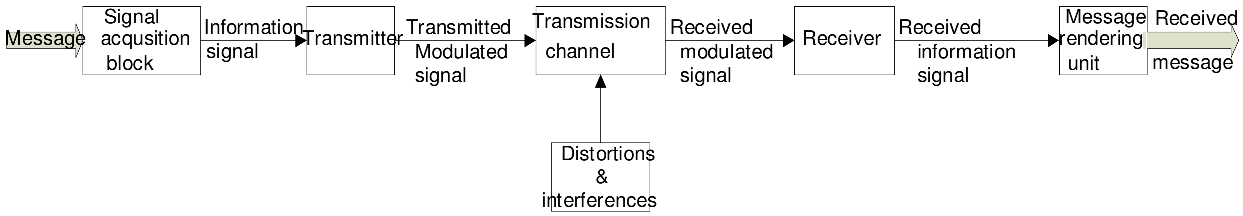
Introductory Notions

- The block diagram of a transmission link, which conveys information by means of electromagnetic signals, is depicted in the figure below.



Block diagram of a transmission link

- the original message (sound, image,..) is acquired and translated into electromagnetic signals by the ***message acquisition block.*** This signal represents the signal that has to be conveyed at destination.
- the information (modulating) signal (usually a time-variable voltage) may be expressed as:

$$g(t) = g_c + g_M \cdot f(t) \tag{1}$$

where the real constant g_M denotes the amplitude (**def.** amplitude –the maximum deviation (of the voltage) from the average value), and $f(t)$ is a real function which has the following properties:

$$f(t) \in [-1;1] \tag{2}$$

$$\int_{-\infty}^{\infty} f(t)\,dt = 0 \tag{3}$$

Note Relation (2) and (3) shows that $f(t)$ is zero-mean value and unitary amplitude.

- The energy of the informational signal is defined as:

$$E_s = \int_{-\infty}^{\infty} |g(t)|^2\,dt = g_M^2 \cdot \int_{-\infty}^{\infty} |f(t)|^2\,dt \tag{4}$$

Note: the definition of the energy (4) is actually the energy dissipated on a unitary load $\left(E = \frac{E_s}{Z}\right)$

- The power of the informational signal:

$$P_s = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |g(t)|^2\,dt = g_M^2 \cdot \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f^2(t)\,dt = g_M^2 \cdot \widetilde{f^2(t)} \tag{5}$$

Note: (5) defines the power dissipated on a unitary load $\left(P = \frac{P_s}{Z}\right)$

- the spectrum of the informational signal is described by the *complex function* $G(\omega)$ which is obtained by applying the Fourier transform on equation (1):

$$G(\omega) = \mathscr{F}(g(t)); \tag{6}$$

- If the modulating signal is analog (e.g. vocal, audio, video signals) its spectrum has usually a finite bandwidth, e.g.:

$$|G(\omega)| = \begin{cases} \neq 0; \ \omega \in [\omega_m; \omega_M] \\ 0 \ \ ; \omega \notin [\omega_m; \omega_M] \end{cases} \tag{7}$$

Def. the frequency band (FB) of a signal is the frequency range which contains the spectral components with a non-zero (or non-negligible) modulus.

$$FB = \left[\frac{\omega_m}{2\pi}; \frac{\omega_M}{2\pi}\right] = [f_m; f_M] \tag{8}$$

def. frequency bandwidth (BW) – is the width of the frequency band

$$BW = f_M - f_m \tag{9}$$

- the digital information signal have their bandwidths of infinite lengths $\left(BF = \left(-\infty;+\infty\right)\right)$, but usually their energy is concentrated in finite-width band (see lecture 4 on Baseband transmissions).

Transmitter – using the informational signal, it generates the *modulated signal,* which is adapted to the characteristics of the employed channel. The modulated signal should be generated such that the receiver would be able to extract the information (modulating) signal out of it, while having a transmission system as efficient as possible.

Communication channel (e.g. radio channel, twisted wires (UTP cable), telephony system (PSTN),…) might be regarded as a circuit which distorts the transmitted signal and adds to it a random signal called *noise*.

- the signal obtained at the channel output (i.e. the received modulated signal) $s_r(t)$ maybe expressed as:

$$s_r(t) = \int_{-\infty}^{\infty} s_t(t-\tau)\cdot h(\tau)\,d\tau + n(t) = s_t(t) \odot h(t) + n(t) \tag{10}$$

where $s_t(t)$ denotes the transmitted modulated signal, $h(t)$ is the channel's impulse response, $n(t)$ is the noise signal, while $\odot$ is the convolutional product.

- the used communications channel also imposes some additional constraints to the transmitted channel, such as the average and maximum power, bandwidth, spectral distribution, etc.

Receiver – is intended to extract the informational signal out of the received modulated signal, which is affected by the interferences and distortions inserted by the channel.

The message rendering block (e.g. speaker, display,...) is the device that transforms the electromagnetic informational signal received into the received message.

The *Modulation Techniques* course presents some of the basic techniques and methods to generate the transmitted signal, in terms of various informational signals, and the basic techniques of extracting the informational signals out of the received signals.

Def. Modulation **–** the modification, in accordance to a certain rule, of some magnitudes that are characteristic to the carrier signal, in order to facilitate the transmission of the information-carrying signals.

- The carrier signal is usually a cosine signal, defined by three characteristic magnitudes: amplitude, frequency and phase. Depending on the parameter that is modified within the modulation process, the modulation techniques (modulations) can be divided into three basic types:

- ***Amplitude Modulations*** – the information is transmitted by varying the amplitude of the carrier signal
- ***Frequency Modulation*** – the information is transmitted by varying the momentary frequency of the carrier signal
- ***Phase Modulation*** – the information is transmitted by varying the momentary phase of the carrier signal
- ***Amplitude+Phase Modulation -*** the information is transmitted by jointly varying the momentary phase and amplitude of the carrier signal

- According to the type of processing required to generate the modulated signal, the modulations can be divided in two categories:

- ***Linear Modulations*** – the modulated signal can be generated by using linear processes (addition, multiplication) – in this category we find the amplitude modulation.
- ***Nonlinear (or Exponential) Modulations*** – in this case the modulated signal cannot be obtained by linear processes. In this category we may include the phase and frequency modulations, as well as all the combined modulations (e.g. amplitude+phase).

