

**Metode Numerice  
2017-2018**

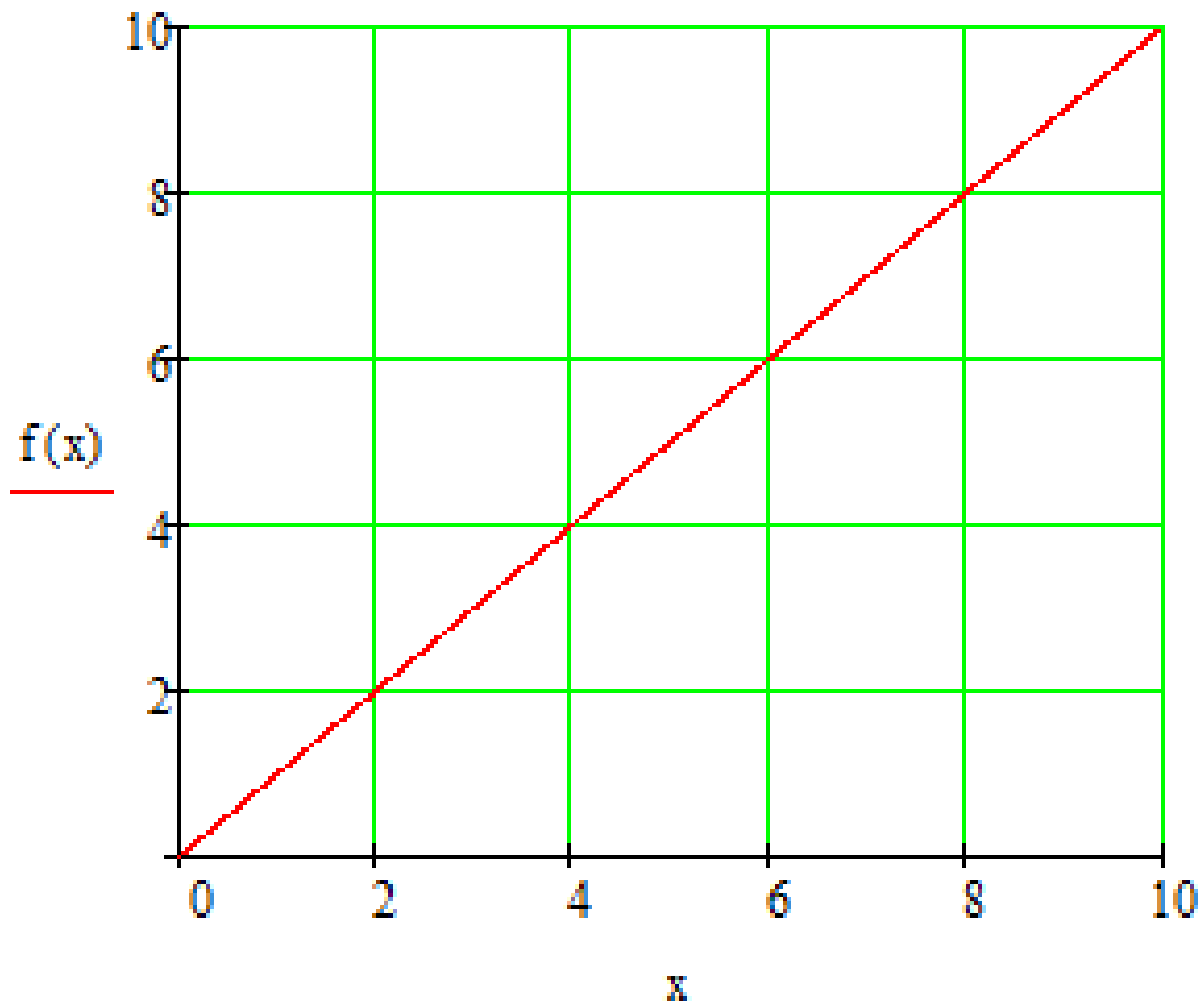
# **Metode numerice de aproximare a funcțiilor**

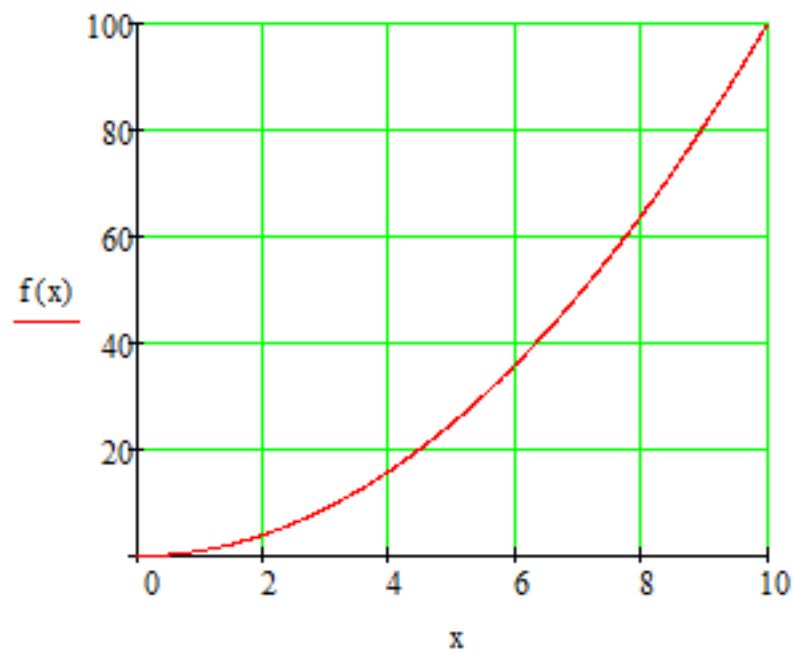
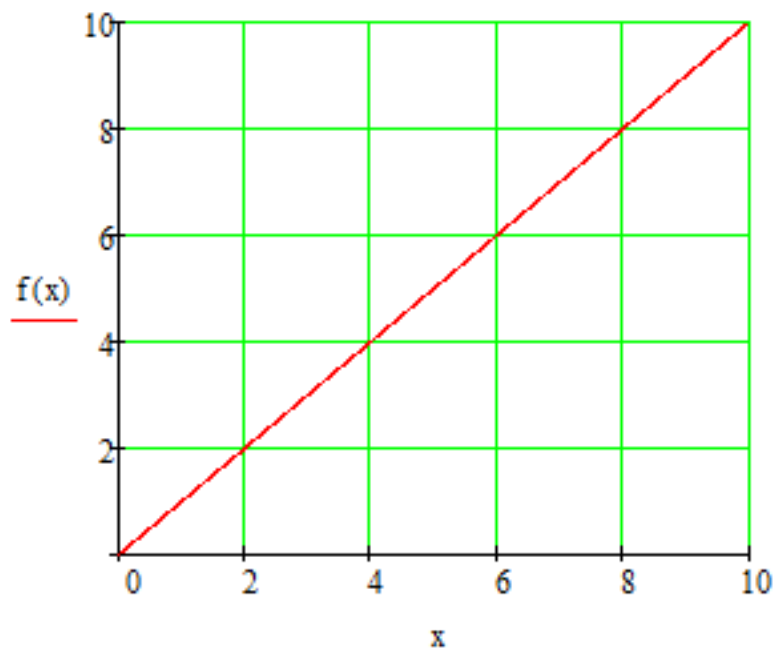
**Noțiuni de aproximare analitică**

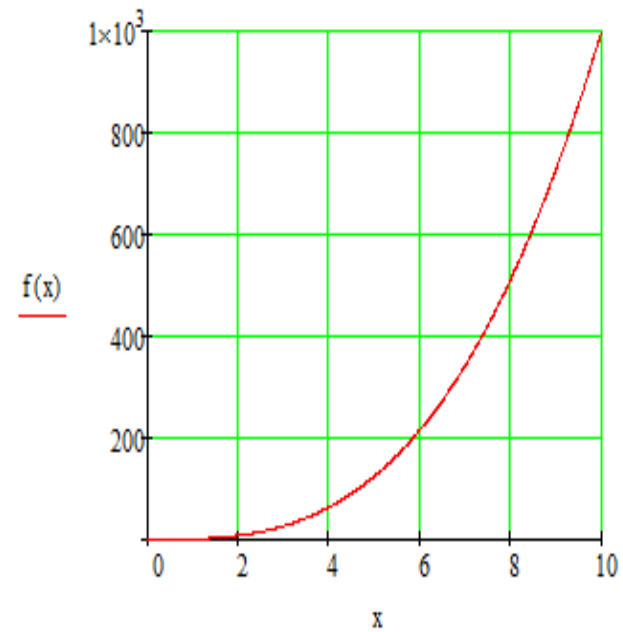
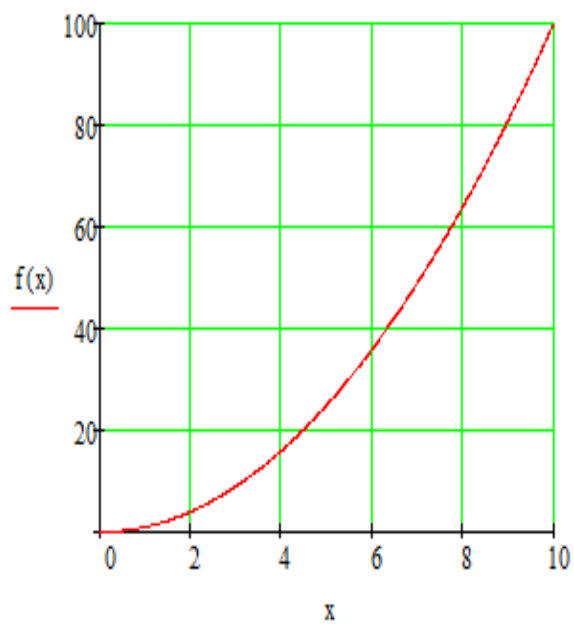
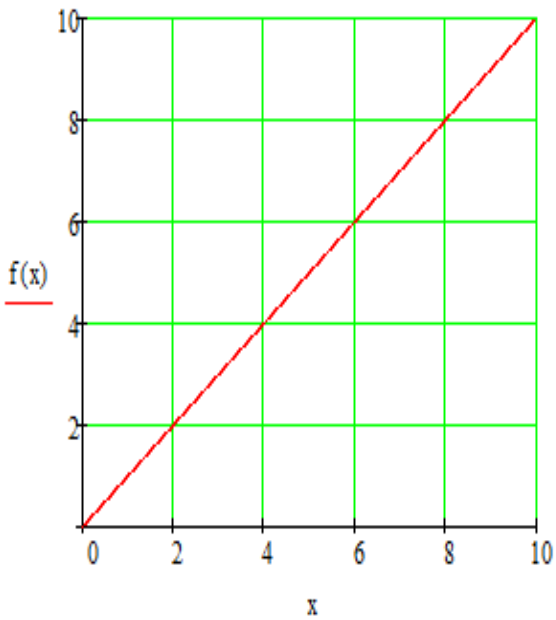
**Prof. Dr. Ing. Dan MICU**

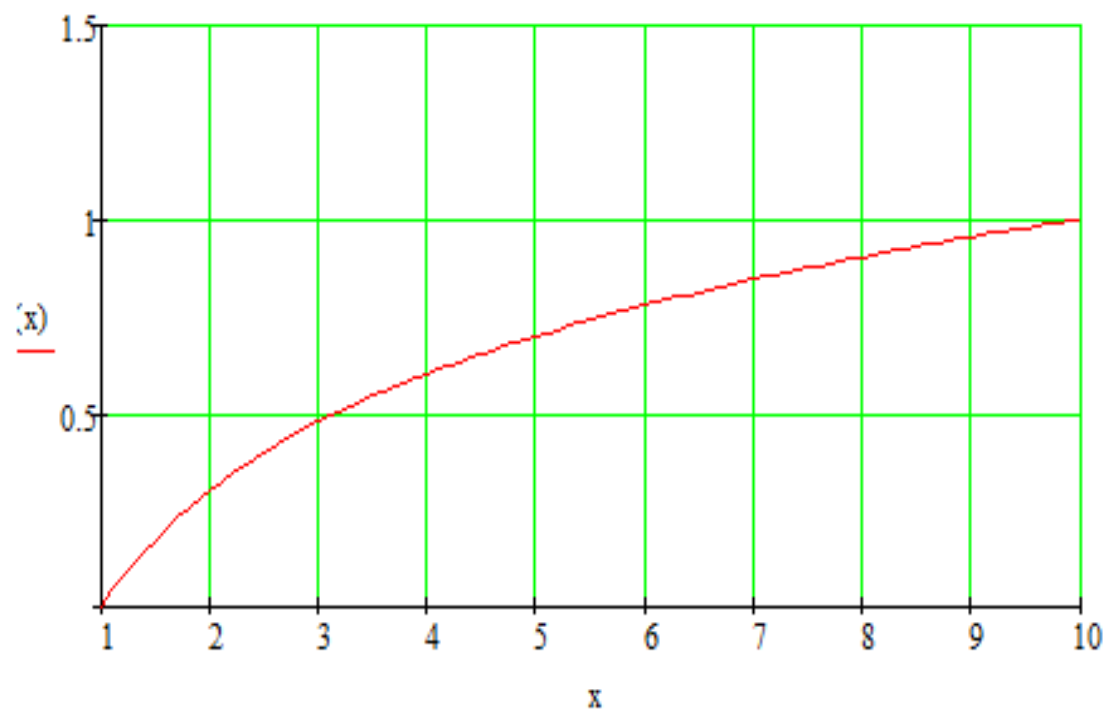
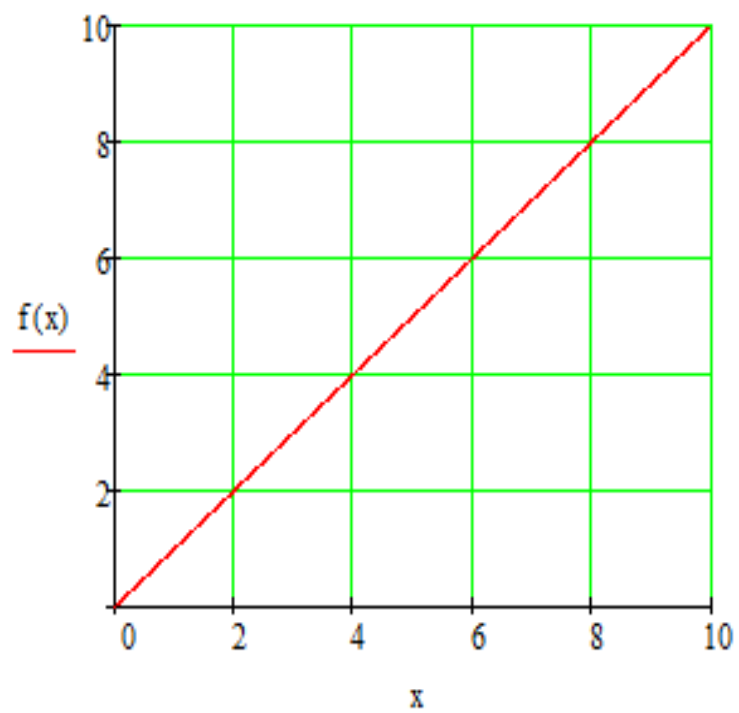
**As. Dr. Ing. Andrei CECLAN**

