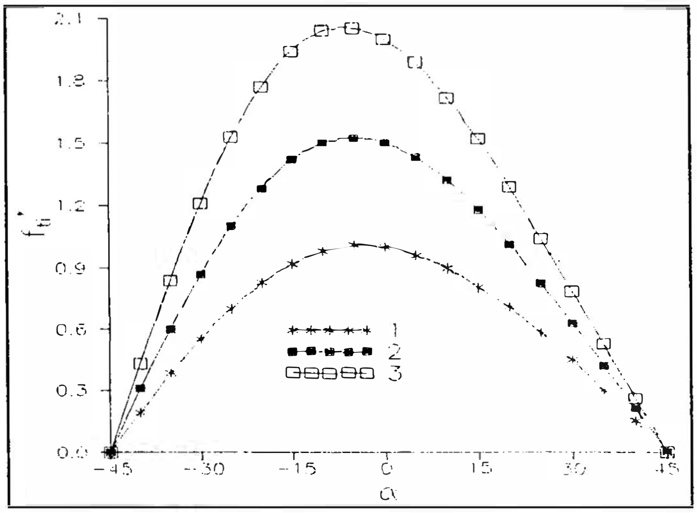

Figure 7

$$(\alpha_0)_{op} = 0 \qquad (30)$$

The maximum value of the unitary tangential force is obtained for the displacement which cheek the equation:

$$\frac{df^{*}_{ti}}{d\alpha} = 0 \qquad (31)$$

which leads to:

$$\sin 2\alpha + ak_i(1 - 2\sin^2 2\alpha) \qquad (32)$$

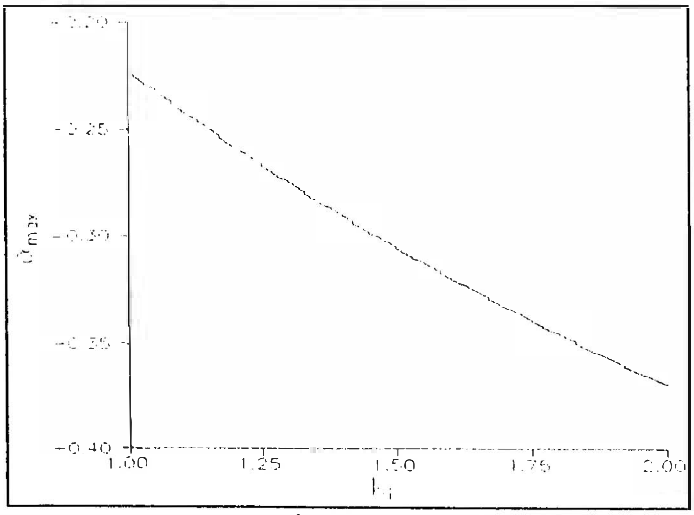

Figure 8

and the solution is:

$$\alpha_{\max} = a\sin\left(\frac{1 - \sqrt{1 + 8(ak_i)^2}}{4ak_i}\right) \qquad (33)$$

The variation of the displacement α_{max} function of k_i is given in Fig.8 for the same sample motor.

In the second case the command coils are supplied with a square wave pulse sequence. Let consider the LSM variant given in Fig.2 with four command coils ant the supplying sequence conducting to the flux's pattern presented in Fig.9. In order to obtain different cases the coefficients c_i will be introduced for the m.m.f. produced by command coils.

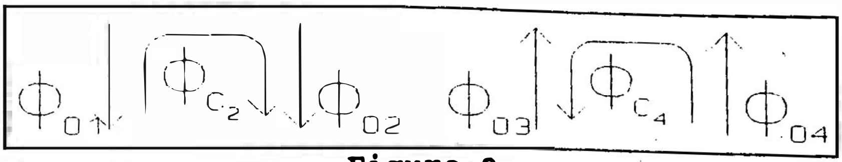

Figure 9

These coefficients can have two values, 1 and 0 that means that there is a voltage pulse on the considered coil, or there is not. Supplying the command coils 2 and 4 the m.m.f. developed are