- According to the nature of the modulating (informational) signal the modulations can be classified as:

- ***Analog Modulations*** – the informational signal is continuous in amplitude and time, whose voltage (or current) level may take infinity of values.
- ***Digital Modulations*** – the informational signal is digital (e.g. a stream of bits), its voltage level taking discreet values out of a finite set. The informational signals are also discreet in time, the level values of the signals being constant during time intervals equaling integer multiples of an elementary period T.

Linear Modulations (LM)

- the useful information is contained in the amplitude of the modulated carrier signal

$$s_{LM} = A(t)\cdot\cos 2\pi f_c t = A(t)\cdot\cos\omega_c t \quad (11)$$

- the modulating signal g(t) may take several forms, depending on the type of LM modulation:

$$\begin{aligned} &g(t) = g_c + g_M\cdot f(t);\quad f(t)\in[-1,+1];\, g_c \ge g_M \quad \text{DSB}-\text{C (AM)}; \quad \text{a.}\\ &g(t) = g_M\cdot f(t);\quad f(t)\in[-1,+1];\quad \text{DSB}-\text{SC},\quad \text{b.}\\ &\text{"special" forms for SSB, VSB; – to be discussed later;}\quad \text{c.}\end{aligned} \quad (12)$$

- the carrier signal is:

$$s_c(t) = V_0\cdot\cos\omega_c t; \quad (13)$$

The Amplitude modulation (AM) - Double Sideband with Carrier DSB-C

- the expression of the modulated signal:

$$s_{ML}(t) = \frac{g(t)\cdot V_0\cos\omega_c t}{V_{ref}} \quad (14)$$

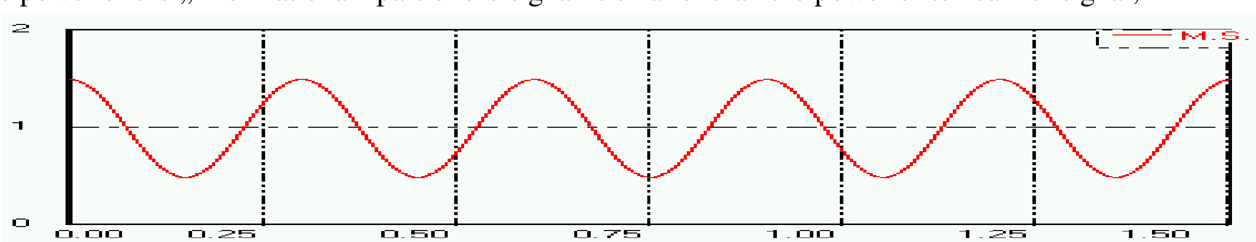
which can be particularized for DSB-C (AM) by using (12).a:

$$s_{AM} = \frac{V_0\cdot g_c}{V_{ref}}(1 + m\cdot f(t))\cos\omega_c t;\quad m = \frac{g_M}{g_c} - \text{modulation index}; \quad (15)$$

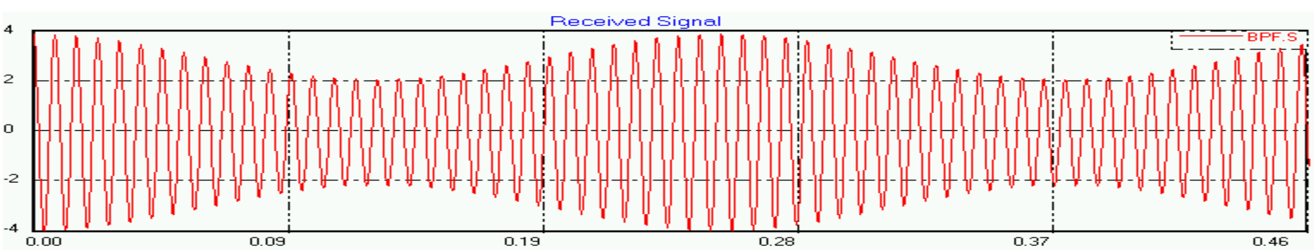
- for $V_0 = V_{ref}$ the average power of the AM signal is:

$$P = \frac{g_c^2}{2} + \frac{g_c^2 m^2 \tilde{f}^2(t)}{2};\quad \frac{g_c^2 m^2 \tilde{f}^2(t)}{2} < \frac{g_c^2}{2} \quad (16)$$

- the power of the „informational” part of the signal is smaller than the power of teh carrier signal;



AM modulating signal – centered on g_C



AM modulated signal

- the DSB-C (AM) signal is the only LM signal for which the envelope of the modulated signal is directly proportional to the modulating signal, allowing for a very simple demodulation**.**

Spectral Composition of the AM Signal

- taking relation (17) into account, relation (15) can be rewritten as (18), if we assume $V_{ref} = 1$:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2} \quad (17)$$

$$\begin{aligned} s_{AM} &= V_0 g_c\left(1 + m\cdot f(t)\right)\cos\left(2\pi f_c t\right) = V_0 g_c\left(1 + m\cdot f(t)\right)\cdot\frac{e^{i2\pi f_c t} + e^{-i2\pi f_c t}}{2} = \\ &= \frac{V_0}{2}\left(g_c + g_M\cdot f(t)\right)e^{i2\pi f_c t} + \frac{V_0}{2}\left(g_c + g_M\cdot f(t)\right)e^{-i2\pi f_c t}\end{aligned} \quad (18)$$

- the spectrum of the modulated signal is obtained by applying the Fourier transform to relation (15) or (18):

$$S(\omega)_{AM} = \mathscr{F}\left(s_{AM}(t)\right) \quad (19)$$

- The spectrum of the AM modulated signal can be computed by the following steps:

- assuming that the spectrum of the modulating signal is expressed by function $G(\omega)=\mathscr{F}\left(g(t)\right)$; and that the modulating signal has a limited frequency bandwidth, i.e.:

$$G(\omega)=\begin{cases}\neq 0;\ \omega\in[\omega_{mm};\omega_{mM}]\\ 0\ \ ;\omega\notin[\omega_{mm};\omega_{mM}]\end{cases} \tag{20}$$

- using the property of the Fourier transform expressed by (21), one could compute the spectrum of the DSB-C (AM) signal

$$\begin{gathered} f(x),h(x)-\text{int}\,egrable\ functions\\ F(\omega)=\mathscr{F}\left(f(x)\right);\\ H(\omega)=\mathscr{F}\left(h(x)\right);\\ if\ h(x)=e^{i2\pi x\omega_0}\cdot f(x)\quad then\ H(\omega)=F(\omega-\omega_0)\ \ for\ \ \omega_0-real \end{gathered} \tag{21}$$

- knowing that $\omega=2\pi f$

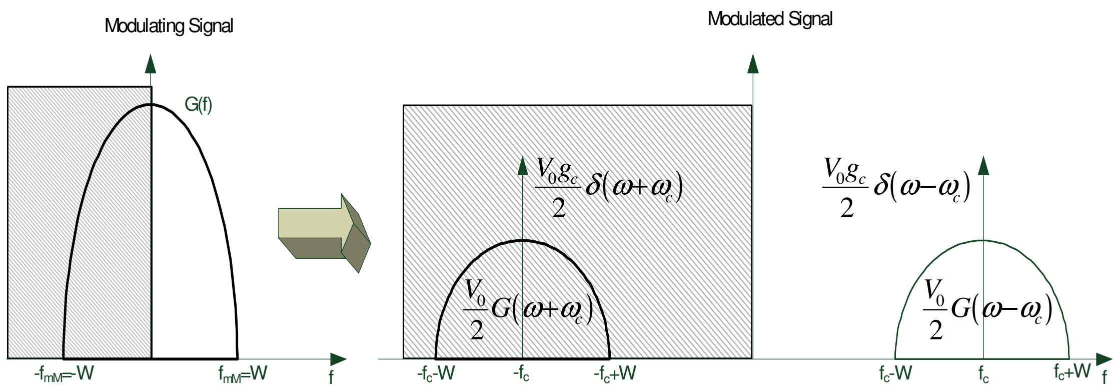
- then equation (19) becomes:

$$S(\omega)_{AM}=\frac{V_0 g_c}{2}\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]+\frac{V_0}{2}\left[G(\omega+\omega_c)+G(\omega-\omega_c)\right] \tag{22}$$

where $\delta(x)$ denotes the Dirac function:

$$\delta(x)=\begin{cases}1;\ x=0\\ 0;\ \ x\neq 0\end{cases} \tag{23}$$

- note: since there are one spectral component on the carrier frequency f_c and two sidebands →the modulation is named “double sideband with carrier – DSB-C” (BLD-P) – see the figure below



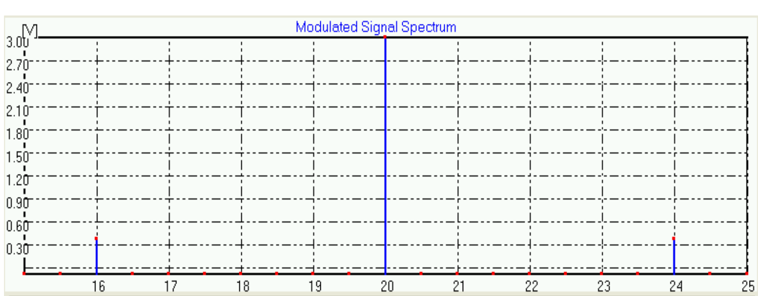
MA Spectrum acc. to relation (22)

- rewriting (15) for $V_0 = V_{ref}$ and for $g(t) = g_M \cos\omega_m t$ we get:

$$s_{AM}=g_c(1+m\cdot\cos\omega_m t)\cos\omega_c t=g_c\cos\omega_c t+\frac{g_c m}{2}\cos(\omega_c-\omega_m)t+\frac{g_c m}{2}\cos(\omega_c+\omega_m)t \tag{24}$$

- equation (24) indicates 2 sidebands placed symmetrically around the carrier
- the frequency band FB and the bandwidth BW of the AM signal are:

$$FB=[f_c-f_{mM},\ f_c+f_{mM}];\quad BW=2\cdot f_{mM}; \tag{25}$$



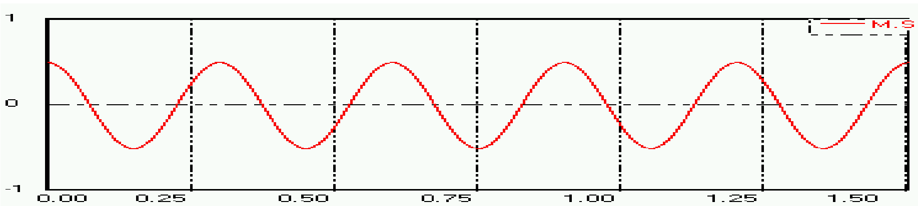
Spectrum of the AM for f_m = 4 Hz and f_c = 20 Hz

- the major disadvantage of AM is the significant amount of power contained in the carrier signal; also note that $BW_{AM}=2BW_{g(t)}$
- the major advantage is the very simple demodulation – to be discussed later

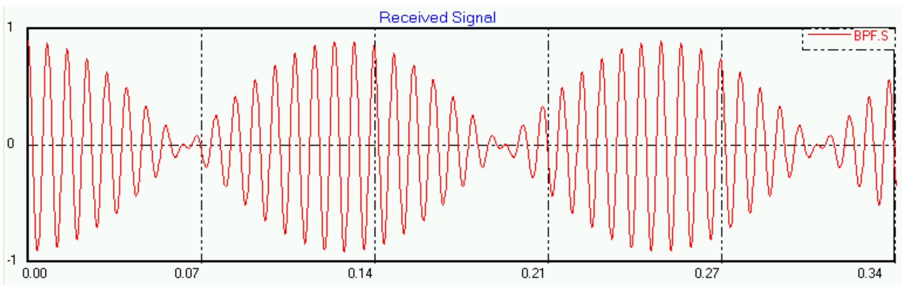
Linear Modulation with double sideband and suppressed carrier – DSB-SC

