The Harris Corner Detector

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In this report the derivation of the Harris corner detector [1] is presented. The Harris corner detector is a popular interest point detector due to its strong invariance to [3]: rotation, scale, illumination variation and image noise. The Harris corner detector is based on the local auto-correlation function of a signal; where the local auto-correlation function measures the local changes of the signal with patches shifted by a small amount in different directions. A discrete predecessor of the Harris detector was presented by Moravec [2]; where the discreteness refers to the shifting of the patches.

Given a shift $(\triangle x, \triangle y)$ and a point (x, y), the auto-correlation function is defined as,

$$c(x,y) = \sum_W [I(x_i, y_i) - I(x_i + \triangle x, y_i + \triangle y)]^2 \tag{1}$$

where $I(\cdot,\cdot)$ denotes the image function and (x_i, y_i) are the points in the window W (Gaussian[1]) centered on (x, y).

The shifted image is approximated by a Taylor expansion truncated to the first order terms,

$$I(x_i + \triangle x, y_i + \triangle y) \approx I(x_i, y_i) + [I_x(x_i, y_i) \; I_y(x_i, y_i)] \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix} \tag{2}$$

where $I_x(\cdot,\cdot)$ and $I_y(\cdot,\cdot)$ denote the partial derivatives in x and y, respectively.

[1]For clarity in exposition the Gaussian weighting factor $e^{-(x^2+y^2)/(2\sigma^2)}$ has been omitted from the derivation.

Substituting approximation Eq. (2) into Eq. (1) yields,

$$c(x,y) = \sum_W [I(x_i, y_i) - I(x_i + \triangle x, y_i + \triangle y)]^2 \tag{3}$$

$$= \sum_W \left(I(x_i, y_i) - I(x_i, y_i) - [I_x(x_i, y_i) \; I_y(x_i, y_i)] \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix} \right)^2 \tag{4}$$

$$= \sum_W \left(-[I_x(x_i, y_i) \; I_y(x_i, y_i)] \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix} \right)^2 \tag{5}$$

$$= \sum_W \left([I_x(x_i, y_i) \; I_y(x_i, y_i)] \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix} \right)^2 \tag{6}$$

$$= [\triangle x \; \triangle y] \begin{bmatrix} \sum_W (I_x(x_i, y_i))^2 & \sum_W I_x(x_i, y_i) I_y(x_i, y_i) \\ \sum_W I_x(x_i, y_i) I_y(x_i, y_i) & \sum_W (I_y(x_i, y_i))^2 \end{bmatrix} \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix} \tag{7}$$

$$= [\triangle x \; \triangle y] C(x,y) \begin{bmatrix} \triangle x \\ \triangle y \end{bmatrix} \tag{8}$$

where matrix $C(x,y)$ captures the intensity structure of the local neighborhood. Let λ_1, λ_2 be the eigenvalues of matrix $C(x,y)$. The eigenvalues form a rotationally invariant description. There are three cases to be considered:

1. If both λ_1, λ_2 are small, so that the local auto-correlation function is flat (i.e., little change in $c(x,y)$ in any direction), the windowed image region is of approximately constant intensity.

2. If one eigenvalue is high and the other low, so the the local auto-correlation function is ridge shaped, then only local shifts in one direction (along the ridge) cause little change in $c(x,y)$ and significant change in the orthogonal direction; this indicates an edge.

3. If both eigenvalues are high, so the local auto-correlation function is sharply peaked, then shifts in any direction will result in a significant increase; this indicates a corner.

References

[1] C. Harris and M.J. Stephens. A combined corner and edge detector. In *Alvey Vision Conference*, pages 147–152, 1988.

[2] H. Moravec. Obstacle avoidance and navigation in the real world by a seeing robot rover. Technical Report CMU-RI-TR-3, Carnegie-Mellon University, Robotics Institute, 1980.

[3] C. Schmid, R. Mohr, and C. Bauckhage. Evaluation of interest point detectors. *International Journal of Computer Vision*, 37(2):151–172, June 2000.