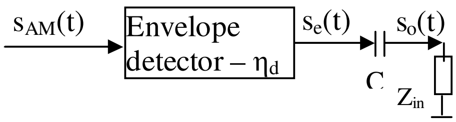
Demodulation of the LM signals – principles

1. ***Non-coherent demodulation of the DSB-C (AM) signals***

$$s_{AM} = \frac{V_0 \cdot g_c}{V_{ref}}(1+\frac{g_M \cdot f(t)}{g_c})\cos\omega_c t = g_c(1+mf(t)\cos\omega_c t; \quad \text{for } V_0 = V_{ref}; \tag{1}$$

- the demodulation is performed by an envelope detector (see annex 2) followed by the suppression of the d.c. component (HP filtering);



MA non-coherent demodulator with envelope detector

- the envelope detector extracts a signal that is proportional to amplitude variation (i.e. the envelope) of the input AM modulating signal:

$$s_e(t) = g_c \cdot \eta_e(1+m\cdot f(t)) = g_c \cdot \eta_e + g_c \cdot \eta_e \cdot m \cdot f(t) \tag{2}$$

- after the HP-filtering of the d.c. component by the capacitor C and the Z_{in} of the audio amplifier, the demodulated signal is:

$$s_o(t) = g_c \cdot \eta_e \cdot m \cdot f(t)) = A \cdot f(t) \tag{3}$$

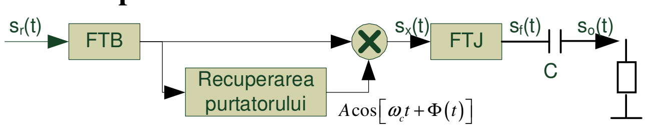
- the operation principles of the main types of envelope detectors are presented in Annex 2

2. Demodulation of the LM signals

- the general expression of the LM signals (4) does not allow for the demodulation with the simple envelope detection, but more elaborate methods are required;

$$s_{LM}(t) = \frac{\alpha}{2}g(t)\cos\omega_c t \mp \frac{1}{2}g_q(t)\sin\omega_c t; \tag{4}$$

Coherent-product LM demodulator



- it consists of a multiplication between the received modulated signal and a locally recovered carrier followed by a LP filtering (with a cut-off frequency $f_t > f_{mM}$) and the removal of the d.c. component (if any), performed by the HPF made of capacitor C and Z_{in}, the input impedance of the audio amplifier.

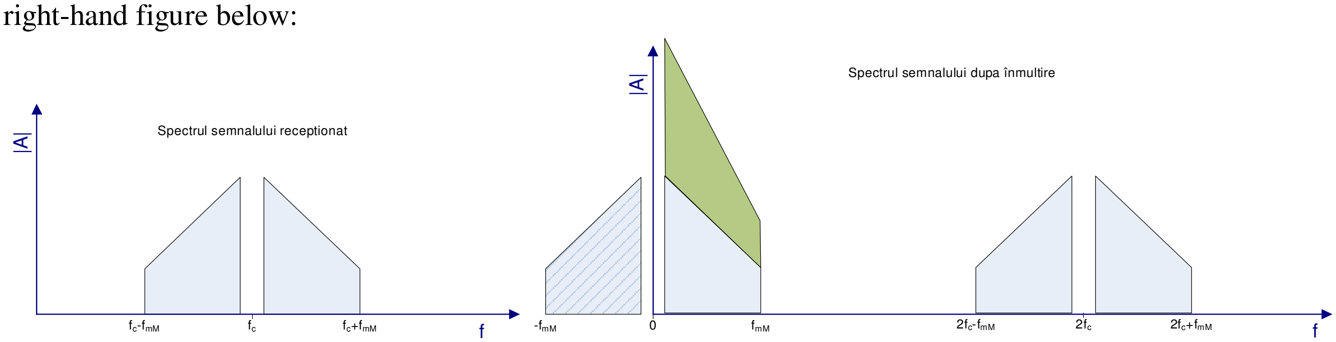
- the local carrier has a frequency offset $d\omega = 2\pi \cdot df$ and an initial phase offset Φ_0, which generate a time-varying phase offset $\Phi(t)$ expressed as:

$$\Phi(t) = 2\pi \cdot df \cdot t + \Phi_0 = \Delta\omega \cdot t + \Phi_0 \tag{5}$$