$$\theta_{C2} = \theta_{C2M}C_2$$
$$\theta_{C4} = \theta_{C4M}C_4 \qquad (34)$$

and the total tangential force, obtained in the same manner as in previous case, is:

$$F_t=4K_F\left[k_{i2}C_2\sin\alpha+k_{i4}C_4\cos\alpha+\frac{a}{2}\sin2\alpha\,(k_{i2}^2C_2-k_{i4}^2C_4)\right] \qquad (35)$$

where the m.m.f's factors k_{i2} and k_{i4} are:

$$k_{i2} = \frac{\theta_{C2M}}{\theta_{MP}} \; ; \; k_{i4} = \frac{\theta_{C4M}}{\theta_{MP}} \qquad (36)$$

In Fig.10 the variation of the unitary total tangential force function of displacement α is given, when the m.m.f's factors k_{i2} and k_{i4} are equal, for three different combinations of C_2 and C_4 coefficients (the same sample motor).

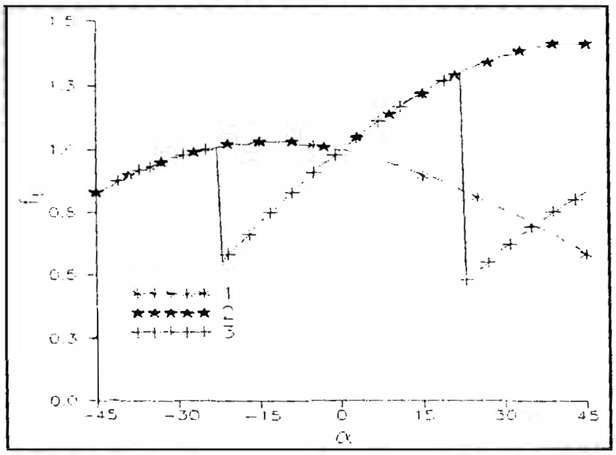

Figure 10

The LSM's variant with four command coils is a very interesting one because the supplying converter is quite uncomplicated. Each coil is independent, as in the usual SRM. In this case there is no need of sinusoidal voltage, but as it was considered, there is the possibility to have two coils supplied simultaneously. Because the control of the LSM's variant with two command coils has been discussed in some previous papers [5,8] here the control of the LSM's variant with four command coils will be detailed a little bit more.

4. MOTOR CONTROL

The control of the mover has to assure the imposed velocity profile and the positioning precision. Mover's speed is depending on the resulting tangential force and the control has to be in a certain relationship with mover position, acting also on the command coil currents. Let discuss these aspects a little bit deeper. As it was already proved [5] when the iron core is not affected by saturation the global air-gap permeance of one side LSM's electromagnet is constant. Also the command coil m.m.f. produces feeble or no-changes at all in permanent-magnet m.m.f. It means that with no saturation effects the permanent magnet operating point does not change and therefore the permanent-magnet m.m.f. can be considered constant. If the command coil currents have also an ideally square shape the tangential force depends mainly on the mover position. Therefore the control system has to assure a corresponding pulse pattern at every mover position in order to keep a certain value of the tangential force. With four independently supplied command coils this can be quite possible. In fact the control system will assure the change of excitation through the command coils at a certain value of the mover's displacement. Which is this optimum value of the displacement? As it was stated previously, [6], this optimum is that one that assure the maximum value of the average tangential force. In the case of the LSM variant with four command coils, when the total unitary tangential force is given by the equation (35) the unitary average tangential force is:

$$f_{tm} = \frac{2}{\pi} \int_{\alpha_0 - \frac{\pi}{4}}^{\alpha_0 + \frac{\pi}{4}} f_t d\alpha \,, \tag{36}$$

which gives:

$$f_{tm} = \frac{2}{\pi} [\sqrt{2}(C_4 k_{i4} \cos\alpha_0 - C_2 k_{i2} \sin\alpha_0) - \\ - \frac{a}{2} \sin 2\alpha_0 (C_2 k_{i2}^2 - C_4 k_{i4}^2)] \tag{37}$$

and its derivative function of α_0,

$$\frac{df_{tm}}{d\alpha_0} = -\frac{2}{\pi} [\sqrt{2}(C_4 k_{i4} \sin\alpha_0 + \\ + C_2 k_{i2} \cos\alpha_0) + a \cdot \cos 2\alpha_0 (C_2 k_{i2}^2 - C_4 k_{i4}^2)] \tag{38}$$