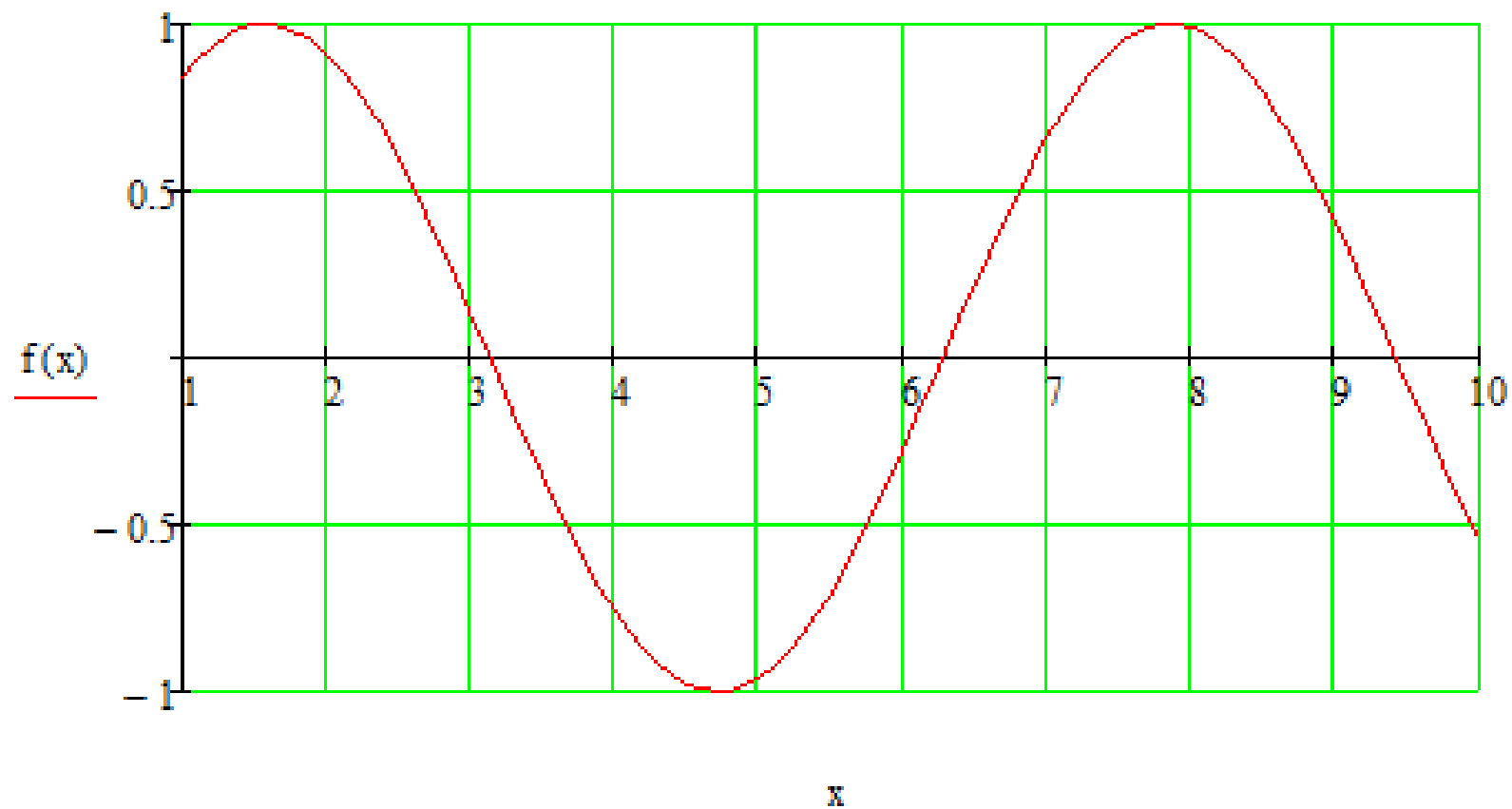
**Director - Laborator de Cercetare în Metode Numerice  
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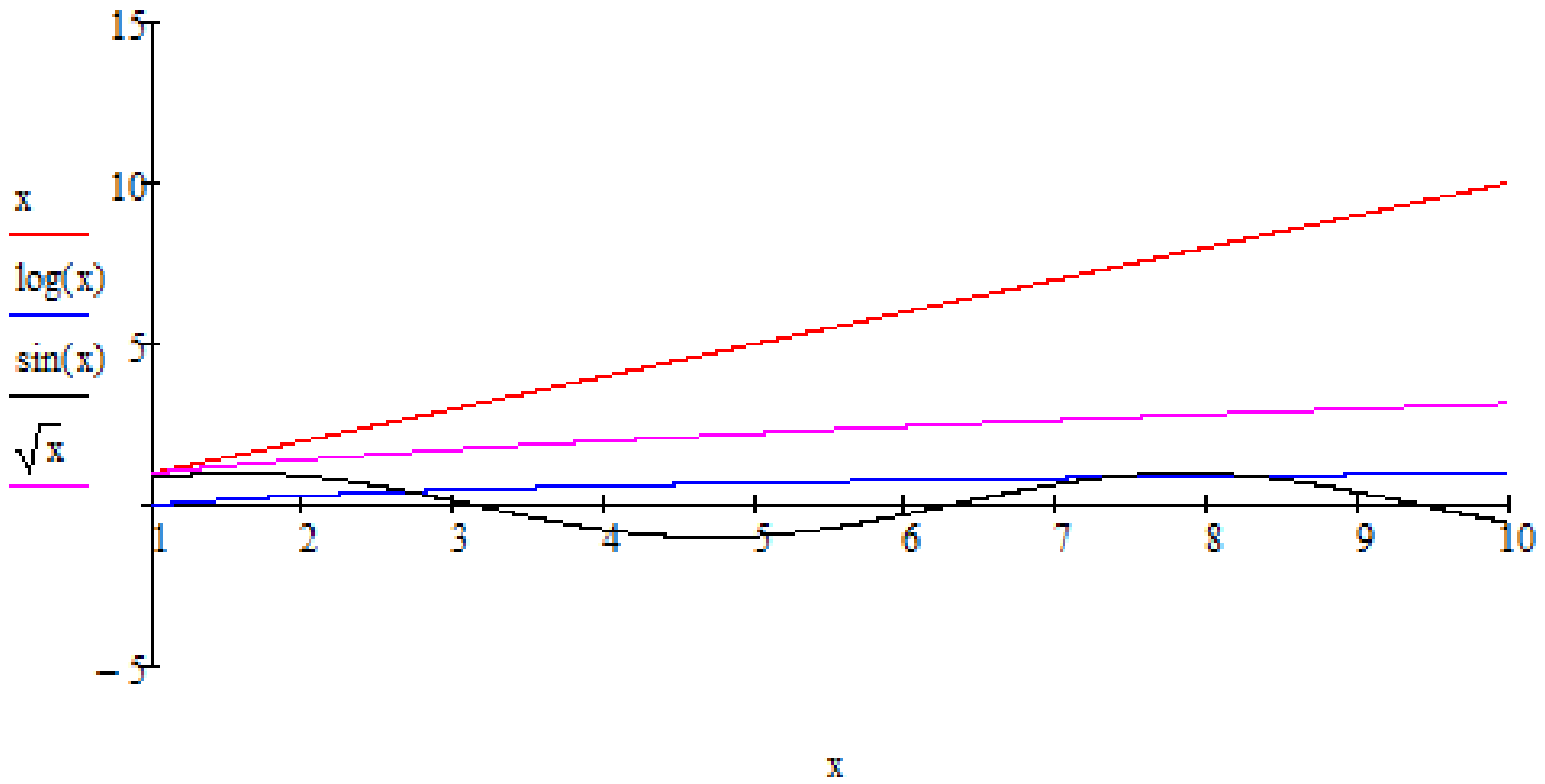












# Cuprins

Seria Taylor – instrument matematic - **Enel**

Abaterăa celor mai mici pătrate - **Lidl**

Seria Fourier – analiza armonică - **Petrom**

# Seria Taylor – instrument matematic

Teorema lui Taylor

$$f(x) = \sum_{k=0}^n \frac{(x-x_0)^k}{k!} \cdot f(x_0)^{(k)} = f(x_0) + \frac{x-x_0}{1!} \cdot f(x_0)^{(1)} + \frac{(x-x_0)^2}{2!} \cdot f(x_0)^{(2)} + \frac{(x-x_0)^3}{3!} \cdot f(x_0)^{(3)} + \dots$$

Nucleu din care se pot deduce metodele numerice!



# Caz numeric

$$f(x) := e^x \quad \backslash\text{ functia test;}$$

$$x := 0, 0.1..4.5 \quad F(x, n) := \frac{d^n}{dx^n} f(x)$$

$$x_0 := 1 \quad \backslash\text{ punctul in jurul caruia se realizeaza aproximarea;}$$

$$T_1(x) := f(x_0) + \frac{(x - x_0)}{1!} \cdot F(x_0, 1) \quad \backslash\text{ aproximarea in serie Taylor de gradul 1;}$$

$$T_2(x) := f(x_0) + \frac{(x - x_0)}{1!} \cdot F(x_0, 1) + \frac{(x - x_0)^2}{2!} \cdot F(x_0, 2) \quad \backslash\text{ aproximarea in serie Taylor de gradul}$$

$$T_3(x) := f(x_0) + \frac{(x - x_0)}{1!} \cdot F(x_0, 1) + \frac{(x - x_0)^2}{2!} \cdot F(x_0, 2) + \frac{(x - x_0)^3}{3!} \cdot F(x_0, 3)$$

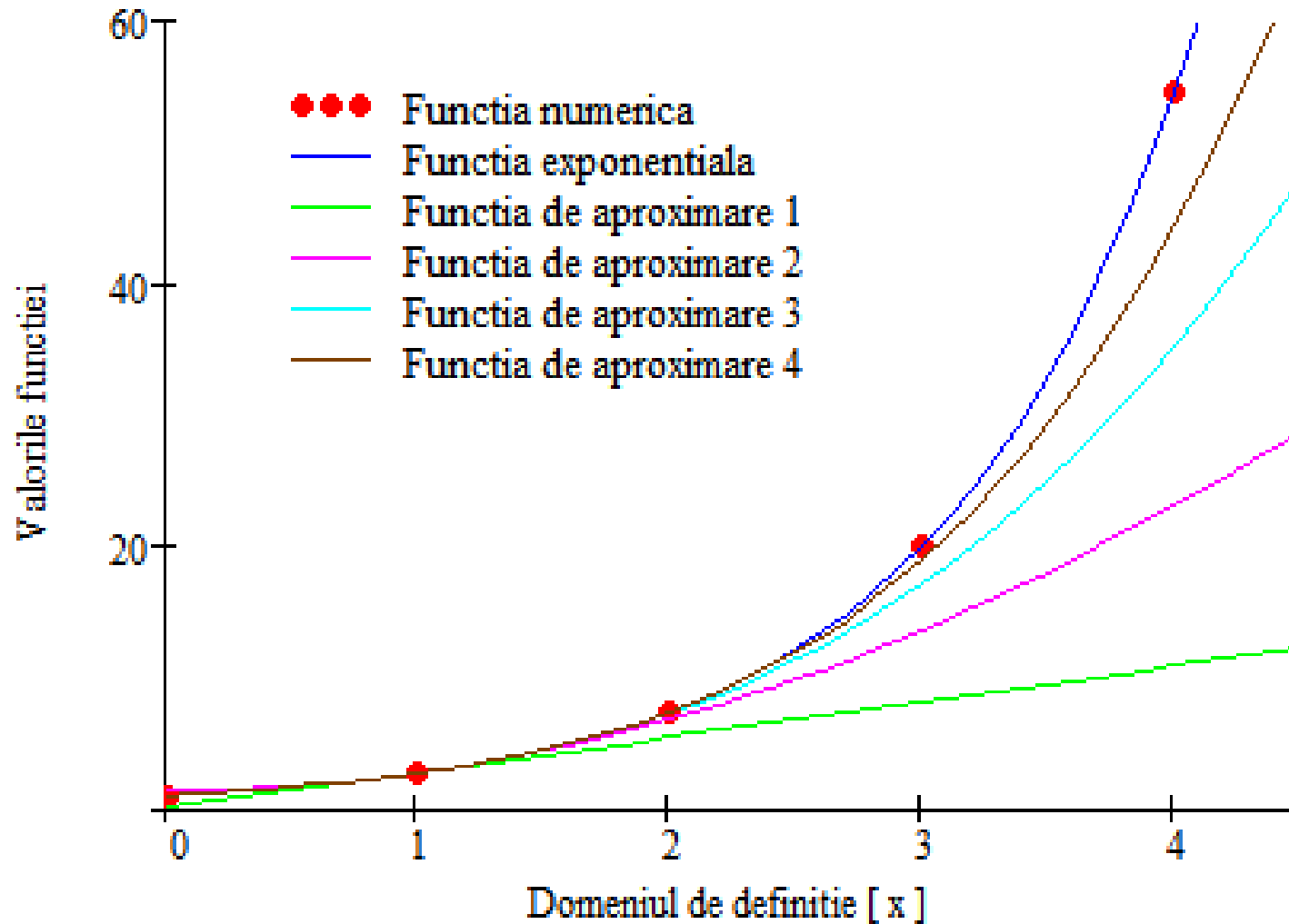
$\backslash\text{ aproximarea in serie Taylor de gradul}$

$$T_4(x) := f(x_0) + \frac{(x - x_0)}{1!} \cdot F(x_0, 1) + \frac{(x - x_0)^2}{2!} \cdot F(x_0, 2) + \frac{(x - x_0)^3}{3!} \cdot F(x_0, 3) + \frac{(x - x_0)^4}{4!} \cdot F(x_0, 4)$$