- the principle of this demodulation method is described by:

$$\begin{aligned} s_x(t) &= (\frac{\alpha g(t)}{2}\cos\omega_c t \mp \frac{g_q(t)}{2}\sin\omega_c t)\cdot A_0\cos(\omega_c t+\Phi(t))/V_{ref} = \\ &= \frac{A_0\alpha g(t)}{4V_{ref}}[\cos\Phi(t)+\cos(2\omega_c t+\Phi(t)] \mp \frac{A_0\alpha g_q(t)}{4V_{ref}}[-\sin\Phi(t)+\sin(2\omega_c t+\Phi(t)] \end{aligned} \tag{6}$$

- the multiplication splits the spectrum of the modulated signal, which is around f_c, into two spectra, see the right-hand figure below:



Spectra of the modulated LM signal s_{LM} (left) and local-carrier multiplied LM signal s_x (right)

- one in the baseband, limited by $f_{mM,}$ the first term in both square brackets;
- one centered around $2f_c$, which has the same BW as the modulated signal.
- if we impose that: $\qquad \omega_{mM} < 2\,\omega_c - \omega_{mM} \leftrightarrow \omega_{mM} < \omega_c \qquad (7)$

then the spectrum around $2f_c$ can be removed by LP filtering from the $s_x(t)$ signal.

- the filtered signal $s_f(t)$ is:

$$s_f(t) = \frac{A_0\alpha g(t)}{4V_{ref}}\cos\Phi(t) \mp \frac{A_0\alpha g_q(t)}{4V_{ref}}(-\sin\Phi(t)); \Rightarrow s_f(t) = \frac{A_0\alpha g(t)}{4V_{ref}} \quad \text{for } \Phi(t) \to 0 \qquad (8)$$

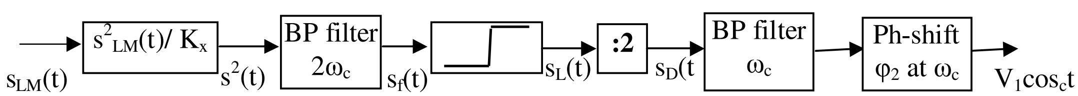
- the corresponding efficiency factor of this demodulator equals: $\eta_e = A_0/(2V_{ref})$ (9)

Methods employed for the carrier recovery

- the locally generated carrier signal should have the same phase as the incoming modulated signal so that the demodulation should deliver an undistorted signal (except for AM signals)
- recalling (5), we have: $\omega_l t = \omega_c t + d(\omega t) + \Phi_0 = \omega_c t + \Phi(t)$; for a correct demodulation we need $\Phi(t) \to 0$.
- this operation called "carrier recovery" involves the "extraction" of a phase-reference signal, followed sometimes by an initial synchronization to remove Φ_0 and a dynamic synchronization to compensate for $d(\omega t)$
- the carrier recovery methods are divided into: methods employing only the received modulated signal and methods employing additional pilot signals sent by the transmitter.

The Quadratic method for the carrier recovery

- it employs only the received modulated signal;
- the operating principle is described below considering (for simplicity) $s_{LM}(t) = g(t)\cos\omega_c t$



Quadratic method carrier recovery – block diagram

$$s^2(t) = \frac{g^2(t)}{2K_x} + \frac{g^2(t)}{2K_x}\cos 2\omega_c t\,; \qquad (10)$$

$$s_f(t) = \frac{g^2(t)}{2K_x}\cos 2\omega_c t\,; \qquad (11)$$

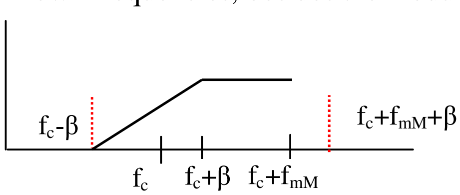
- after a narrow-band BP filtering the signal transformed into a rectangular one by a clamping amplifier:

$$s_L(t) = \frac{4V}{\pi}\sum_{k=0}^{n}\frac{\sin 2(2k+1)\omega_c t}{2k+1} \qquad (12)$$

- then, being regarded as a digital signal, it is divided by 2 in frequency:

$$s_D(t) = \frac{4V}{\pi}\sum_{k=0}^{n}\frac{\sin(2k+1)\omega_c t}{2k+1} \qquad (13)$$