- the expression of DSB-SC, is obtained from (12).b and (13):

$$s_{BLD-PS} = \frac{V_0}{V_{ref}} \cdot g_M \cdot f(t) \cdot \cos\omega_c t; \quad \text{for} \quad V_0 = V_{ref} \Rightarrow s_{LM} = g_M \cdot f(t) \cdot \cos\omega_c t;$$
$$P = \frac{g_M^2 \cdot \tilde{f}^2(t)}{2}; \qquad (26)$$



LM modulating signal
– no d.c. component inserted



DSB-SC modulated signal

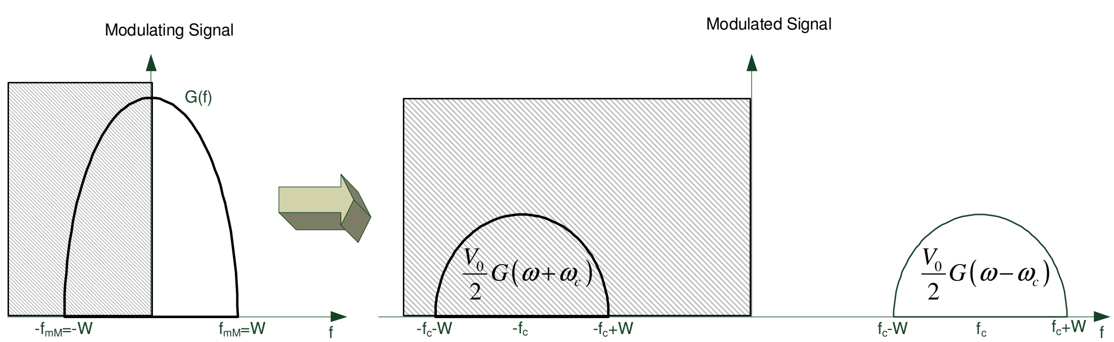
- the envelope of the DSB-SC signal no longer follows the modulating signal
– it inserts a 180º uncertainty
- requires a more elaborated demodulation

- advantage: the modulated signal has smaller power; still the $BW_{DSB-SC}=2\cdot BW_{g(t)}$

Spectral Composition of the DSB-SC Signal

- the spectrum of the DSB-SC signal is obtained in a manner similar to (22) and is expressed by (27), see the figure below:

$$S(\omega)_{BLD-PS} = \frac{V_0}{2}\left[G(\omega+\omega_c)+G(\omega-\omega_c)\right] \qquad (27)$$

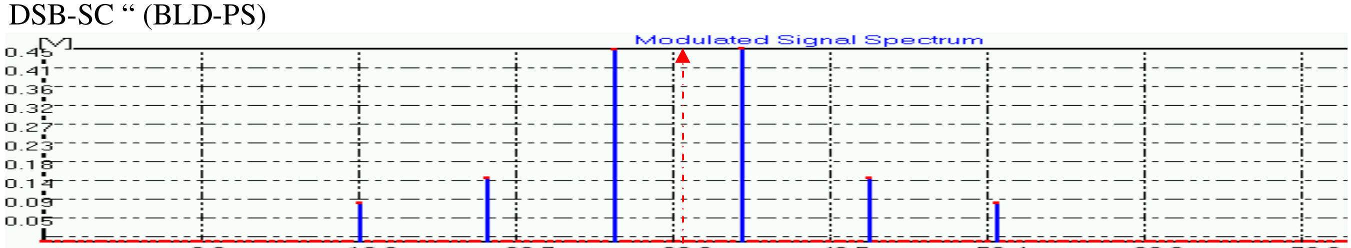


DSB-SC Spectrum acc. to (27)

- rewriting (26) for $V_0 = V_{ref}$ and for $g(t) = \sum_{i=1}^{3} g_i \cdot \sin(i\cdot\omega_m \cdot t) = \sum_{i=1}^{3} g_i \cdot \sin(\omega_i \cdot t)$ we get:

$$s_{AM} = \cos\omega_c t \cdot \sum_{i=1}^{3} g_i \cdot \sin(\omega_i \cdot t) = \frac{1}{2}\sum_{i=1}^{3} g_i \cdot \sin(\omega_c - \omega_i)t + \frac{1}{2}\sum_{i=1}^{3} g_i \cdot \sin(\omega_c + \omega_i)t \qquad (28)$$

- (27) and (28) indicate 2 sidebands placed symmetrically around the carrier, but no component inserted intentionally on the carrier frequency – the modulation is named "Double SideBand with Suppressed Carrier– DSB-SC " (BLD-PS)



Spectrum of the DSB-SC for rectangular signal with f_m = 4Hz and f_c = 40 Hz

- the frequency band FB and the bandwidth BW of the above signal are:

$$FB = [f_c - f_{mM}, f_c + f_{mM}]; \quad BW = 2 \cdot f_{mM}; \tag{29}$$

- Advantage of DSB-SC – no power inserted on the carrier frequency f_c; still the transmitted power is high
- Disadvantages: 1. More elaborate demodulation; 2. Poor spectral efficiency, since the DSB requires a $BW=2 \cdot f_m$ to transmit a modulating signal with a $BW = f_m$.