From the equation:

$$\frac{df_{tm}}{d\alpha_0} = 0 \,, \tag{39}$$

if $C_2 = 0$ and $C_4 = 1$, the optimum α_0 is

$$(\alpha_0)_{op} = asin\left[\sqrt{2}\,\frac{1 - \sqrt{1 + (2ak_{i4})^2}}{4ak_{i4}}\right] \tag{40}$$

and its variation function of k_{i4} is given on Fig.11 for the sample motor under consideration.

In order to detect the position of the mover the induced e.m.f. through the command coils can be of real help.

The induced e.m.f. in a command coil is

$$e_j = -N\frac{d\phi_j}{dt} \tag{41}$$

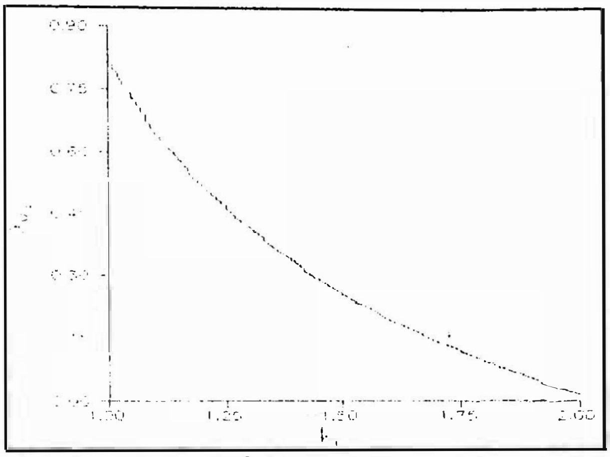

Figure 11

Let consider the flux's pattern given in Fig.9, and compute the induced e.m.f. through the first of all four command coils. It will come to:

$$e_1 = +N\cdot K_\phi\left[[a\cdot\sin\alpha\,(1+2ak_{i2}\cos\alpha)]\,\frac{d\alpha}{dt} + (1-a\cdot\cos\alpha)\,\frac{dk_{i2}}{dt}\right] \tag{42}$$

where the flux coefficient K_ϕ is:

$$K_\phi = \frac{1}{4}\,(\theta_{MP}\cdot P_{me}) \tag{43}$$

The induced e.m.f. through the command coil number two, e_2, is equal to the induced e.m.f. through the number one command coil and of oposite sense,

$$e_2 = -e_1 \tag{44}$$

On the left side electromagnet the induced e.m.f. through the command coils are:

$$e_3 = -e_4 = NK_\Phi \left[a \cdot \cos\alpha \, (1 - 2ak_{i4}C_4 \sin\alpha) \frac{d\alpha}{dt} + \right. \left. + C_4 (1 - a^2 \sin^2\alpha) \frac{dk_{i4}}{dt} \right] \tag{45}$$

If there is no command current through the coil number two, $C_2 = 0$, the induced e.m.f. through this coil is:

$$e_2 = -NK_\Phi a \cdot \sin\alpha \frac{d\alpha}{dt} \tag{46}$$

where

$$\frac{d\alpha}{dt} = \frac{2\pi}{t_d} \frac{dx}{dt} = \frac{2\pi}{t_d} v , \tag{47}$$

v being the mover speed.

The equations (42), (45) and (46) offer the possibility to calculate the mover's speed and displacement when the command currents are measured and the permanent-magnet m.m.f. is known.

The theoretical results obtained can provide reliable tools in controlling LSM of such construction. Considering that the control basics are already stated, even here is no aim to detail a control system, in Fig.12 such a system is suggested.