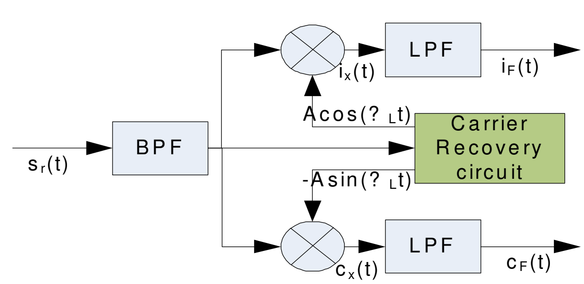
- a last narrow-band BP filtering, centered around ω_c, retains the first term of the sum (13), delivering the *sinusoidal* recovered carrier, *$4V(\sin\omega_c t)/\pi$.*
- the processing chain described above inserts a constant phase shift φ_1 of the recovered carrier, refered to the received signal. The output phase-shift circuit inserts an additional phase-shift of $\varphi_2 = \pi/2 - \varphi_1$, to deliver the recovered carrier signal ($-\cos\omega_c t$). Optionally, this phase-shift can be removed by the synchronization, see the lectures on Baseband transmissions.
- another class of carrier recovery methods employs the transmission of additional pilot signals, on a priori known frequencies, besides the modulated signal



- as an example, the complex TV signal includes two pilot signals;
- the VSB signal lies between $[f_c-\beta, f_c+f_{mM}]$ with the vestige $[f_c-\beta, f_c]$ of the suppressed sideband; used in analog TV broadcast
- two additional pilot signals are transmitted on $f_c-\beta$, and $f_c+f_{mM}+\beta$. This carrier recovery method is described in [Ed.Nicolau].

Demodulation of the QAM signals

- the QAM demodulator uses the fact that the two carrier signals are relatively orthogonal (see eq (23) in the previous LM lecture)
- the demodulator's block diagram is shown in the neighbouring figure.



QAM demodulator – block diagram

- assuming that the transmission channel is ideal, the received signal (after the input BP filtering) should be written as (14):

$$s_{rf}(t) = g_I(t)\cos(\omega_c t) - g_Q(t)\cdot\sin(\omega_c t) \tag{14}$$

- the locally generated carrier signals exhibit a pulsation shift $d\omega$ and an initial phase shift Φ_0 relative to the incoming carrier signals and hence are as:

$$s_{IL}(t) = A\cos((\omega_c + d\omega)t + \Phi_0); \quad s_{QL}(t) = -A\sin((\omega_c + d\omega)t + \Phi_0) \tag{15}$$

- the signal after the multiplier on the *in-phase branch* $i_x(t)$ is expressed by:

$$\begin{aligned} i_x(t) &= \frac{s_{rf}(t)}{V_{ref-I}}\cdot s_{IL}(t) = \frac{g_I(t)\cos(\omega_c t) - g_Q(t)\cdot\sin(\omega_c t)}{V_{ref-I}} A\cos((\omega_c + d\omega)t + \Phi_0) = \\ &= \frac{A}{V_{ref-I}}\left[g_I(t)\cos(\omega_c t)\cdot\cos((\omega_c + d\omega)t + \Phi_0)\right] - \frac{A}{V_{ref-I}}\left[g_Q(t)\cdot\sin(\omega_c t)\cdot\cos((\omega_c + d\omega)t + \Phi_0)\right] = \\ &= \frac{A}{V_{ref-I}}\left\{g_I(t)\cdot\left[\frac{\cos(d\omega t + \Phi_0) + \cos((2\omega_c + d\omega)t + \Phi_0)}{2}\right]\right\} - \\ &- \frac{A}{V_{ref-I}}\left\{g_Q(t)\cdot\left[\frac{\sin(d\omega t + \Phi_0) + \sin((2\omega_c + d\omega)t + \Phi_0)}{2}\right]\right\} = \\ &= \frac{A}{2\cdot V_{ref-I}}\left[g_I(t)\cos(d\omega t + \Phi_0) - g_Q(t)\sin(d\omega t + \Phi_0)\right] + \\ &+ \frac{A}{2\cdot V_{ref-I}}\left[g_I(t)\cos((2\omega_c + d\omega)t + \Phi_0) - g_Q(t)\sin((2\omega_c + d\omega)t + \Phi_0)\right] \end{aligned} \tag{16}$$