Quadrature Amplitude Modulation - QAM

- the spectral efficiency of the DSB modulations is rather small, since they require a bandwidth of $2 \cdot BW_0$ to transmit a modulating signal with a BW_0 bandwidth, where $BW_0 = [0, f_{mM}]$.
- to use more efficiently the frequency band, the QAM transmits two independent modulating signals in the same frequency band, by modulating them on two orthogonal carrier signals.
- the orthogonality of the two carrier signals allows for the separation of the two modulating signals at the receiving end.
- two real functions are orthogonal if:

$$\frac{1}{T}\int f(x) \cdot g(x) dx = \begin{cases} 0 & if \quad f(x) \neq g(x) \\ ct. if & f(x) = g(x) \end{cases} \tag{30}$$

- it can be easily shown that the signals $s_I(t)=V_0\cos(\omega_c t)$ and $s_Q(t)=V_0\sin(\omega_c t)$ observe the condition (30) when integrated over a period $T_c = 1/f_c$.
- assuming that the signals $g_I(t)$ and $g_Q(t)$ are two real modulating signals with a limited bandwidth, the expression of the QAM signal modulated on the orthogonal carriers $s_I(t)$ and $s_Q(t)$ is :

$$s_{MAQ}(t) = \frac{g_I(t) \cdot s_I(t)}{V_{ref-I}} - \frac{g_Q(t) \cdot s_Q(t)}{V_{ref-Q}} \tag{31}$$

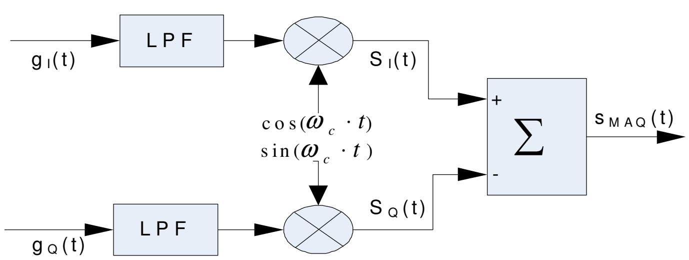
- imposing the conditions (32) to simplify the expressions:

$$V_{ref-I} = V_{ref-Q} = V_0 \tag{32}$$

the expression (31) of the MAQ signal becomes:

$$s_{MAQ}(t) = g_I(t)\cos(\omega_c t) - g_Q(t) \cdot \sin(\omega_c t) \tag{33}$$

- the block diagram of the QAM modulator is:



Block diagram of the QAM modulator for two analogue modulating signals

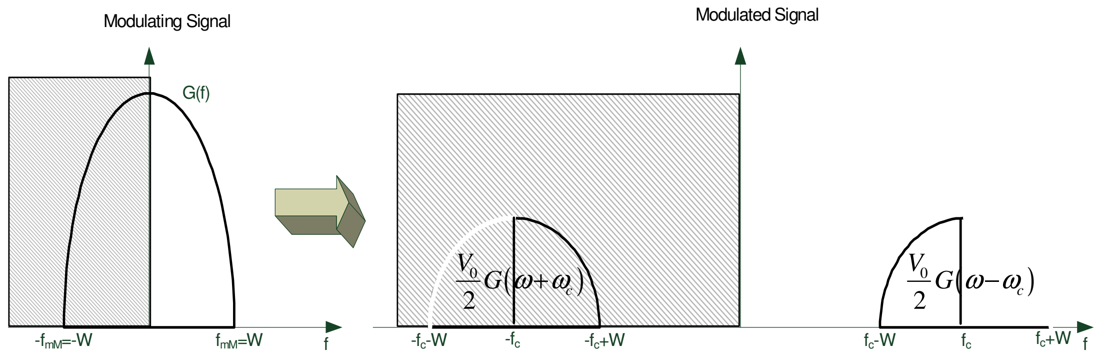
- the QAM modulation may be regarded as a sum of two DSB-SC signals within which, if the two modulating signals $g_I(t)$ and $g_Q(t)$ have the same frequency band, the modulated signals $S_I(t)$ and $S_Q(t)$ would occupy the same frequency band (not the same as the modulating signals!).
- assuming that the modulating signals $g_I(t)$ and $g_Q(t)$ have non-null spectral components in the frequency band $(0, f_{mM}]$, the frequency band FB and the bandwidth BW of the QAM signal would be:

$$\begin{aligned} FB_{MAQ} &\in [f_c - f_{mM}; f_c + f_{mM}] \\ BW_{MAQ} &= 2 \cdot f_{mM} \end{aligned} \tag{34}$$

- though the BW_{QAM} equals the BW_{DSB}, within it one can transmit two modulating signals with $BW = \omega_{mM}$, and so the BW of the modulated signal can be considered equal to ω_{mM}/modulating signal
- the LP filters placed at the inputs of the QAM modulator ensure that the frequency spectra of the modulating signals are upper-limited to ω_{mM}.

Linear Modulation with Single Sideband – SSB

- the whole information of the DSB-SC modulated signal is contained in one sideband
- the DSB-SC uses redundantly the second sideband and transmits a significant amount of additional power
- in order to decrease the frequency band employed and the transmitted power, only one sideband is transmitted, see the figure below



SSB-inf Spectrum

- the SSB signal could be obtained by two methods: a. by filtering the DSB-SC signal; b. by phase-shifting the modulating signal

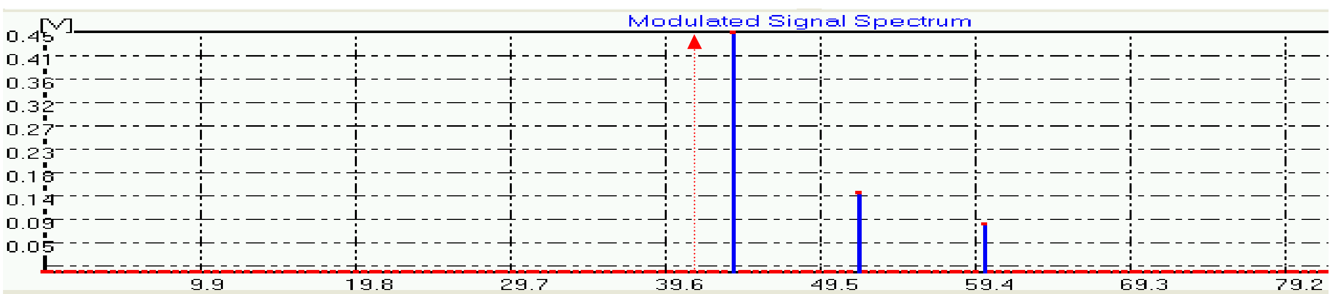
a. the filtering method: the DSB-SC signal is BP filtered suppressing the undesired sideband