$\backslash\text{ aproximarea in serie Taylor de gradul}$

$$X := \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \quad Y := f(X) \quad Y = \begin{pmatrix} 1 \\ 2.718 \\ 7.389 \\ 20.086 \\ 54.598 \end{pmatrix} \quad \backslash\text{ valorile functiei numerice;}$$

# Rezultate numerice



# Aplicații - exemple



enel

# Aplicații - exemple

Discriminarea între curenții interni de defect și curenții de magnetizare într-un transformator;

Aplicație: implementare curbă analitică în relele digitale de protecție;





# Conceptul de aproximare

An important area in approximation is the problem of fitting a curve to experimental data.

Since the data is experimental, we must assume that it is polluted with some degree of error – most commonly measurement error – so we do not necessarily want to construct a curve that goes through every data point.

Rather, we want to construct a function that represents “the sense of the data” and is, in some sense, a close approximation to the data.

# Abaterea celor mai mici pătrate

$$y_i \quad g(x)$$

$$FO = \sum_{i=0}^n (y_i - g(x_i))^2$$

## Funcțională de abatere medie pătratică

$$g(x) = \sum_{k=0}^m a_k \cdot g_k(x)$$

$$\frac{\partial FO}{\partial a_k} = \frac{\partial}{\partial a_k} \sum_{i=0}^n \left( y_i - \sum_{k=0}^m a_k \cdot g_k(x_i) \right)^2 \quad k=0..m$$



In cazul particular in care functiile  $g_k(x)$  sunt un set de functii independente de forma:

$$g_k(x) = x^{k-1}$$

atunci relatiile de derivare partiala capata forma particulara:

$$\frac{\delta}{\delta a_k} \left[ \sum_{i=0}^n \left( y_i - a_0 - a_1 \cdot x_i - a_2 \cdot x_i^2 - a_3 \cdot x_i^3 \dots a_m \cdot x_i^m \right)^2 \right] = 0$$

Astfel in cazul particular al aproximarii cu o functie liniara de gradul 1, se obtine sis

$$n \cdot a_0 + \left( \sum_{i=0}^n x_i \right) \cdot a_1 = \sum_{i=0}^n y_i$$

$$\left( \sum_{i=0}^n x_i \right) \cdot a_0 + \left( \sum_{i=0}^n x_i^2 \right) \cdot a_1 = \sum_{i=0}^n y_i \cdot x_i$$

Just an Example

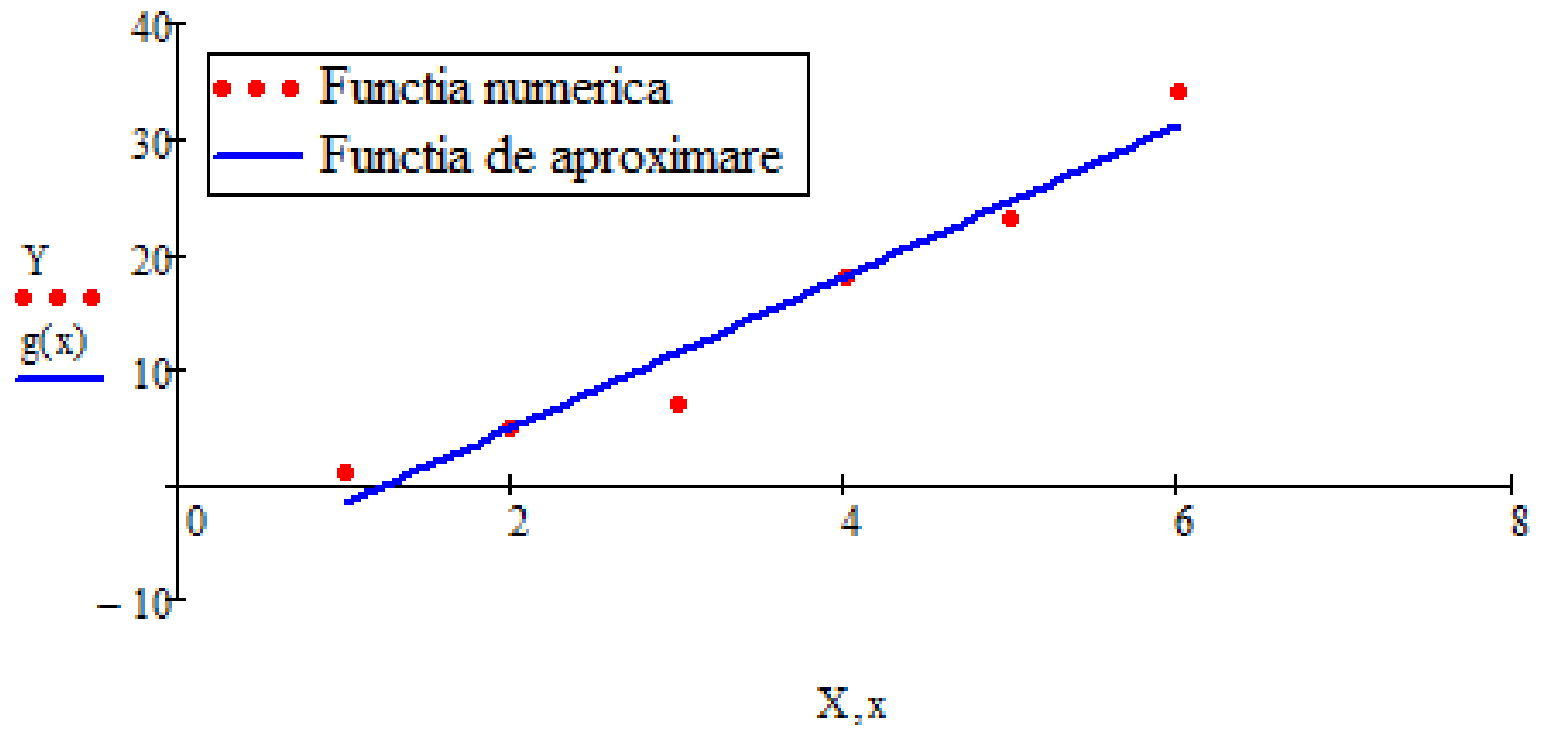
$$X := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix}$$

$$Y := \begin{pmatrix} 1 \\ 5 \\ 7 \\ 18 \\ 23 \\ 34 \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} := \text{line}(X, Y)$$

$$g(x) := a_0 + a_1 \cdot x$$

$$x := 1, 1.1, \dots, 6$$



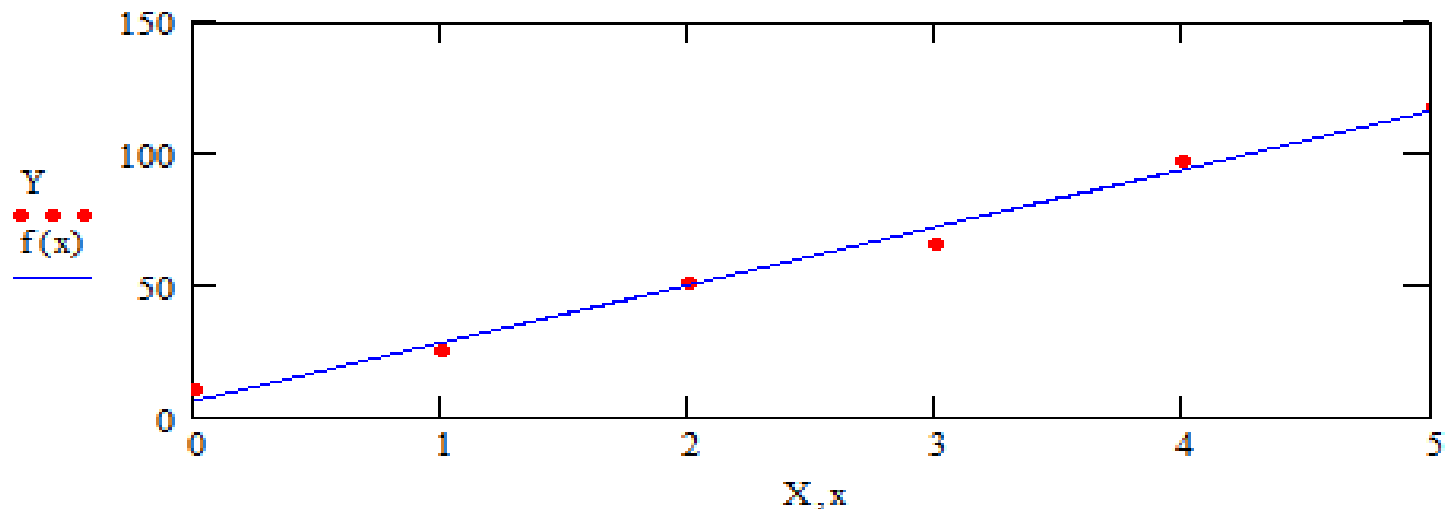
# Caz numeric

just an  
**Example**

$$\mathbf{X} := \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \quad \mathbf{Y} := \begin{pmatrix} 10 \\ 25 \\ 51 \\ 66 \\ 97 \\ 118 \end{pmatrix} \quad \backslash \text{functia numerica;}$$

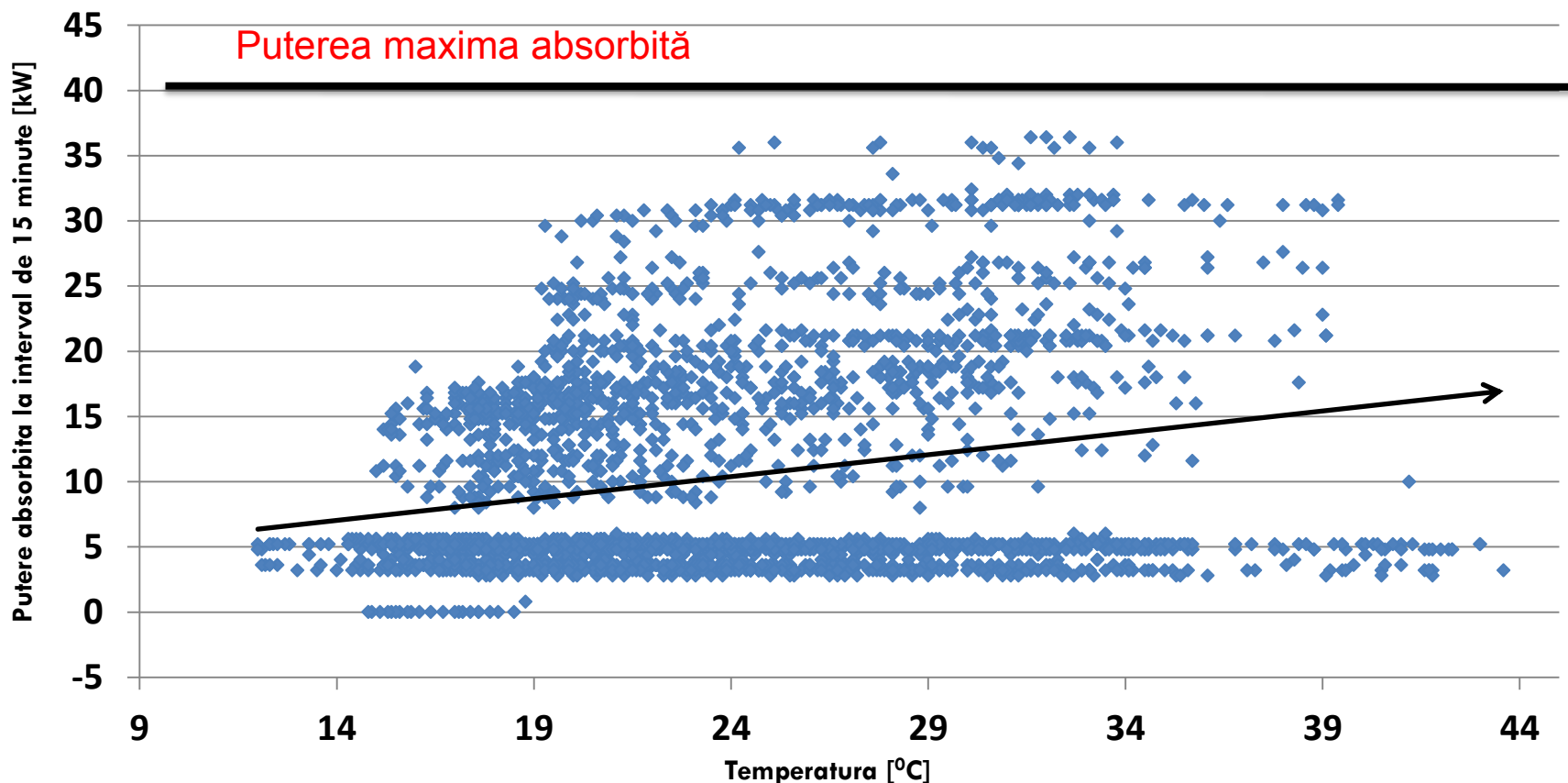
$$f(x) := 22.02 \cdot x + 6.09 \quad \backslash \text{functia de aproximare;}$$

$$x := 0, 0.01 \dots 5 \quad \backslash \text{intervalul de definitie al functiei de aproximare;}$$