- the signal after the multiplier on the *quadrature branch* $c_x(t)$ is expressed by:

$$\begin{aligned} c_x(t) &= \frac{s_{rf}(t)}{V_{ref-Q}}\cdot s_{QL}(t) = \frac{g_I(t)\cos(\omega_c t) - g_Q(t)\cdot\sin(\omega_c t)}{V_{ref-Q}}\cdot\left(-A\sin((\omega_c + d\omega)t + \Phi_0)\right) = \\ &= \frac{A}{V_{ref-Q}}\left[-g_I(t)\cos(\omega_c t)\cdot\sin((\omega_c + d\omega)t + \Phi_0)\right] - \frac{A}{V_{ref-Q}}\left[-g_Q(t)\cdot\sin(\omega_c t)\cdot\sin((\omega_c + d\omega)t + \Phi_0)\right] = \\ &= \frac{A}{V_{ref-Q}}\left\{-g_I(t)\cdot\left[\frac{\sin((2\omega_c + d\omega)t + \Phi_0) + \sin(d\omega t + \Phi_0)}{2}\right]\right\} - \\ &- \frac{A}{V_{ref-Q}}\left\{-g_Q(t)\cdot\left[\frac{\cos(d\omega t + \Phi_0) - \cos((2\omega_c + d\omega)t + \Phi_0)}{2}\right]\right\} = \\ &= \frac{A}{2\cdot V_{ref-Q}}\left[-g_I(t)\sin(d\omega t + \Phi_0) + g_Q(t)\cos(d\omega t + \Phi_0)\right] + \\ &+ \frac{A}{2\cdot V_{ref-Q}}\left[-g_I(t)\sin((2\omega_c + d\omega)t + \Phi_0) - g_Q(t)\cos((2\omega_c + d\omega)t + \Phi_0)\right] \end{aligned} \tag{17}$$

- the signals $\boldsymbol{i_x(t)}$ (16) and $\boldsymbol{c_x(t)}$ (17) are fed to the LP filters. These filters are intended to suppress the spectral components placed around the 2·fc frequency.
- the signals resulting at the outputs of the two LP filters are:

$$\begin{aligned} i_F(t) &= \frac{A}{2\cdot V_{ref-I}}\left[g_I(t)\cos(d\omega t + \Phi_0) - g_Q(t)\sin(d\omega t + \Phi_0)\right] \\ c_F(t) &= \frac{A}{2\cdot V_{ref-Q}}\left[-g_I(t)\sin(d\omega t + \Phi_0) + g_Q(t)\cos(d\omega t + \Phi_0)\right] \end{aligned} \tag{18}$$

- choosing the amplitude of the local carrier signals and the value of the reference voltage, so that:

$$\frac{A}{2\cdot V_{ref-I}} = \frac{A}{2\cdot V_{ref-Q}} = 1 \tag{19}$$

and assuming a perfect recovery of the local carrier signals, i.e.:

$$d\omega = 0; \quad \Phi_0 = 0; \tag{20}$$

- the signals at the outputs of the LP filters become (substituting (19) and (20) in (18)):

$$\begin{aligned} i_F(t) &= g_I(t)\cos(0) - g_Q(t)\sin(0) = g_I(t) \\ c_F(t) &= -g_I(t)\sin(0) + g_Q(t)\cos(0) = g_Q(t) \end{aligned} \tag{21}$$

Effects of the incorrect recovery of the locally generated carriers

a. the local carriers have the same frequency as the received ones, but exhibit a phase-shift relative to the received ones, i.e.:

$$d\omega = 0; \quad \Phi_0 = ct. \tag{22}$$

- the signals at the LP filters' outputs would be (substituting (19) and (22) in (18)):

$$\begin{aligned} i_F(t) &= g_I(t)\cos(\Phi) - g_Q(t)\sin(\Phi) \\ c_F(t) &= -g_I(t)\sin(\Phi) + g_Q(t)\cos(\Phi) \end{aligned} \tag{23}$$

- since the phase-shift Φ_0 is constant, and the functions $\cos(\Phi_0)$ and $\sin(\Phi_0)$ would have constant values, the signal demodulated on one of the two branches would be composed of the signal transmitted on that branch, attenuated by $\cos(\Phi_0)$, summed with the signal sent on the other branch, weighted by $\sin(\Phi_0)$ – the phenomenon is denoted by *inter-carrier interference*

- since $|\cos\Phi_0| < 1$, for $\Phi \neq 0°$, the demodulated signal might suffer a significant attenuation

b. the locally generated carriers have a frequency offset w.r.t. the frequency of the received carriers

- in this case, $d\omega \neq 0$, the demodulated signals would have the expression (18), for $(\Phi_0) = 0°$, i.e. the demodulated signals are no longer proportional to the signals transmitted on the two branches. They are weighted by time-variable signals and other time-variable signals are added to them. →

- for a correct demodulation of the QAM signals, the locally generated carriers should be synchronized to the transmitted ones, $d\omega = 0$, as for any coherent demodulator.