- the FB and the BW of the SSB are:

FB = [f_c - f_{mM}, f_c] – the inferior-SSB; FB = [f_c, f_c + f_{mM}] – the superior-SSB;

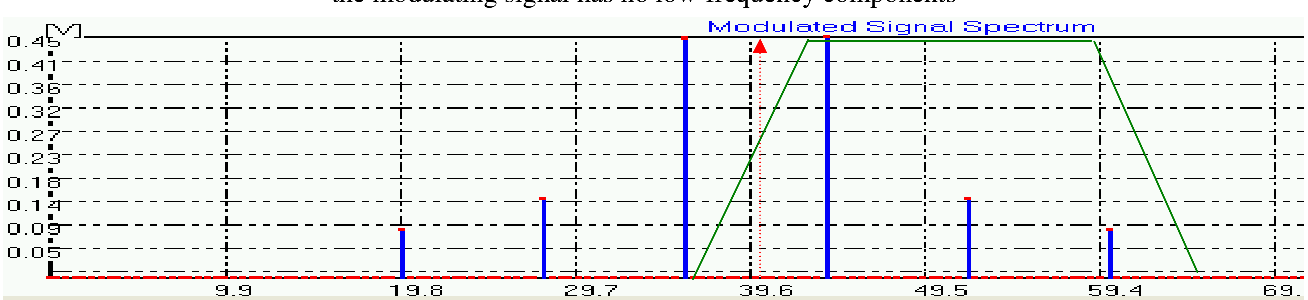
$$BW = f_{mM} \qquad P = \frac{g_M^2 \cdot \tilde{f}^2(t)}{4}; \tag{35}$$

Filtering method for SSB-superior; the modulating signal has no d.c.

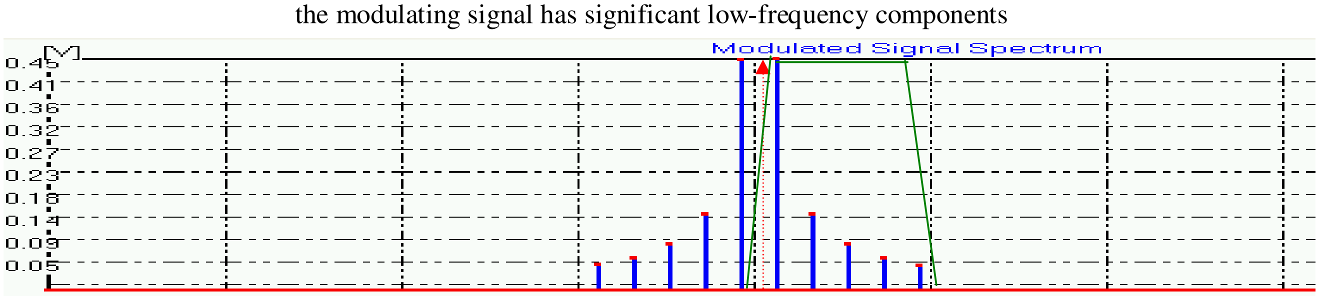


SSB – sup

the modulating signal has no low-frequency components



the modulating signal has significant low-frequency components



- the required filter should have a higher slope; more complicated to implement and sometimes impossible

b. SSB by phase-shifting the modulating signal

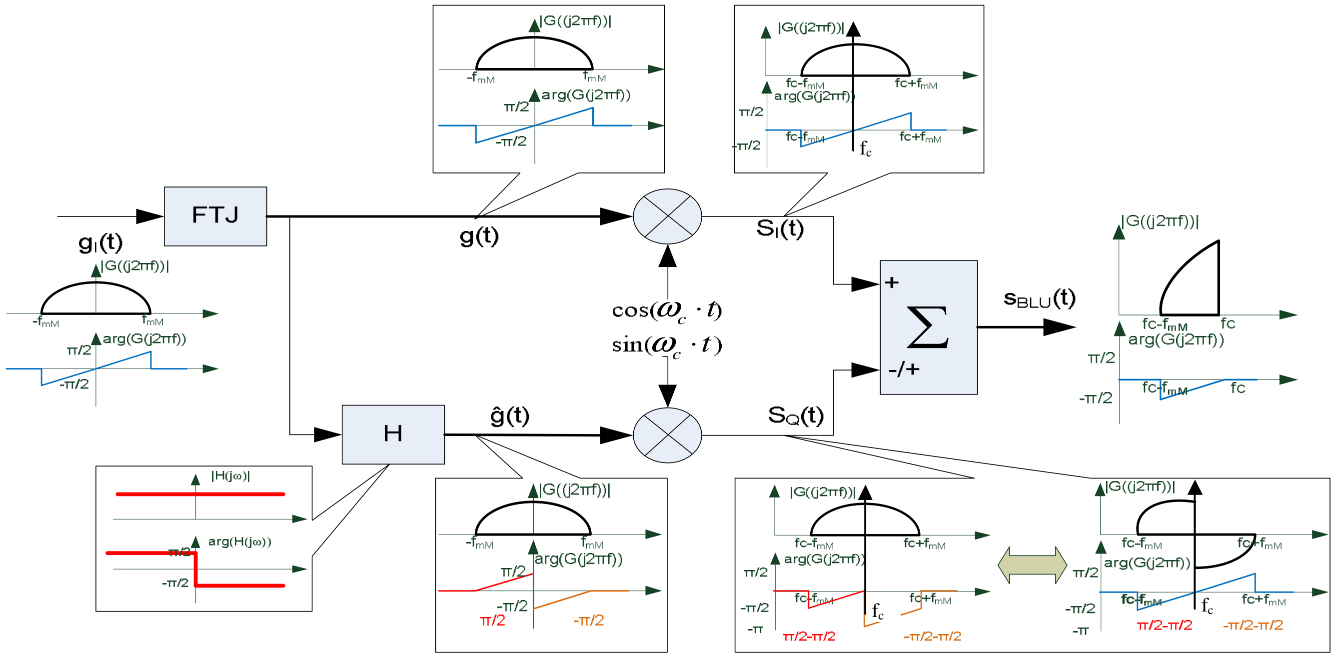
- it employs the Hilbert transform of the modulating signal