## Corelare temperatura exterioara - putere absorbita chiller – Iulie 2016

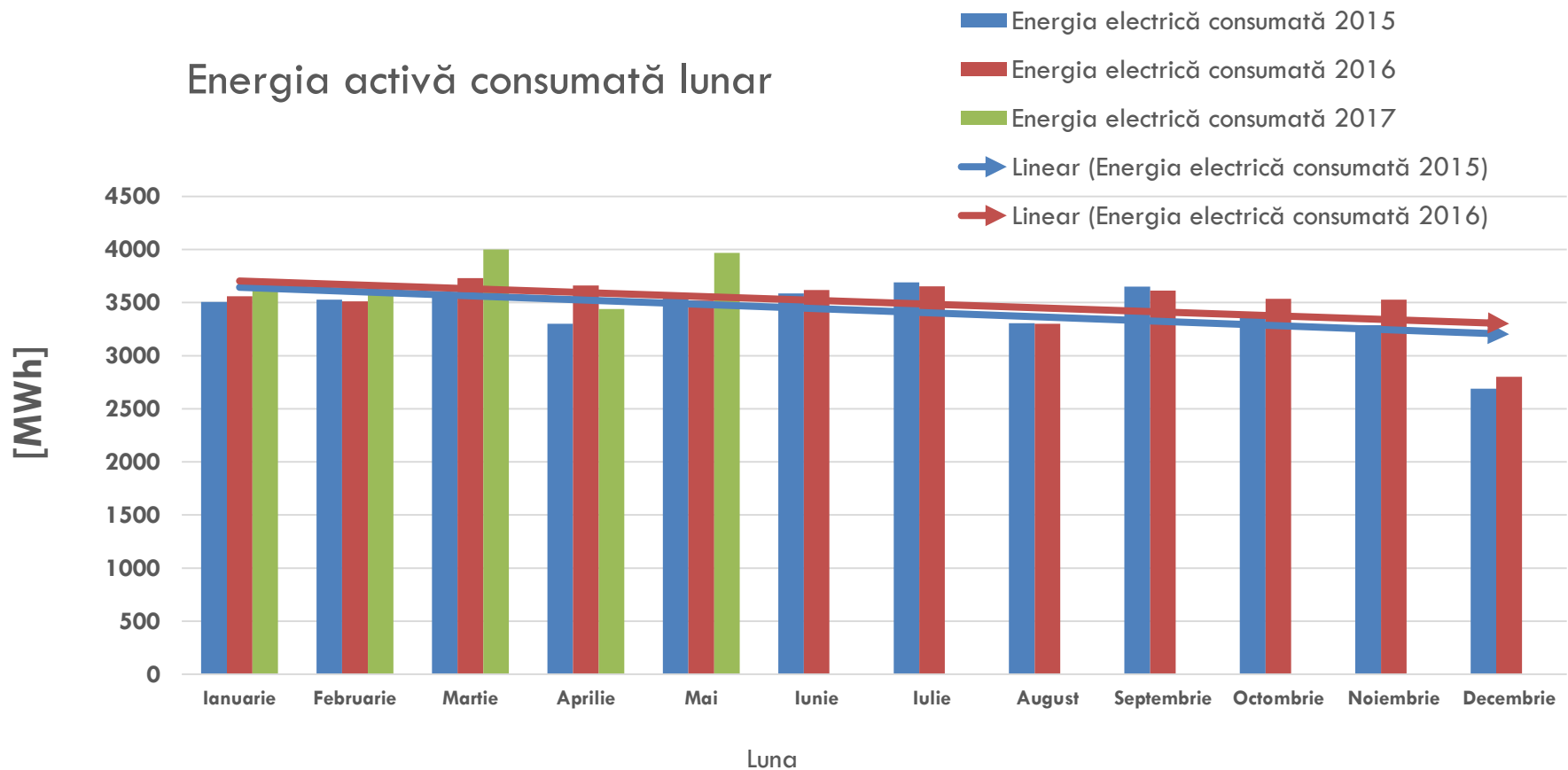


Regimul de funcționare nu este corelat cu temperatura ambientală. În schimb este evident regimul de recirculare a agentului de răcire, unde avem o putere absorbită de 5 kW.

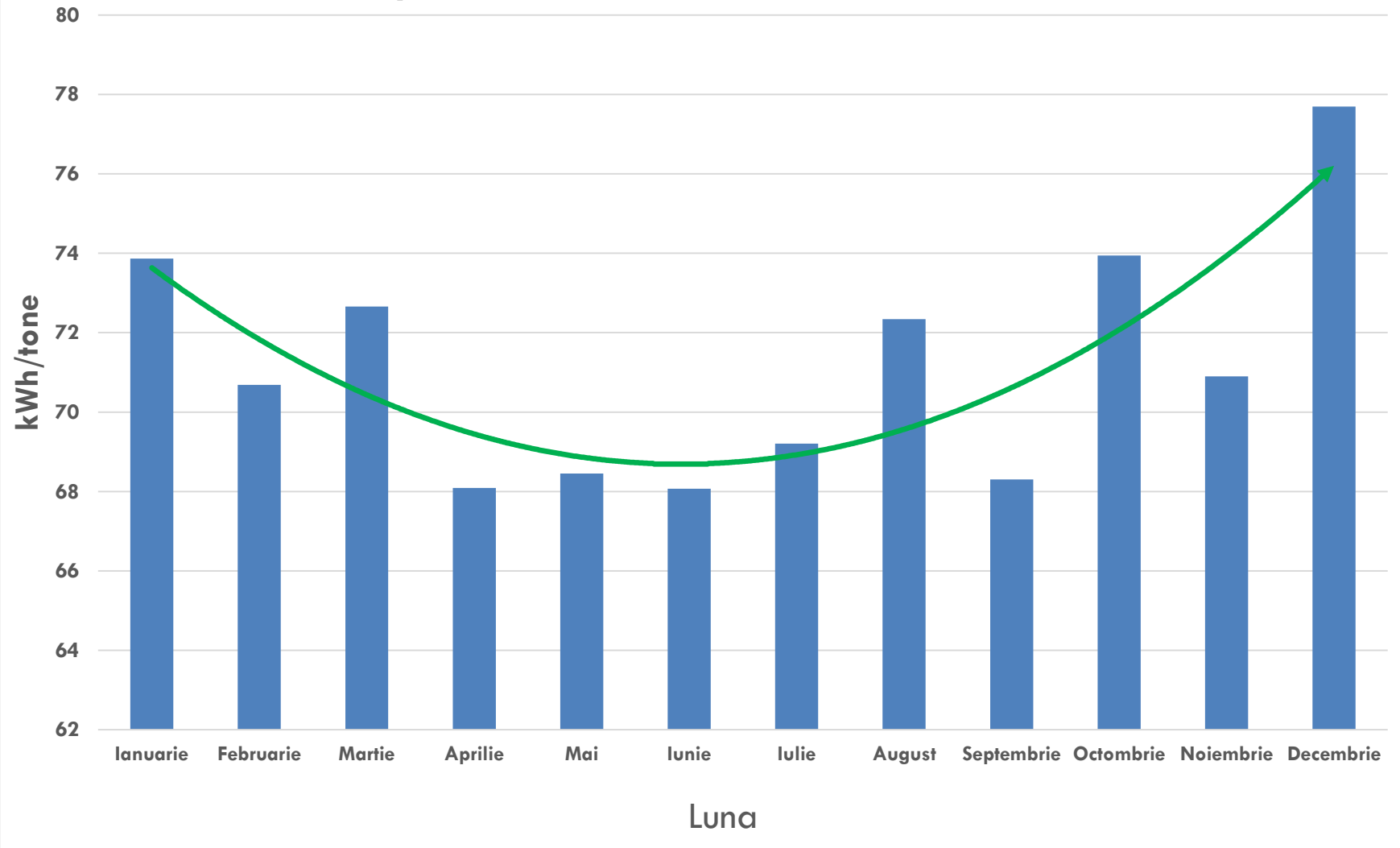




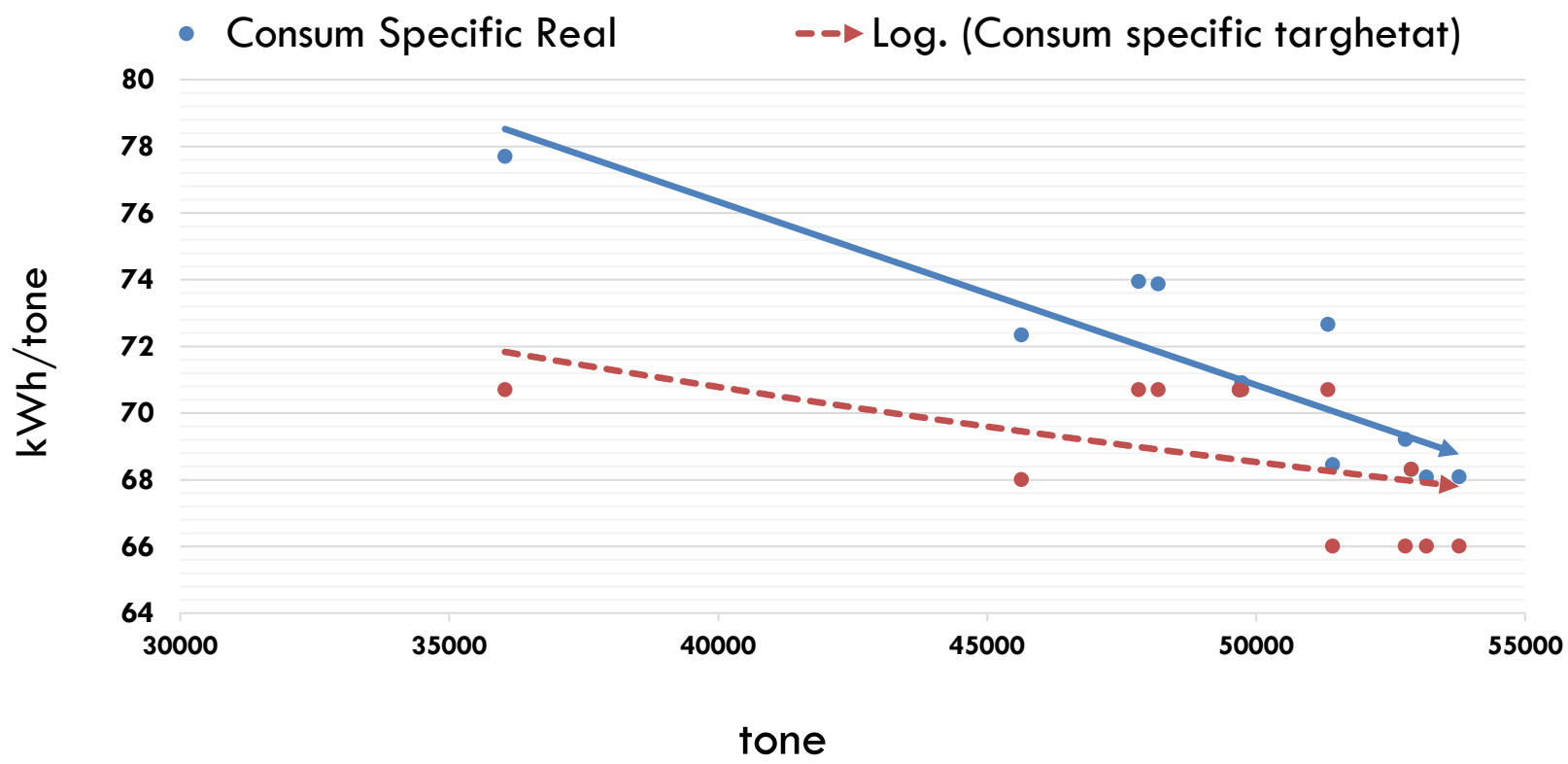
## Energia activă consumată lunar



## Consum specific de energie electrică - 2016 OTELINOX



## Analiza Consum Energie Electrică 2016



# Eficiența aproximării liniare

$$a_C = \frac{\left[ \sum_{i=0}^n (x_i \cdot y_i) \right] - \frac{1}{n} \cdot \left( \sum_{i=0}^n x_i \right) \cdot \left( \sum_{i=0}^n y_i \right)}{\sqrt{\left[ \sum_{i=0}^n (x_i)^2 - \frac{1}{n} \cdot \left( \sum_{i=0}^n x_i \right)^2 \right] \cdot \left[ \sum_{i=0}^n (y_i)^2 - \frac{1}{n} \cdot \left( \sum_{i=0}^n y_i \right)^2 \right]}}$$

Coeficientul de corelație – cât mai apropiat de valoarea 1.