- note that in practice a small value of Φ_0 ($\neq 0$) is acceptable

Remark: The quadratic method for the carrier recovery cannot be applied to the QAM modulated signals due to the fact that they are bi-dimensional signals. Prove this statement!

Signal-to-noise ratio performances of the linear modulations

Considerations regarding the Gaussian (White) Noise

- assuming the modulated signal is transmitted across a channel with a flat frequency characteristic, the recived signal can be expresed as:

$$s_r(t) = s_t(t) + n(t) \tag{24}$$

- in most theoretical analyses of the performance provided by various modulations, the noise signal $n(t)$ is modeled as a „white noise" having a normal (Gaussian) distribution of its amplitude, i.e. a AWGN (Additive White Gaussian Noise) signal.

def. A noise signal is denoted as ***white*** **if** its power spectral distribution is uniform (constant) over a very large frequency range.

- the white noise power spectral density is denoted by N_0, and defines the average power of this signal within a given bandwidth BW, i.e. $P_{n\text{-}av} = BW \cdot N_o$. Usually the measuring unit of N_0 is:

$$[N_0] = \frac{dBm}{kHz} \tag{25}$$

def. Gaussian noise – a random signal whose amplitude has a normal (Gaussian) distribution, i.e. the probability that the value on $n(t)$, at time instant t to be x is given by its probability density function $pdf(x,\sigma,\mu)$, expressed by::

$$P(n(t) = x) = pdf(x,\sigma,\mu) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{26}$$

where μ denotes the average (mean) value, while σ denotes its variance.

Note: the power of the noise signal is directly proportional to σ^2, i.e. $\sigma^2 = N_0 \cdot BW$.

- the mean value of AWGN is zero, i.e. $\mu = 0$.

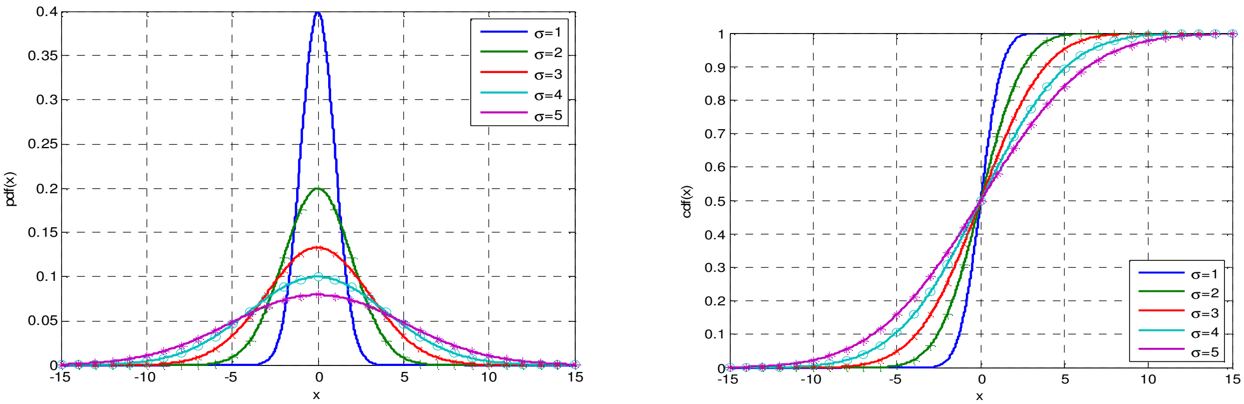
- the probability that the noise value would be smaller than a given value *x*, at time instant *t*, which is the cumulative distribution function (*cdf(x)*), is expressed as:

$$P\left(n(t)<x\right)=cdf\left(x,\sigma,\mu\right)=\int_{-\infty}^{x} pdf\left(\tau,\sigma,\mu\right)d\tau=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{(\tau-\mu)^2}{2\sigma^2}\right)d\tau=\frac{1}{2}\left[1+erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right] \quad (27)$$

wher erf(x) (*error function*) is the function defined as::

$$erf\left(x\right)=\frac{2}{\sqrt{\pi}}\int_{-\infty}^{x} e^{-\tau^2}d\tau \quad (28)$$

- the pdf(x) and cdf(x) of a Gaussian random variable are presented in the figures below for various values of its variance **σ** and a null mean value



PDF and CDF of the Gaussian noise for different values of σ ($\mu=0$)