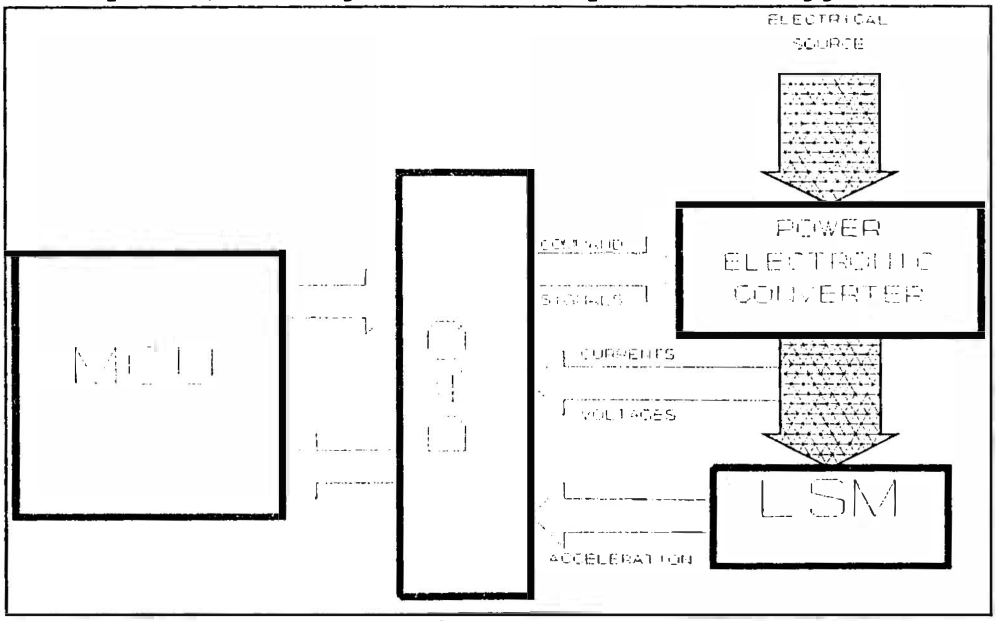

Figure 12

It consists of a microcontroller (MCU), a data acquisition card (DAQ), a power electronic converter and the needed sensors to measure currents and voltages on the command coils and the mover's acceleration.

The control strategy which can be developed on the obtained theoretical results, looks to come up freely and means commutation at the optimum value of the control angle and velocity control via coil's current.

LIST OF MAIN SYMBOLS

A - mover pole area [m^2]
a - motor constant
e_1, e_2, e_3, e_4 - induced e.m.f. [V]
F_t - resultant tangential force [N]
F_{tj} - tangential force (j=1÷4) [N]
F_{tm} - medium tangential force [N]
j - pole number
k_c - Carter's factor
k_F - tangential force coefficient
k_i - m.m.f.'s factor (Θ_c/Θ_{MP})
j - pole number
N - coil turns
P_{me} - equivalent magnetic permeance [H]
R - control coil resistance [Ω]
t - time [s]
t_d - mover's tooth pitch [m]
v - mover's velocity [m/s]
Z - mover's pole teeth number
x - horizontal coordinate (displacement) [m]
W_{mj} - magnetic energy under pole *j* (j=1÷4) [Ws]
α_j - angular displacement (j=1÷4) [rad]
α_0 - optimum angular displacement [rad]
δ - motor air-gap [m]
δ' - equivalent air-gap $\delta'=k_c\delta$ [m]
δ_e - equivalent variable air-gap [m]
λ - coefficient of equivalent air-gap permeance
Φ_{CA}, Φ_j, Φ_{01}, Φ_{C2} - flux linkages [Wb]
μ_0 - magnetic constant ($\mu_0=4\pi/10^7$) [H/m]
R_m - magnetic reluctance [1/H]

APPENDIX

The geometrical dimensions and parameters of LSM under consideration:

- tooth width	1 mm
- slot width	1 mm
- tooth pitch (t_d)	2 mm
- nr. of teeth per pole (Z)	5
- airgap (δ)	0.1 mm
- permanent magnet type	VACOMAX-145
- residual flux density	0.9 T
- coercive force	650 KA/m.
- nr. of coil turns	200
- coil resistance	3.5 Ω
- pole area	760 mm^2
- motor width	40 mm

- coefficient of equivalent air-gap permeance (λ) 0.672
- motor's constant (a) 0.244

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