# Sistem de ecuații

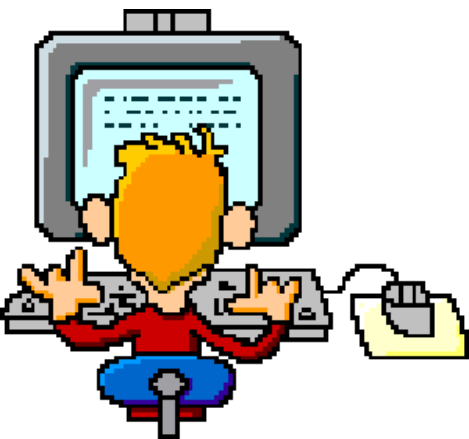
Dacă aproximarea se realizează cu o funcție parabolică de gradul II, se obține sist

$k := 0..2$

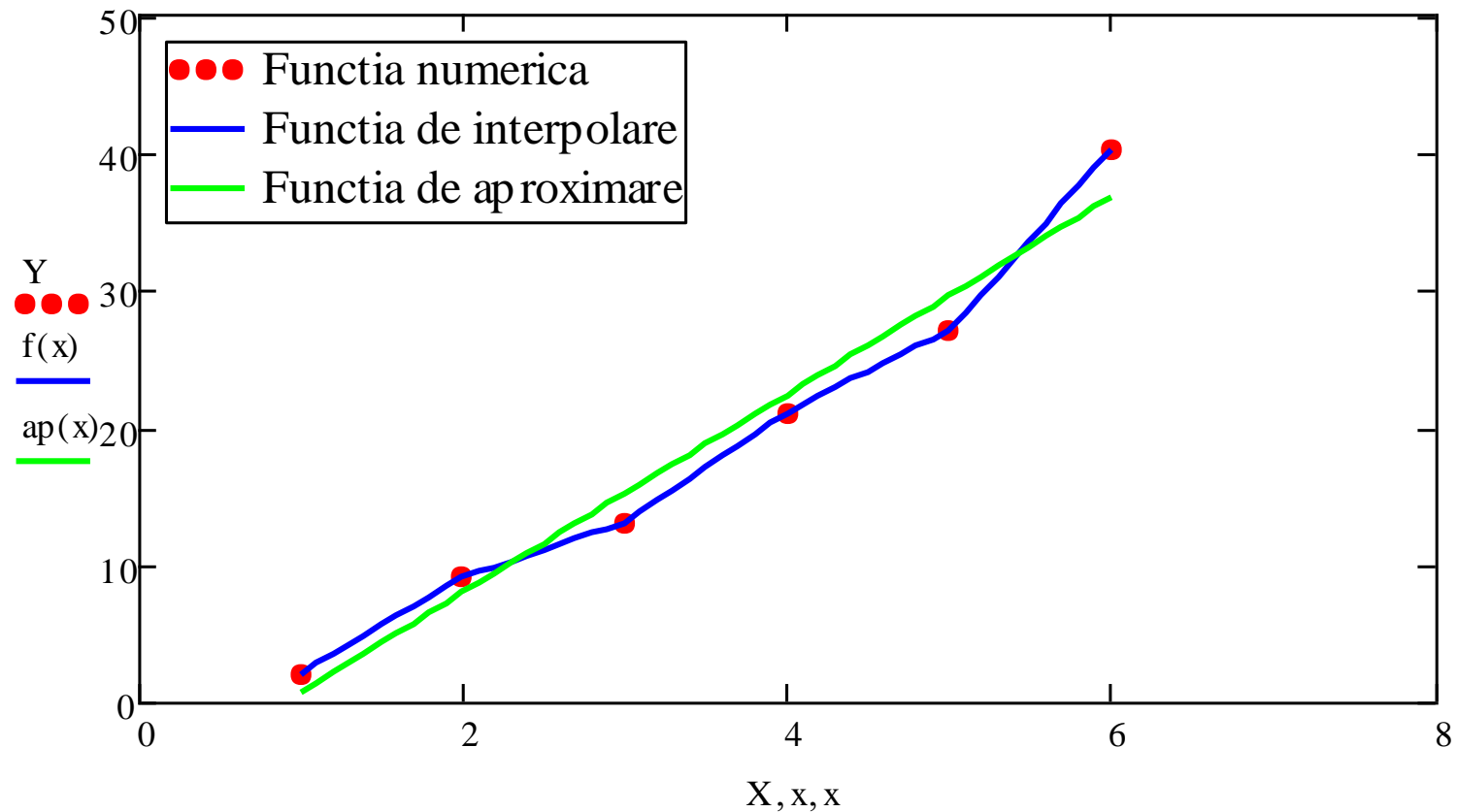
$$n \cdot a_0 + \left( \sum_{i=0}^n x_i \right) \cdot a_1 + \left( \sum_{i=0}^n x_i^2 \right) \cdot a_2 = \sum_{i=0}^n y_i$$

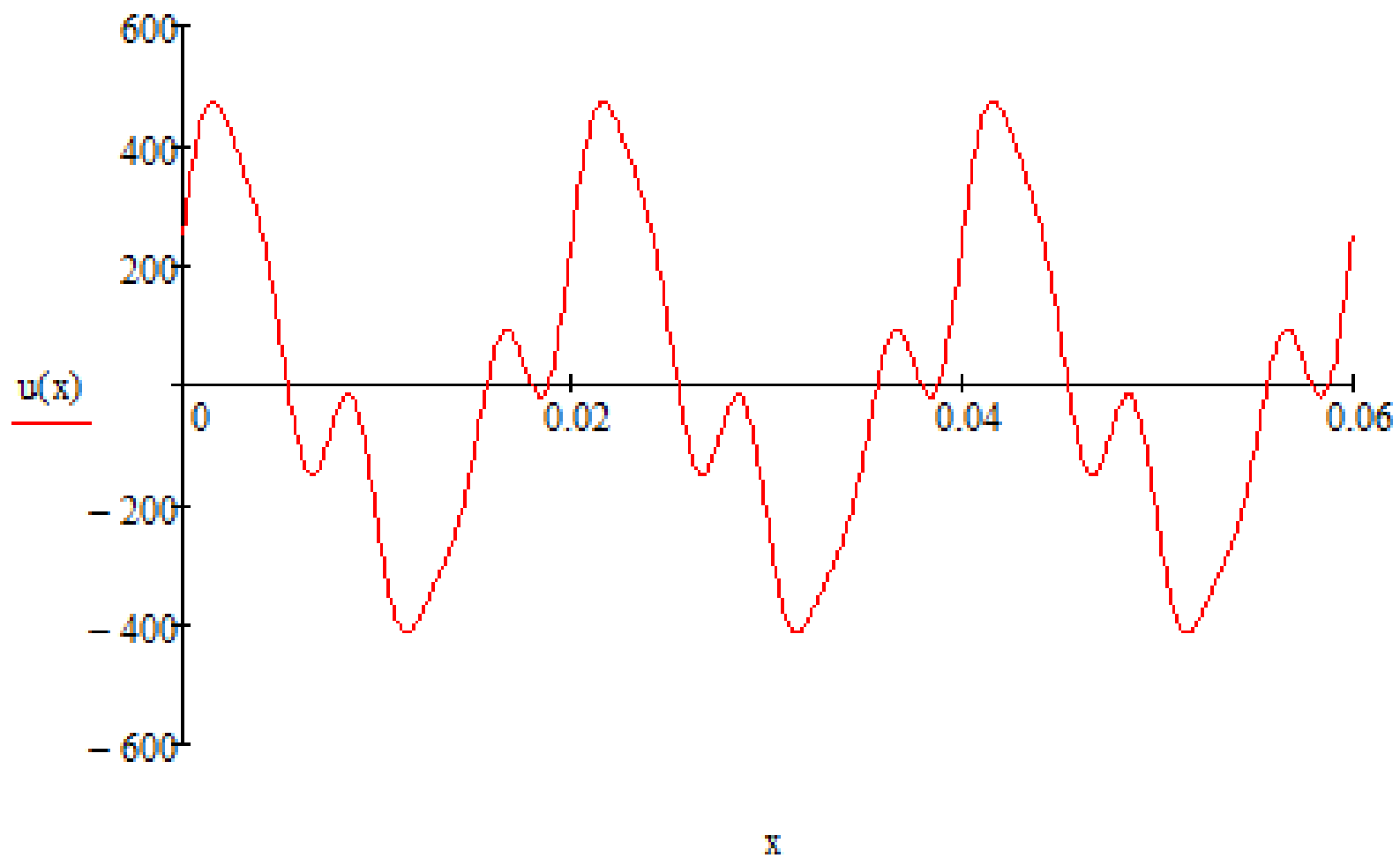
$$\left( \sum_{i=0}^n x_i \right) \cdot a_0 + \left( \sum_{i=0}^n x_i^2 \right) \cdot a_1 + \left( \sum_{i=0}^n x_i^3 \right) \cdot a_2 = \sum_{i=0}^n y_i \cdot x_i$$

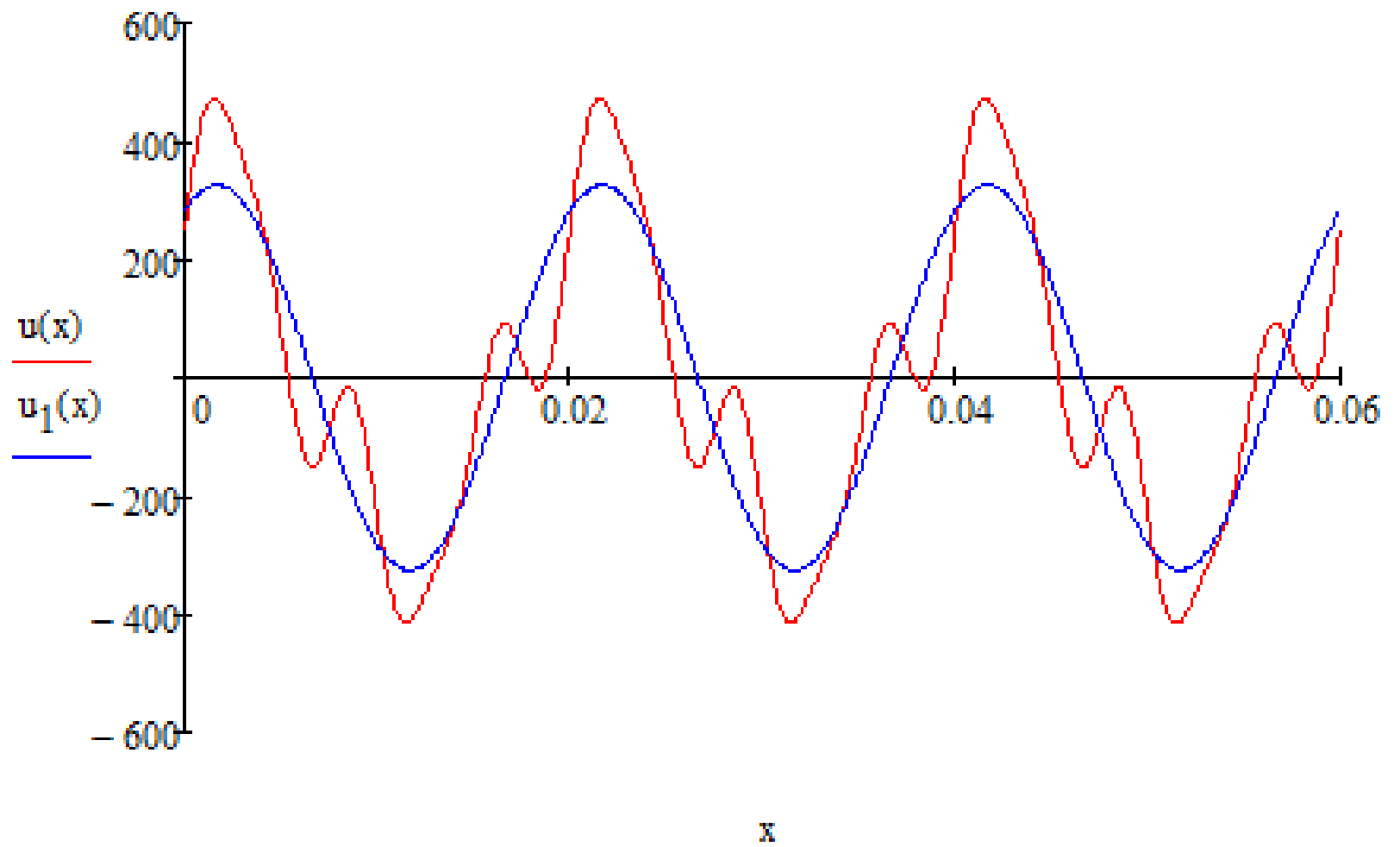
$$\left( \sum_{i=0}^n x_i^2 \right) \cdot a_0 + \left( \sum_{i=0}^n x_i^3 \right) \cdot a_1 + \left( \sum_{i=0}^n x_i^4 \right) \cdot a_2 = \sum_{i=0}^n y_i \cdot x_i^2$$