- the probability of the noise signal to have a value greater or equal to *x*, is of particular importance for communications, this probability being expressed by the so-called Q function, $Q\left(\frac{x-\mu}{\sigma}\right)$, defined as:

$$P\left(n(t)>x\right)=Q\left(\frac{x-\mu}{\sigma}\right)=\int_{x}^{\infty} pdf\left(\tau,\sigma,\mu\right)d\tau=1-cdf\left(x,\sigma,\mu\right)==\frac{1}{2}\left[1-erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right] \quad (29)$$

- since the received signal is band-pass filtered at the receiver's input with a pass-band equaling the bandwidth of the modulated signal BW, the modulated signal is actually summed with narrow-band noise.
- the power of the narrow-band noise (P_N) is then:

$$P_N=N_0\cdot BW=\sigma^2 \quad (30)$$

- a narrow-band noise can be expressed as a QAM signal [Haykin]:

$$n(t)=n_I(t)\cos\left(2\pi f_c t\right)-n_Q(t)\sin\left(2\pi f_c t\right) \quad (31)$$

- in (31) $n_I(t)$ and $n_Q(t)$ are Gaussian-distributed random signals, with zero-mean and variance σ; their bandwidth is BW/2 and their spectral power density equaling N_0.
- knowing that:

$$a\cdot\cos(x)-b\cdot\sin(x)=\sqrt{a^2+b^2}\cdot\cos\left(x-\tan^{-1}\left(\frac{b}{a}\right)\right) \quad (32)$$

the signal defined in (31) can be rewritten as [Proakis1]:

$$n(t)=R(t)\cdot\cos\left(\omega_c t+\Psi(t)\right) \quad (33)$$

where: $$R(t)=\sqrt{n_I^2(t)+n_Q^2(t)};a.\quad \Psi(t)=-\tan^{-1}\left(\frac{n_I(t)}{n_Q(t)}\right);b. \quad (34)$$

- the envelope of the BP-filtered noise, expressed by (34), is a random signal with a Rayleigh distribution, [Lathi], [Haykin]:

$$P_R(x)=\frac{x}{\sigma_R{}^2}e^{-x^2/2\sigma_R^2} \quad (35)$$

- the inserted phase shift $\Psi(t)$ has a uniform distribution in the $[0,2\pi]$ interval.

SNR performance of coherently-demodulated linear modulations on AWGN channels

- considering the general expression of LM signals see (40) in the previous LM lecture, and the narrow-band noise expressed by (31), the expression of the received signal affected by noise is:

$$s_r(t) = s_{ML}(t) + n(t) = \left[\frac{\alpha}{2} g(t) + n_I(t)\right]\cos(\omega_c t) - \left[\pm\frac{1}{2} g_q(t) + n_Q(t)\right]\sin(\omega_c t) \tag{36}$$

- the signal/noise ratio, i.e. the ratio of the received signal's power at the demodulator's input (expressed in the previous LM lecture by (16) for DSB-C, (26) for DSB-SC and (35) for SSB) over the power of the narrow-band noise, expressed by (30), equals:

- **DSB-C (BLD+P)**

$$\rho_{DSBC-i} = \frac{P_{DSBC}}{P_{N-i}} = \frac{\dfrac{g_c^2\left(1+m^2\widetilde{f^2}(t)\right)}{2}}{N_0 \cdot BW} = \frac{g_c^2\left(1+m^2\widetilde{f^2}(t)\right)}{2\cdot N_0 \cdot 2 f_{mM}}; \tag{37}$$

- **DSB-SC (BLD-PS)**

$$\rho_{DSBSC-i} = \frac{P_{DSB-SC}}{P_{N-i}} = \frac{\dfrac{g_M^2\widetilde{f^2}(t)}{2}}{N_0 \cdot BW} = \frac{g_M^2\widetilde{f^2}(t)}{2N_0 \cdot 2 f_{mM}}; \tag{38}$$

- **SSB (BLU)**

$$\rho_{SSB-i} = \frac{P_{SSB}}{P_{N-i}} = \frac{\dfrac{g_M^2\widetilde{f^2}(t)}{4}}{N_0 \cdot BW} = \frac{g_M^2\widetilde{f^2}(t)}{4N_0 \cdot f_{mM}}; \tag{39}$$

- if the received signal (36) is fed into the coherent ML demodulator, the demodulated signal will be, see (6) and (8):

$$s_f(t) = \frac{A_0}{2}\left[\frac{\alpha}{2} g(t) + n_I(t)\right] \tag{40}$$

- provided that the local carrier is synchronized to the received carrier, the SNR at the demodulator's output, after the removal of the d.c. component (when present), would be expressed for different LM modulations (different values of α) by:

- the power of the received demodulated signal is expressed by (41) for a periodic signal g(t) :

$$P_{s-o} = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|\frac{A_0\alpha}{4} g(t)\right|^2 dt = \frac{A_0^2\alpha^2 g_M^2}{16}\cdot\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f^2(t)\,dt = \frac{A_0^2\alpha^2 g_M^2}{16}\cdot\widetilde{f^2}(t) \tag{41}$$

- replacing the value of α, we get the expressions of the average power for different LM modulations at the demodulator's output, see (41) and (42) of the first LM lecture:

  - DSB

$$P_{DSB-o} = \frac{A_0^2\alpha^2 g_M^2}{16}\cdot\widetilde{f^2}(t) = \frac{A_0^2 2^2 g_M^2}{16}\cdot\widetilde{f^2}(t) = \frac{A_0^2 g_M^2}{4}\cdot\widetilde{f^2}(t) \tag{42}$$

  - SSB

$$P_{SSB-o} = \frac{A_0^2\alpha^2 g_M^2}{16}\cdot\widetilde{f^2}(t) = \frac{A_0^2 1^2 g_M^2}{16}\cdot\widetilde{f^2}(t) = \frac{A_0^2 g_M^2}{16}\cdot\widetilde{f^2}(t) \tag{43}$$

- the noise power at the output of the demodulator in DSB transmissions will be: two sidebands!

$$P_{N-o} = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|\frac{A_0}{2} n_I(t)\right|^2 dt = \frac{A_0^2}{4}\cdot\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} n_I^2(t)\,dt = \frac{A_0^2}{4}\cdot 2\cdot N_0\cdot BW = \frac{A_0^2}{2}\cdot N_0\cdot f_{mM} \tag{44}$$

while for the SSB it will be (only one sideband!):

$$P_{N\_BLU-o} = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|\frac{A_0}{2} n_I(t)\right|^2 dt = \frac{A_0^2}{4}\cdot\lim_{T\to\infty}\frac{1}{T}\int_{0}^{\frac{T}{2}} n_I^2(t)\,dt = \frac{A_0^2}{4}\cdot N_0\cdot LB = \frac{A_0^2}{4}\cdot N_0\cdot f_{mM} \tag{45}$$

- using the above relations the signal-to-noise ratio (*in linear representation*) would become:

- **DSB-C** – after the removal of the d.c. component:

$$\rho_{DSBC-o} = \frac{P_{DSBC-o}}{P_{N-o}} = \frac{\frac{A_0^2}{4} g_M^2 \cdot \widetilde{f^2}(t)}{\frac{A_0^2}{4} \cdot N_0 \cdot f_M} = \frac{g_c^2 m^2 \cdot \widetilde{f^2}(t)}{2 \cdot N_0 \cdot f_{mM}}; \tag{46}$$

- the ratio between the two signal-to-noise ratios ρ_{DSBC-o} and ρ_{DSBC-i} is:

$$\eta = \frac{\rho_{DSBC-o}}{\rho_{DSBC-i}} = \frac{\frac{g_c^2 m^2 \cdot \widetilde{f^2}(t)}{2 \cdot N_0 \cdot f_{mM}}}{\frac{g_c^2\left(1 + m^2 \widetilde{f^2}(t)\right)}{2 \cdot N_0 \cdot 2 f_{mM}}} = \frac{2m^2 \widetilde{f^2}(t)}{1 + m^2 \widetilde{f^2}(t)} \tag{47}$$

- the ratio η (47) represents the „demodulator's gain"; for m = 1 and f(t) = cosω_mt, we get η= 2/3.

- **DSB-SC**

$$\rho_{DSBSC-o} = \frac{P_{DSBSC-o}}{P_{N-o}} = \frac{\frac{A_0^2}{4} g_M^2 \cdot \widetilde{f^2}(t)}{\frac{A_0^2}{2} \cdot N_0 \cdot f_M} = \frac{g_M^2 \cdot \widetilde{f^2}(t)}{2 \cdot N_0 \cdot f_{mM}}; \tag{48}$$

and the demodulator's gain for DSB-SC equals:

$$\eta = \frac{\rho_{DSBSC-o}}{\rho_{DSBSC-i}} = \frac{\frac{g_M^2 \cdot \widetilde{f^2}(t)}{2 \cdot N_0 \cdot f_{mM}}}{\frac{g_M^2 \cdot \widetilde{f^2}(t)}{2 \cdot N_0 \cdot 2 f_{mM}}} = 2 \tag{49}$$