- Hilbert transform – transfer function:

$$H(\omega)=\begin{cases}-j & \omega>\omega_0\\ 0 & \omega=\omega_0\\ j & \omega<\omega_0\end{cases}\qquad j \ \text{denotes a phase shift of}\ \pi/2;\qquad |H(\omega)|=1 \tag{36}$$

- the SSB signal is expressed by:

$$s_{SSB}(t)=\frac{1}{2}g(t)\cos\omega_c t\mp\frac{1}{2}\hat{g}(t)\sin\omega_c t;\ -\text{for sup SB};+\text{for inf SB};\quad \hat{g}(t)=H(g(t)) \tag{37}$$



Generating the SSB-sup by using the Hilbert transform

- for an accurate implementation the modulating signal should have no d.c. component and small low-frequency components
- the SSB signal (37) is similar to a QAM signal (33), but the modulating signals are not independent, but are “versions” of the same signal used to produce an uni-dimensional signal
- the QAM approach is used only at the transmitter
- the block diagram of the SSB generation is similar to the one of the QAM modulator (see figure above)
- SSB requires smaller BW and less power than both DSB-(S)C to transmit the same modulating signal

Linear Modulation with Vestigial Sideband - VSB

- it is employed for modulating signals that have low frequency components, e.g. analog TV signals
- the modulated signals have a vestige of the undesired sideband (see the drawing on the blackboard)
- the modulating signal is filtered with a low-pass characteristic K_β and the modulated signal is:

$$s_{VSB}(t)=g(t)\cos\omega_c t\mp g_\beta(t)\sin\omega_c t;\ -\text{for sup SB};+\text{for inf SB}; \tag{38}$$

- the FB and BW of the VSB are:

FB = $[f_c - f_{mM},\ f_c + \beta]$ – the inferior-VSB; FB = $[f_c - \beta,\ f_c + f_{mM}]$ – the superior-VSB; BW = $f_{mM} + \beta$ (39)

General expression of LM modulated signals:

$$s_{LM}(t)=\frac{\alpha}{2}g(t)\cos\omega_c t\mp\frac{1}{2}g_q(t)\sin\omega_c t; \tag{40}$$

Particular cases:

$\alpha = 2$; $g_q(t) = 0$ – DSB: if $g(t) = g_c + g_M f(t)$ – DSB-C; if $g(t) = g_M f(t)$ – DSB-SC; (41)

$\alpha=1$; $g_q(t)=h(t)\odot g(t)$; $h(t)=F^{-1}(H(\omega))$; – SSB; “-“ for SSB_{sup}; “+ “ for SSB_{inf}; (42)

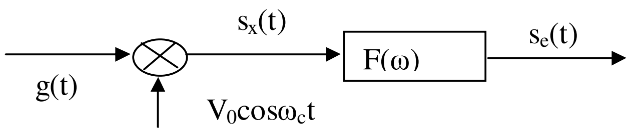
$\alpha=1$; $g_q(t)=K_\beta(t)\odot g(t)$ - VSB (43)

- the general expression employs the Quadrature Amplitude Modulation (QAM) - only at the transmitter end
- the SSB and VSB can be expressed easier using this approach.

LM modulation Methods

a. modulators with analog multipliers
b. modulators with choppers
c. modulators with non-linear circuits – see reference Ed.Nicolau
d. modulators operating directly on the tuned circuit – see reference Ed. Nicolau

a. LM modulator with analog multiplier



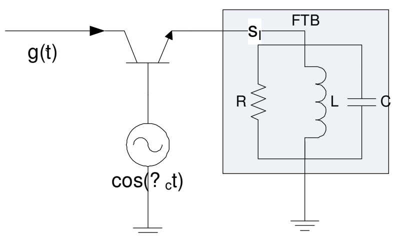
- particular cases will be discussed on the blackboard
- the analog multipliers are not available for very high frequencies

b. modulators with choppers

b.1. Unbalanced Chopper

- the modulating signal is chopped with the interruption function with the frequency f_c:

$$f_i(t) = \begin{cases} 1 & \text{if } V_0 \cos \omega_c t > 0; \\ 0 & \text{if } V_0 \cos \omega_c t \le 0; \end{cases} \quad f_i(t) = \frac{1}{2} + \frac{2}{\pi}\sin \omega_c t + \frac{2}{3\pi}\sin 3\omega_c t ... \quad (44)$$

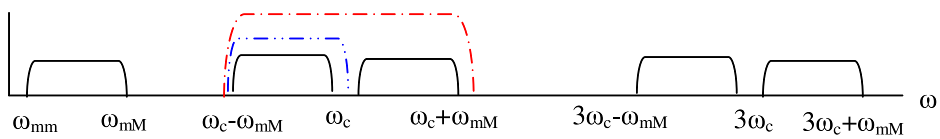


Unbalanced chopper DSB modulator

- the multiplication between the $f_i(t)$ signal and the modulating signal g(t) is accomplished by means of a transistor (see the neighbouring figure):

$$s_x(t) = \frac{k_i g(t)}{2} + \frac{2k_i g(t)}{\pi}\sin \omega_c t + \frac{2k_i g(t)}{3\pi}\sin 3\omega_c t ... \quad (45)$$

- the spectrum of the modulated signal contains spectral components in the baseband
- by BP filtering only the desired bandwidth is selected:



- requirements to make the filtering possible:

$$\omega_c - \omega_{mM} > \omega_{mM}; \quad 3\omega_c - \omega_{mM} > \omega_c + \omega_{mM} \rightarrow \omega_c > 2\omega_{mM} \quad (46)$$