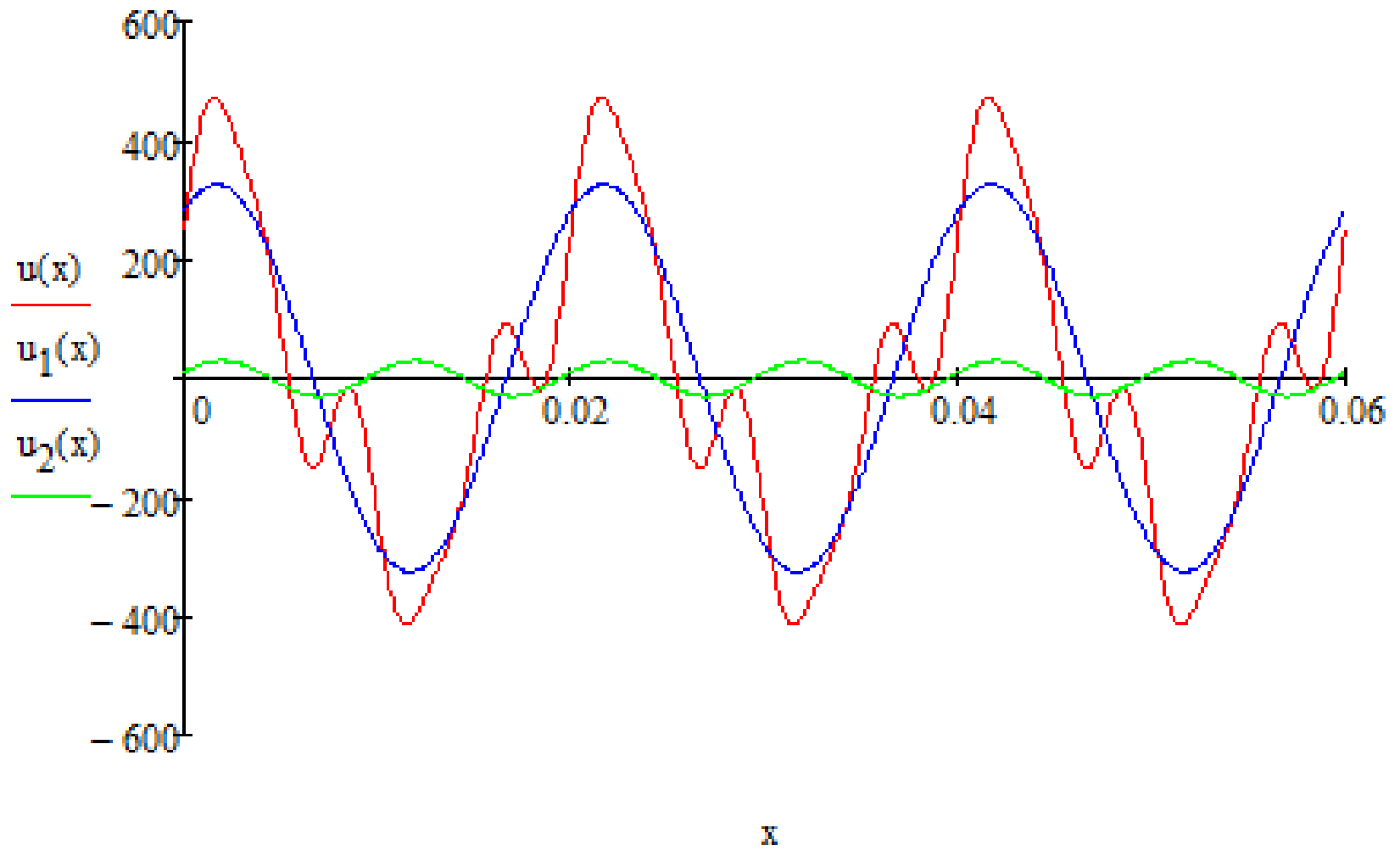


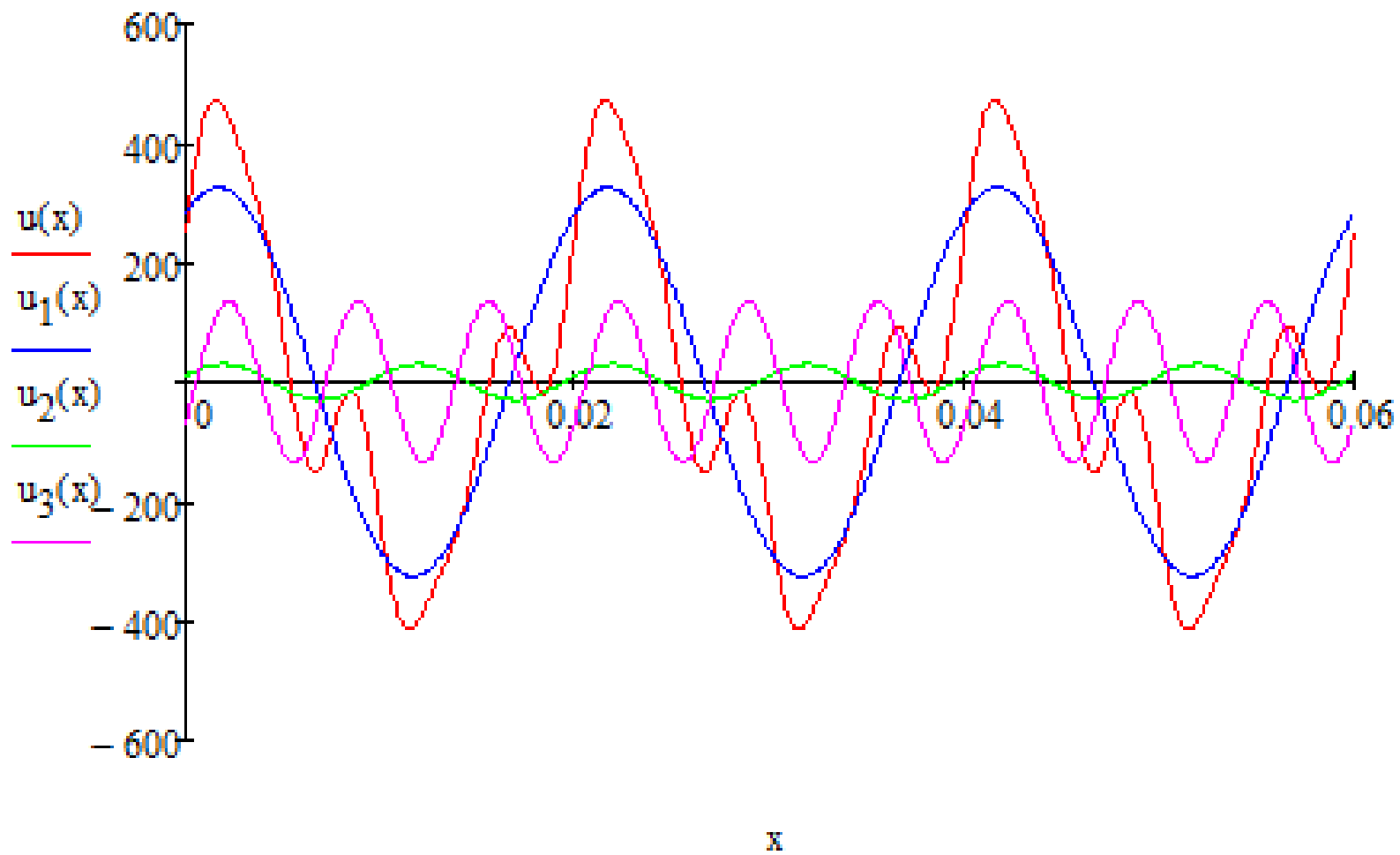
# Diferența dintre aproximare analitică și interpolare numerică

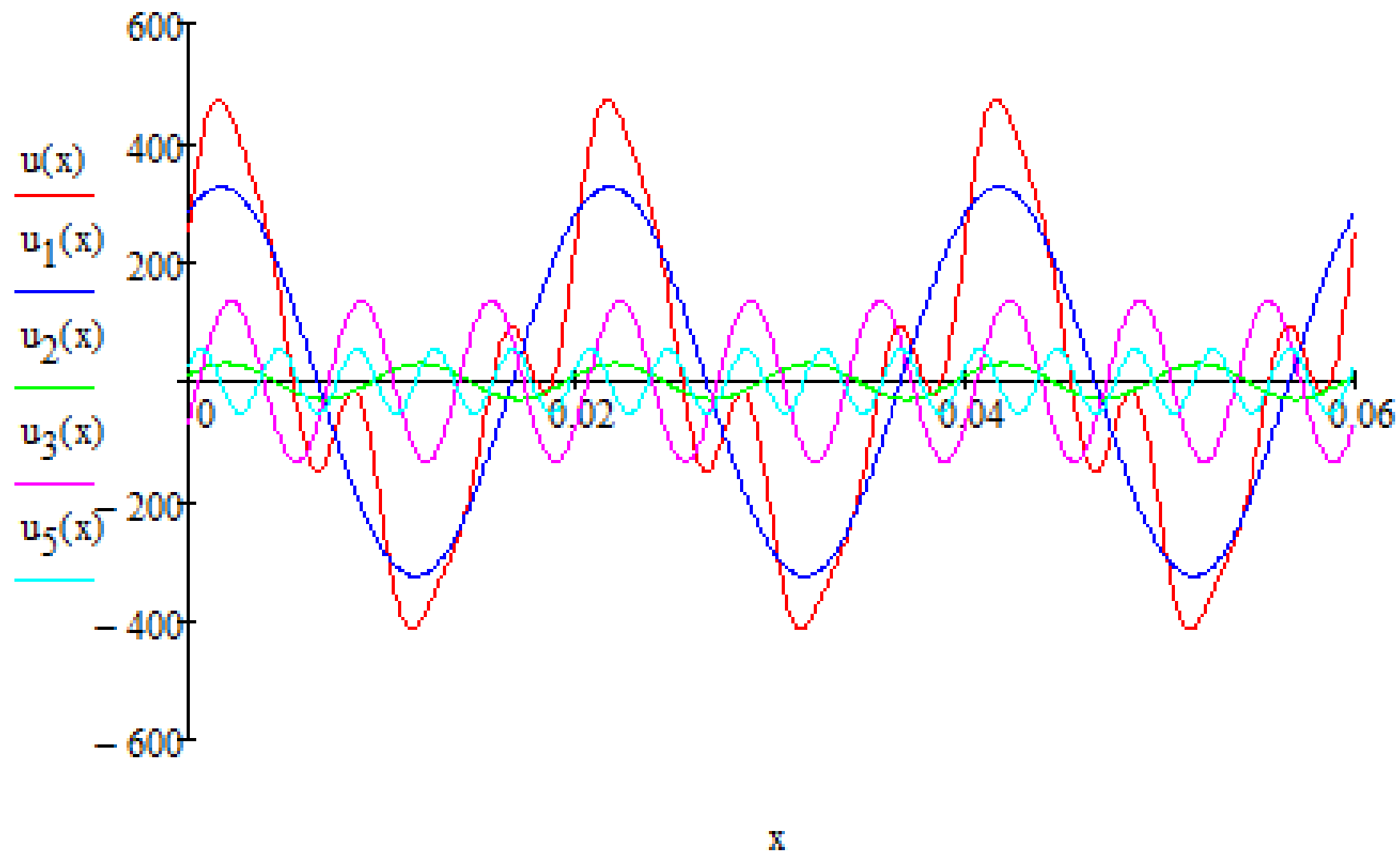












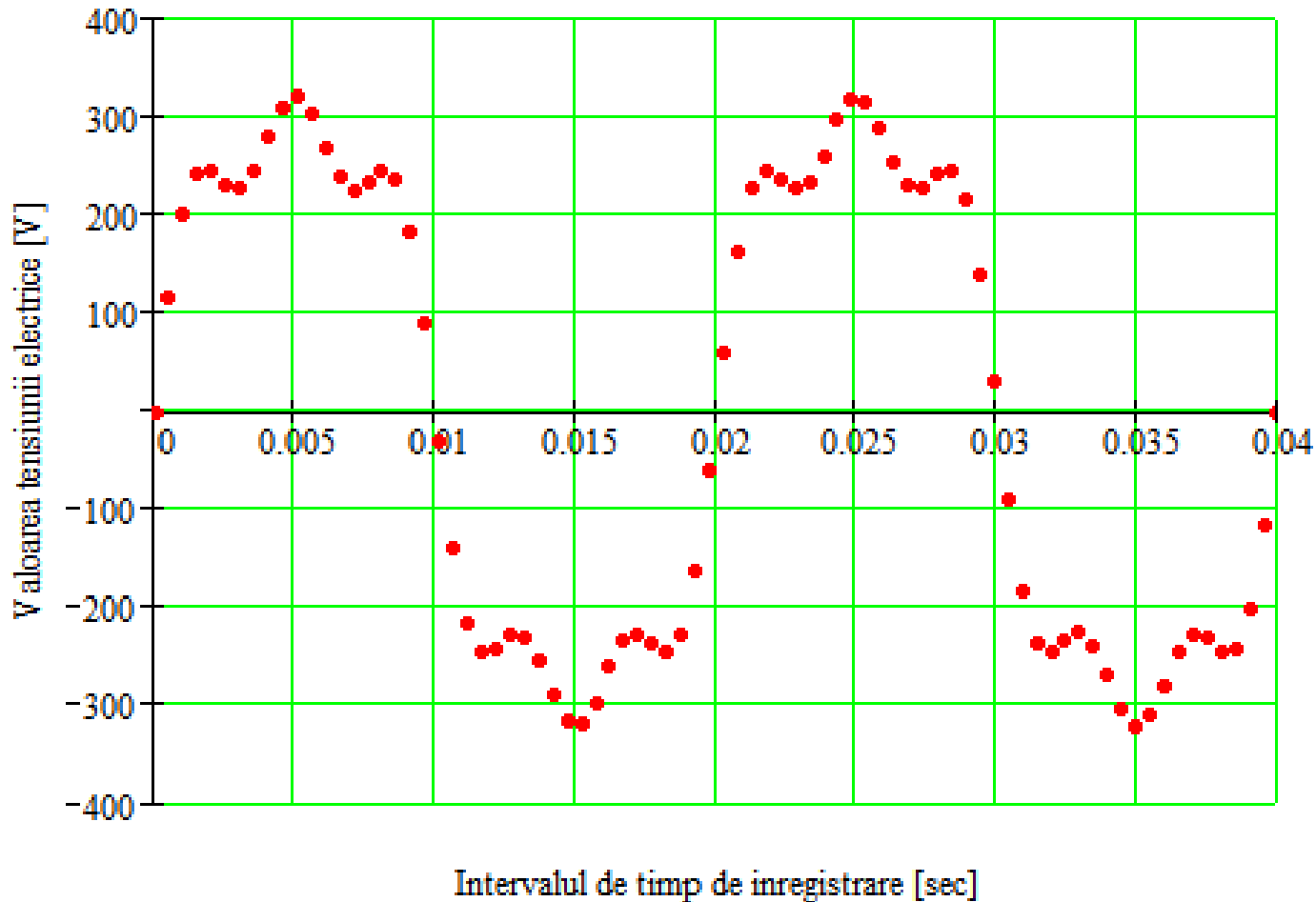
# Analiza armonică – studii practice

- **OMV Petrom**
- NUTRIENTUL Bihor
- DELTA Mioveni
- UPRUC Făgăraș
- CONTINENTAL Sibiu



# Seria Fourier – analiza armonică

Inregistrarea formei de unda cu analizorul ION 8800



# Aproximarea prin serii Fourier

$$u(t) = C_0 + \sum_{k=1}^{\infty} \sqrt{2} \cdot C_k \cdot \sin(k \cdot \omega \cdot t + \gamma_k)$$

$$u(t) = C_0 + \sum_{k=1}^{\infty} \sqrt{2} \cdot C_k \cdot (\cos(\gamma_k) \cdot \sin(k \cdot \omega \cdot t) + \sin(\gamma_k) \cdot \cos(k \cdot \omega \cdot t))$$

$$u(t) = C_0 + \sum_{k=1}^{\infty} \sqrt{2} \cdot A_k \cdot \sin(k \cdot \omega \cdot t) + \sum_{k=1}^{\infty} \sqrt{2} \cdot B_k \cdot \cos(k \cdot \omega \cdot t)$$



$$C_k = \sqrt{A_k^2 + B_k^2} \quad \gamma_k = \text{atan}\left(\frac{B_k}{A_k}\right)$$

# Coeficientii seriei Fourier

Calcul pe baza marimii analitice

$$C_0 = \frac{1}{T} \cdot \int_0^T u(t) dt$$

$$\sqrt{2} \cdot A_k = \frac{2}{T} \cdot \int_0^T (u(t) \cdot \sin(k \cdot \omega \cdot t)) dt$$

$$\sqrt{2} \cdot B_k = \frac{2}{T} \cdot \int_0^T (u(t) \cdot \cos(k \cdot \omega \cdot t)) dt$$

Calcul pe baza marimii numr

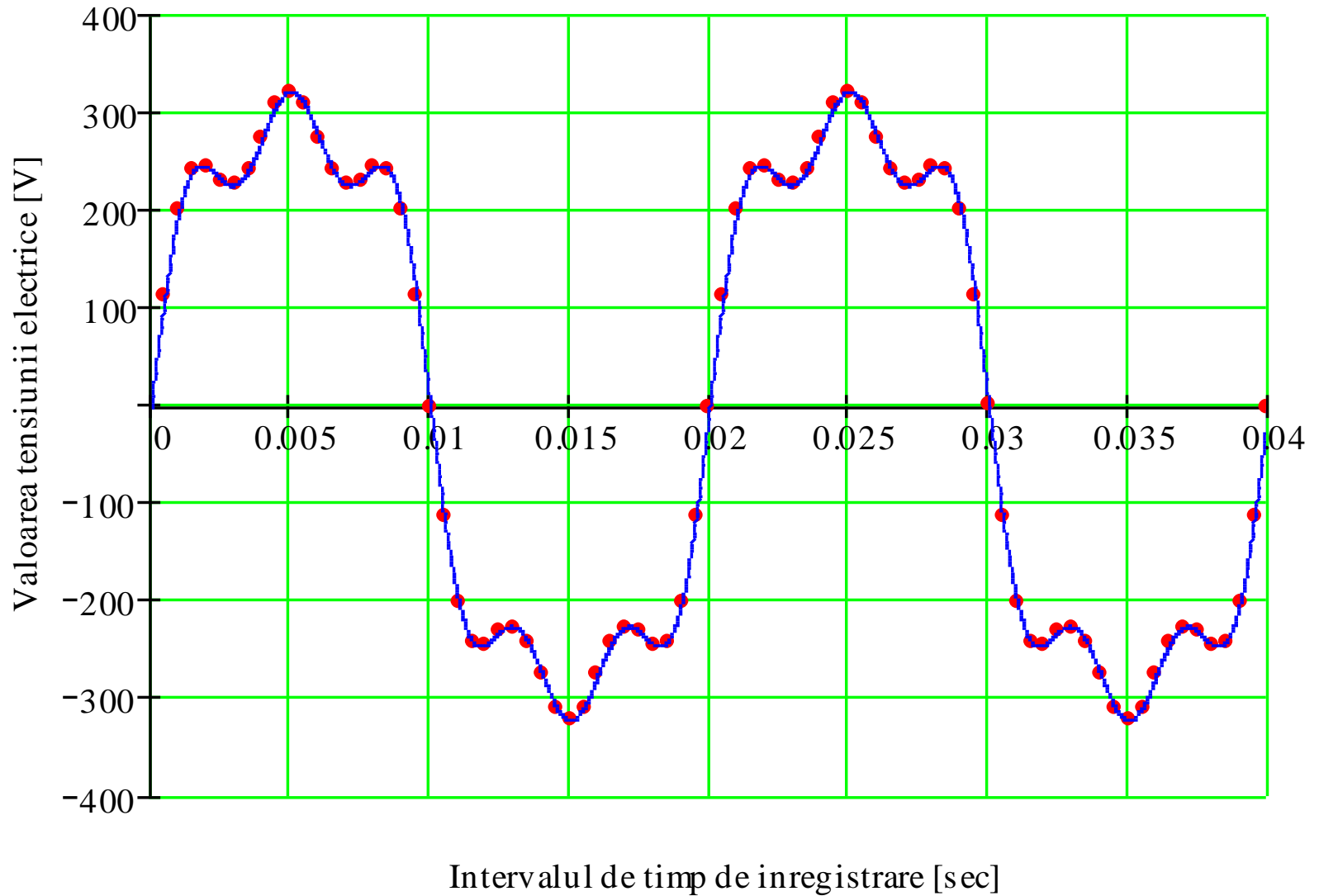
$$C_0 = \frac{1}{2 \cdot p} \cdot \sum_{i=1}^{2 \cdot p} U_i$$

$$\sqrt{2} \cdot A_k = \frac{1}{p} \cdot \sum_{i=1}^{2 \cdot p} U_i \cdot \sin\left(\frac{2 \cdot k \cdot \pi \cdot i}{p}\right)$$

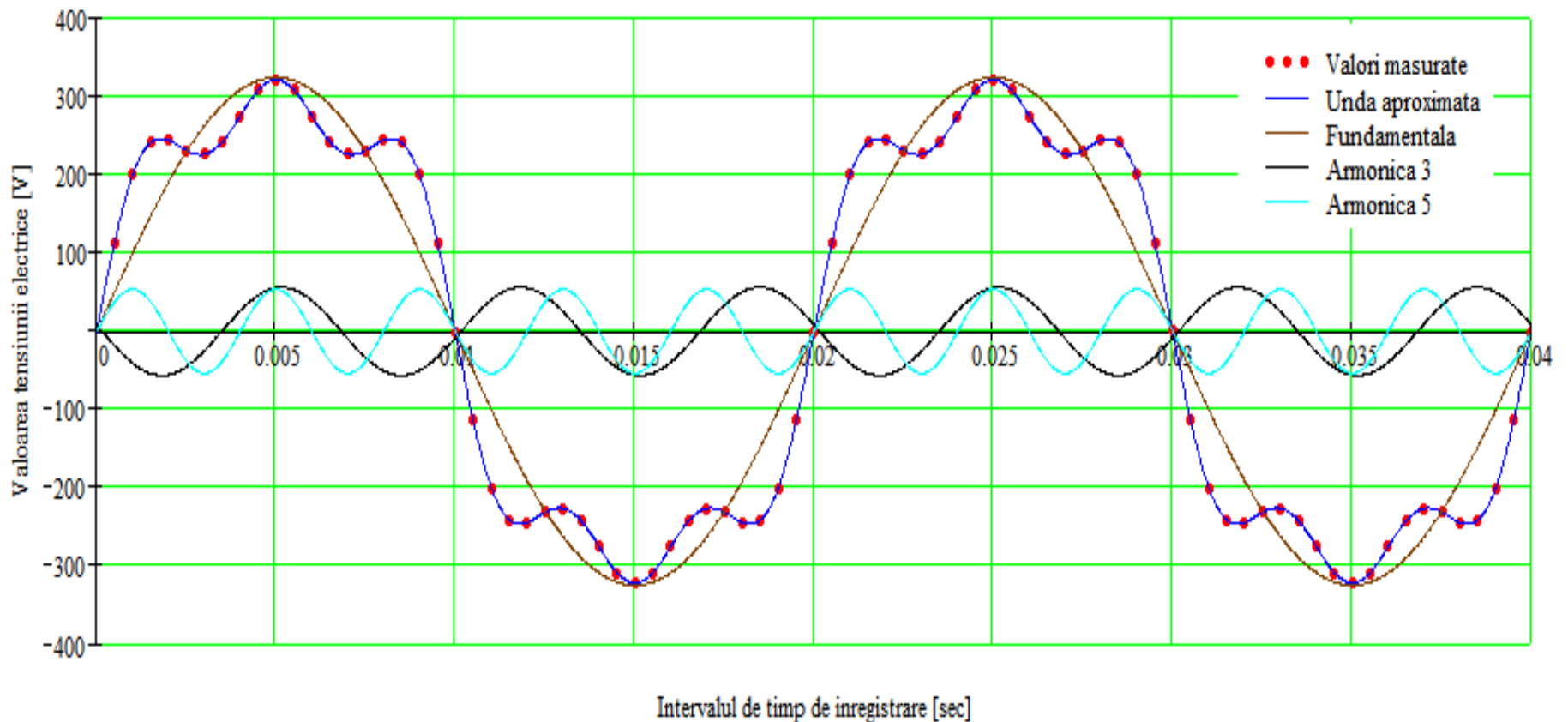
$$\sqrt{2} \cdot B_k = \frac{1}{p} \cdot \sum_{i=1}^{2 \cdot p} U_i \cdot \cos\left(\frac{2 \cdot k \cdot \pi \cdot i}{p}\right)$$

$$p = 80 \quad i = 1..2 \cdot p$$

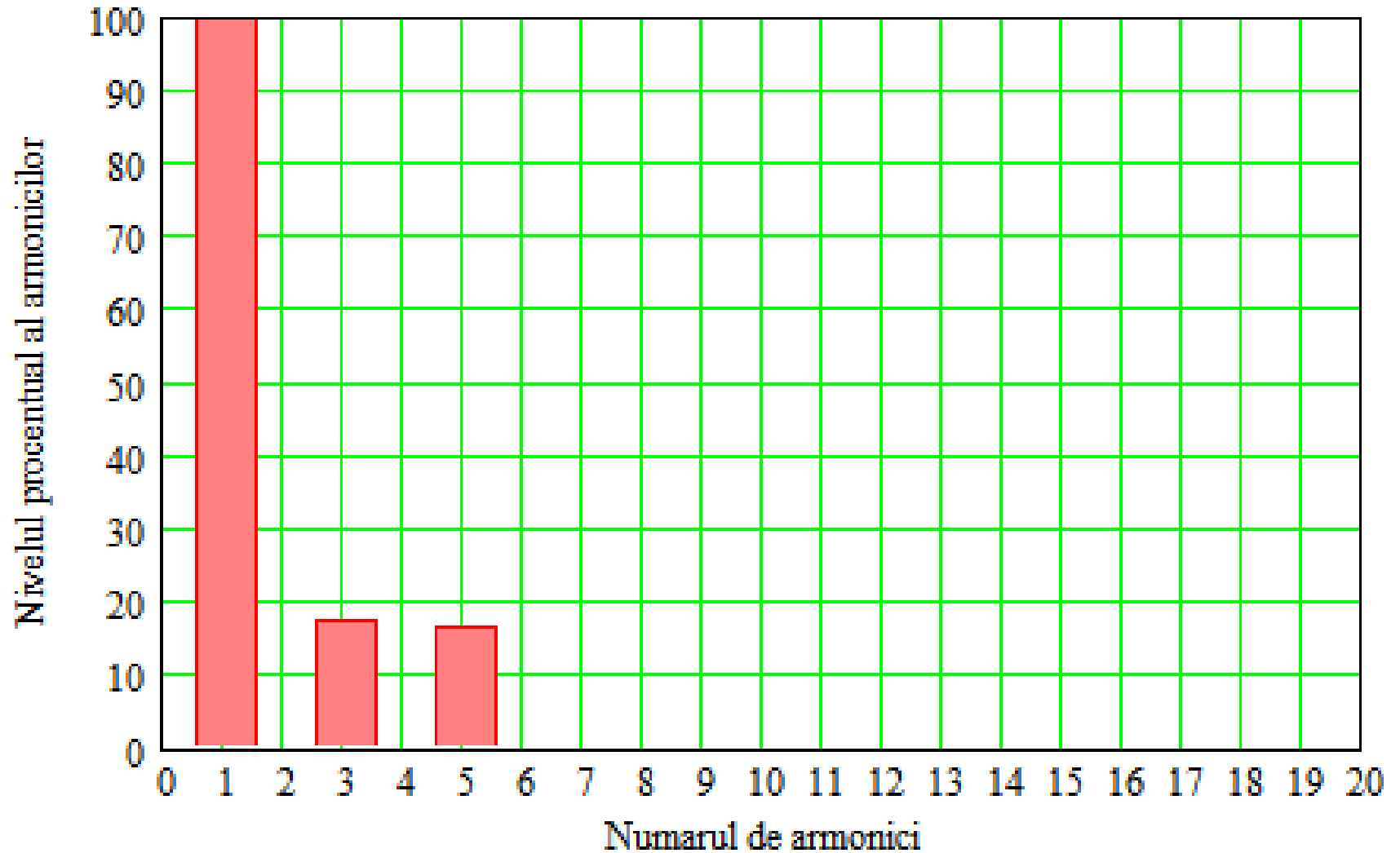
# Inregistrarea formei de unda cu analizorul ION 8800