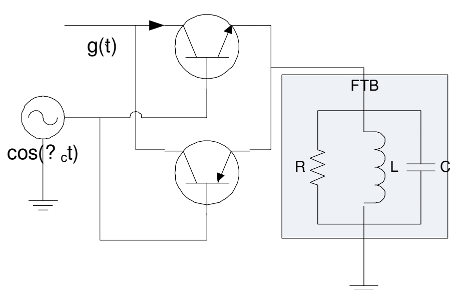
b.2. Balanced Chopper

- the balanced chopper performs the multiplication with switching function $f_s(t)$:

$$f_s(t) = 2f_i(t) - 1; \quad f_s(t) = \begin{cases} 1 & \text{if } V_0 \cos \omega_c t > 0; \\ -1 & \text{if } V_0 \cos \omega_c t \le 0; \end{cases} \quad f_s(t) = \frac{4}{\pi}\sin \omega_c t + \frac{4}{2\pi}\sin 3\omega_c t ... \quad (47)$$

- it is implemented with two complementary transistors;

$$s_x(t) = \frac{4k_i g(t)}{\pi}\sin \omega_c t + \frac{4k_i g(t)}{3\pi}\sin 3\omega_c t ... \quad (47')$$



Balanced chopper DSB modulator

- it does not generate spectral components in the baseband and generates a double amplitude of the modulated signal
- requirements to make the filtering possible:

$$3\omega_c - \omega_{mM} > \omega_c + \omega_{mM} \rightarrow \omega_c > \omega_{mM} \quad (48)$$

Effects of the imperfect implementation of the Hilbert transform upon the phase-shifting method for SSB

- the SSB signal is:

$$s_{SSB}(t) = \frac{1}{2}g(t)\cos \omega_c t \mp \frac{1}{2}\hat{g}(t)\sin \omega_c t; \ - \text{for sup SB}; + \text{for inf SB}; \quad (49)$$

- considering $g(t) = \cos\omega_m t$, if due imperfect implementation of the Hilbert transform the phase shift is expressed by (50) instead of $-\pi/2$:

$$\Phi(\omega_m) = -\pi/2 + \varphi \quad (50)$$

then the SSB-inf would be expressed by (51) as shown in Annex 1.

$$I(t)+Q(t)=\frac{1}{2}\cos\frac{\varphi}{2}\cdot\cos[(\omega_c-\omega_m)t+\frac{\varphi}{2}]+\frac{1}{2}\sin\frac{\varphi}{2}\cdot\sin[(\omega_c+\omega_m)t+\frac{\varphi}{2}]; \quad (51)$$

- the SSB-inf is attenuated by cos(φ/2)/2 and phase-shifted by φ/2, which distorts the signal
- the SSB-sup (which should be suppressed) has a level that depends proportionally on φ/2

- Conclusion: φ should be as small as possible φ→ 0

Considerations regarding the Peak to Average Power Ratio (PAPR) of the LM signals

a. Comparison between the powers of AM (DSB-C), DSB-SC and SSB

$$P_{AM}=\frac{g_c^2}{2}+\frac{g_c^2\cdot m^2\cdot\tilde{f}^2}{2};\quad P_{DSB-SC}=\frac{g_c^2\cdot m^2\cdot\tilde{f}^2}{2}=\frac{g_M^2\cdot\tilde{f}^2}{2};\quad P_{SSB}=\frac{g_M^2\cdot\tilde{f}^2}{4};$$
$$\frac{P_{AM}}{P_{DSB-SC}}=1+\frac{1}{m^2\cdot\tilde{f}^2}\geq 2;\quad \frac{P_{DSB-SC}}{P_{SSB}}=2; \quad (52)$$

b. PAPR considerations -

$$\text{PAPR [dB]} = 10\lg(P_p/P_{av}) \quad (53)$$

- the Peak to Average Power Ratio is depending of the crest-factor of the modulating signal
- for random signals SSB is to be employed since it has the smallest PAPR value
- for deterministic signals DSB-SC should be employed
- the signal BW should also be considered
- a high PAPR leads to spectral and signal distortions in HP-RF amplifiers – this issue will be discussed later

<u>Annex 1</u>

Effects of the imperfect implementation of the Hilbert transform upon the phase-shifting method for SSB

- not required for the exam
- we prove expression (51)
- the SSB signal is:

$$s_{SSB}(t)=\frac{1}{2}g(t)\cos\omega_c t\mp\frac{1}{2}\hat{g}(t)\sin\omega_c t;\ -\text{ for sup SB; }+\text{ for inf SB;} \quad (54)$$

- if due imperfect implementation of the Hilbert transform the phase shift is expressed by (50) instead of –π/2:
- consider $g(t) = \cos\omega_m t$; on the in-phase branch we would have:

$$I(t)=\frac{1}{2}\cos\omega_m t\cdot\cos\omega_c t=\frac{1}{4}[\cos(\omega_c t-\omega_m t)+\cos(\omega_c t+\omega_m t)] \quad (55)$$

- on the quadrature branch the phase shift equals (50) instead of –π/2;
- so the signal on this branch would be:

$$Q(t)=\frac{1}{2}\cos(\omega_m t-\frac{\pi}{2}+\varphi)\cdot\sin\omega_c t=$$
$$=\frac{\cos\varphi}{4}[\cos(\omega_c t-\omega_m t)-\cos(\omega_c t+\omega_m t)]-\frac{\sin\varphi}{4}[\sin(\omega_c t-\omega_m t)+\sin(\omega_c t+\omega_m t)] \quad (56)$$

- by adding the signals of the two branches, (55) and (56), to get the SSB-inf, we obtain:

$$I(t)+Q(t)=\frac{1+\cos\varphi}{4}\cos(\omega_c t-\omega_m t)+\frac{1-\cos\varphi}{4}\cos(\omega_c t+\omega_m t)]-\frac{\sin\varphi}{4}\sin(\omega_c t-\omega_m t)+\frac{\sin\varphi}{4}\sin(\omega_c t+\omega_m t)] \quad (57)$$

- (57) can be easily shown to be equal to (51) by some trigonometric manipulations.