# Reconstrucția formei de undă

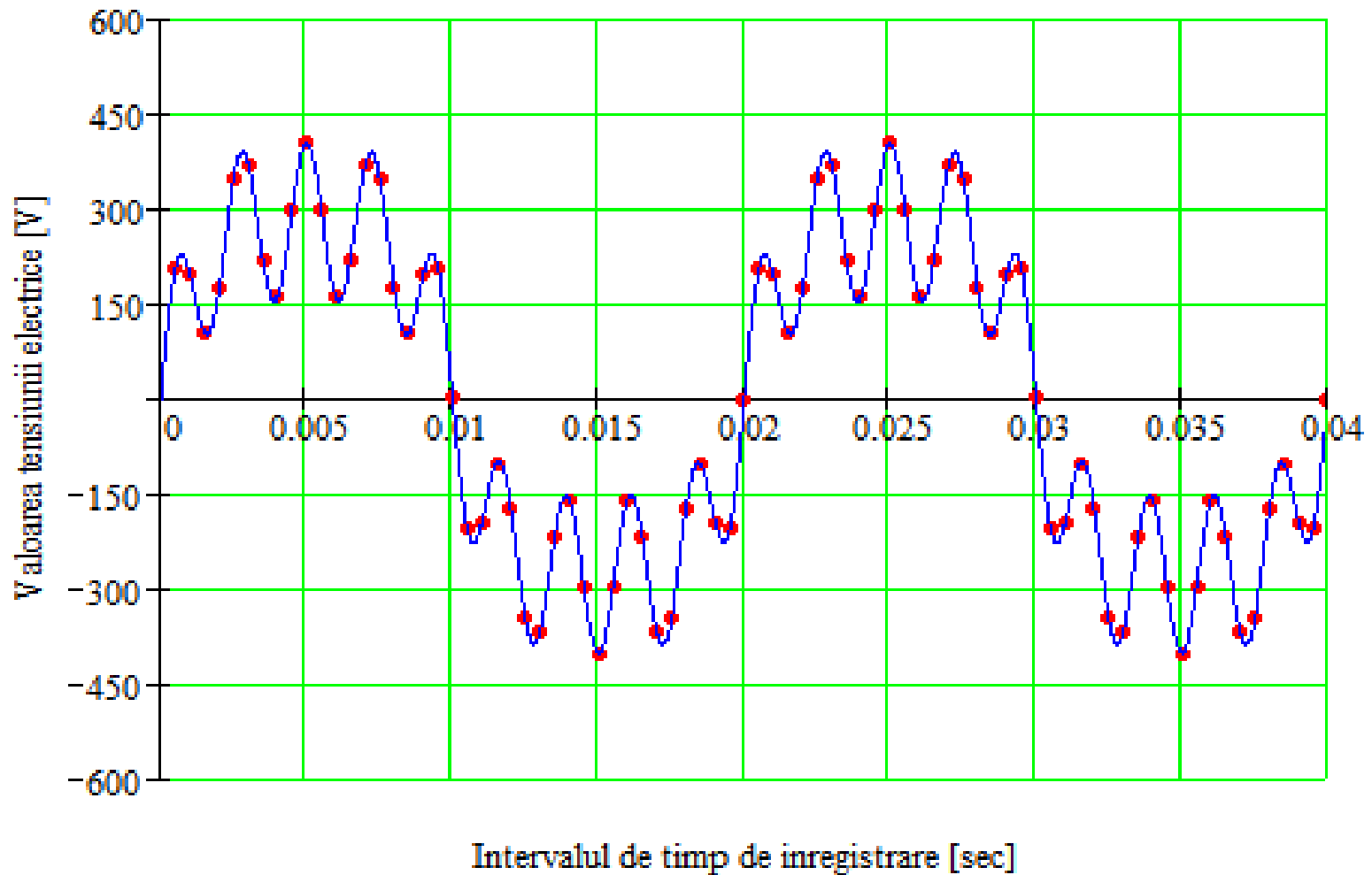


# Analiza armonică

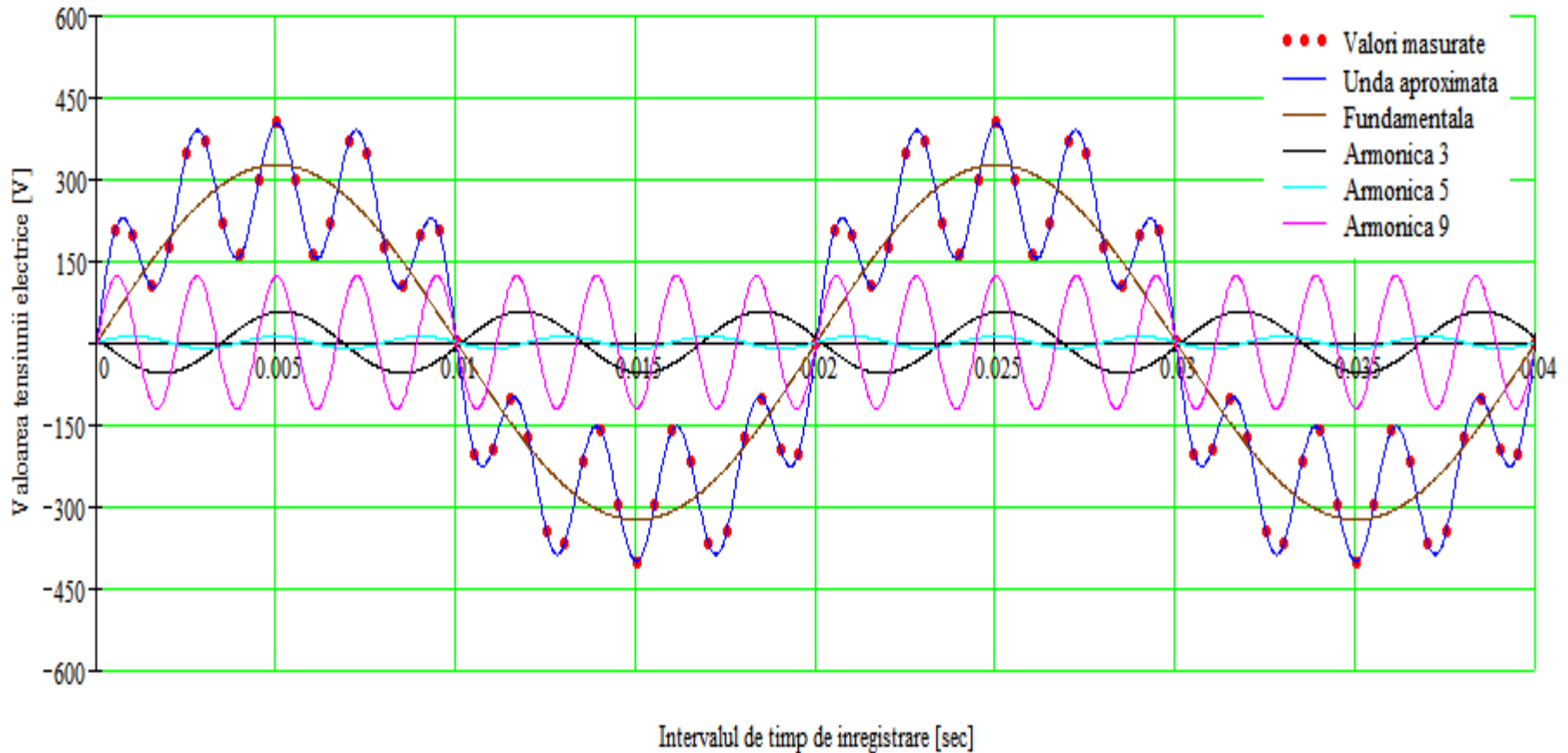


# Caz numeric

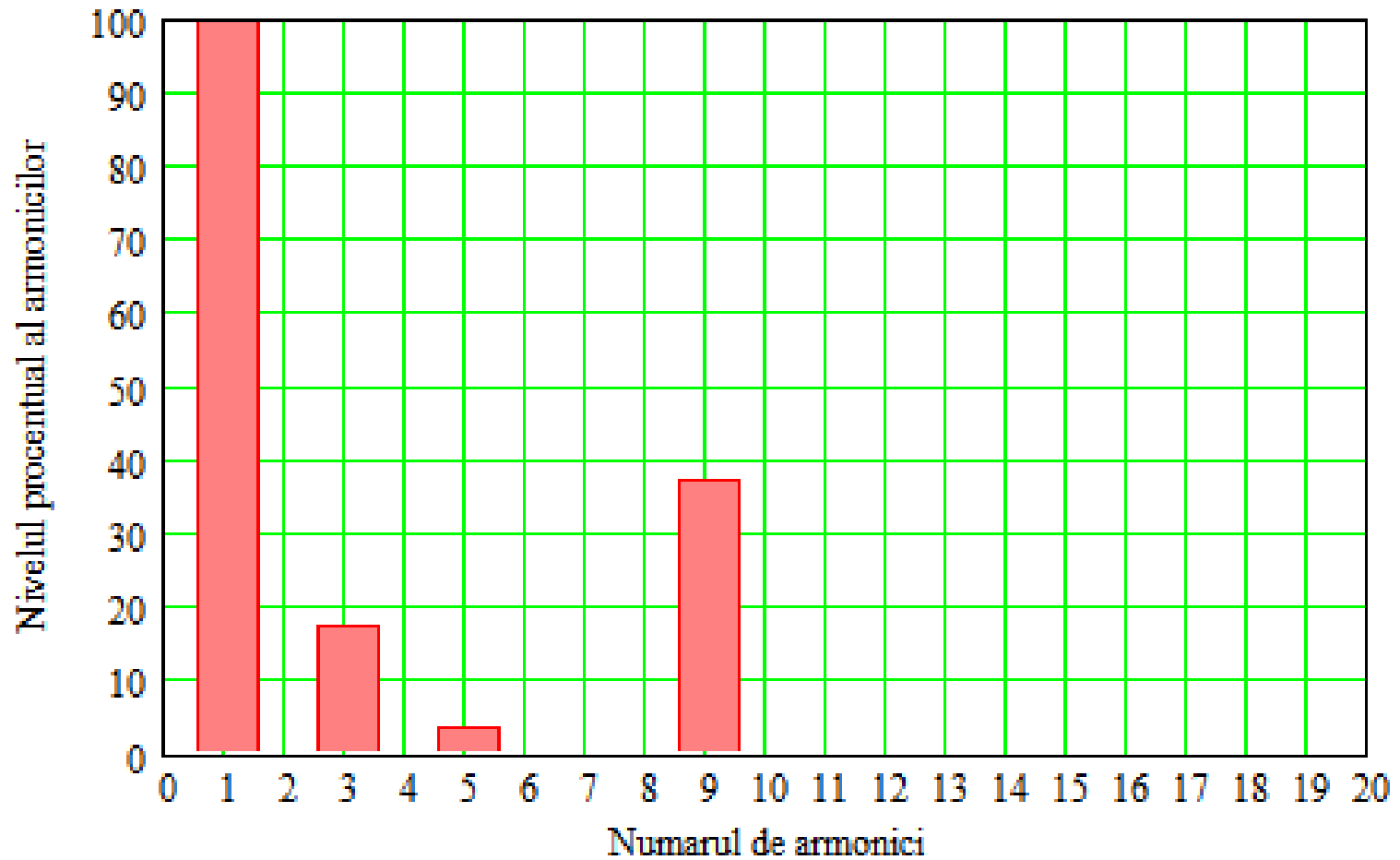
Inregistrarea formei de unda cu analizorul ION 8800



# Evidențierea armonicilor



# Analiza armonică



# Tema de casă



Fiind date valorile  $Y$  în punctele  $X$ , să se aproximeze cu ajutorul unei funcții analitice de tip liniar, apoi să se reprezinte grafic această funcție.



# Sumarizare curs

Seria Taylor – instrument matematic

Abaterăa celor mai mici pătrate

Seria Fourier – analiza armonică