- **SSB**

$$\rho_{SSB-o} = \frac{P_{SSB-o}}{P_{N\_SSB-o}} = \frac{\frac{A_0^2 g_M^2}{16} \cdot \widetilde{f^2}(t)}{\frac{A_0^2}{4} \cdot N_0 \cdot f_M} = \frac{g_M^2 \cdot \widetilde{f^2}(t)}{4 \cdot N_0 \cdot f_{mM}}; \tag{50}$$

and the demodulator's gain is:

$$\eta = \frac{\rho_{SSB-o}}{\rho_{SSB-i}} = \frac{\frac{g_M^2 \cdot \widetilde{f^2}(t)}{4 \cdot N_0 \cdot f_{mM}}}{\frac{g_M^2 \cdot \widetilde{f^2}(t)}{2 \cdot N_0 \cdot f_{mM}}} = 1 \tag{51}$$

- the DSB-SC's gain is two times greater than SSB's gain due to the two side bands! This is its only advantage

SNR performance of DSB-C demodulated with an envelope detector on AWGN channels

- the DSB-C (AM) signal affected by the narrow-band noise is expressed by:

$$s_r(t) = s_{MA}(t) + n(t) = \left\{g_c\left[1 + m \cdot f(t)\right] + n_I(t)\right\} \cdot \cos(\omega_c t) - n_Q(t)\sin(2\pi f_c t) \tag{52}$$

- by using (32) equation (52) can be expressed as:

$$s_r(t) = \sqrt{\left\{g_c\left[1 + m \cdot f(t)\right] + n_I(t)\right\}^2 + n_Q^2(t)} \cdot \cos(\omega_c t + \Theta(t)) \tag{53}$$

- the envelope detector extracts the envelope of this signal, proving an output signal expressed by:

$$A_r(t) = \sqrt{\left\{g_c\left[1 + m \cdot f(t)\right] + n_I(t)\right\}^2 + n_Q^2(t)} \tag{54}$$

- we will consider now the two extreme cases of the ratio between the level of the modulated signal's envelope and the noise level. Then we get:

a) Envelope level significantly greater than noise level, i.e. the probability that the signal level is greater than noise level is close to unity, $P\left(g_c\left[1 + m \cdot f(t)\right] >> n(t)\right) \approx 1$

- in this case the signal at the envelope-detector's output can be approximated by:

$$A_r(t) \approx \sqrt{\left\{g_c\left[1 + m \cdot f(t)\right] + n_I(t)\right\}^2} = g_c\left[1 + m \cdot f(t)\right] + n_I(t) \tag{55}$$

- (55) shows that at the demodulator's output we get approximately the same signal as at the output of a coherent demodulator, the difference being given by a multiplicative constant $A_0\alpha/(4V_{ref})$, see (8) for DSB-C. Therefore, the signal-to-noise ratio at the demodulator's output would be expressed by (46), while the demodulator's gain is expressed by(47).
- the above considerations and (55) show that if the signal-to-noise ratio's value at the demodulator' input is great, the non-coherent demodulator provides about the same performance as the coherent demodulator.

b) ***Envelope level significantly smaller than the noise level***, i.e. the probability that the signal level is smaller than noise level is close to unity, $P\left(g_c\left[1+m\cdot f\left(t\right)\right]<<n\left(t\right)\right)\approx 1$

- the analysis of this particular case is presented in Annex 3
- the expression of the signal at the output of the envelope detector is, see Annex 3:

$$A_r\left(t\right)\approx R\left(t\right)+\frac{g_c n_I\left(t\right)}{R\left(t\right)}\left[1+m\cdot f\left(t\right)\right] \tag{56}$$

- expression (56) shows that the demodulated signal is no longer proportional to the modulating function $f\left(t\right)$; the noise is no longer added to the useful signal, but it is multiplied to the useful signal (*due to the non-linear processing performed by the demodulator)*. Moreover, this product is added to a noise-signal R(t) with a Rayleigh distribution. In this case the output signal is „dominated" by noise generating the so-called *"noise captured" demodulation*.
- note that AM demodulated with a non-coherent demodulator is the only LM modulation that exhibits such a phenomenon, which is common for the FM – to be discussed in the FM lectures.

Conclusions regarding the SNR performance of linear modulations

- the "noise captured" demodulation occurs only for non-coherently demodulated AM.
- the demodulator's gain (or SNR improvement factor) η, has the greatest value for DSB-SC, $\eta = 2$ (but it uses a double BW); for SSB $\eta = 1$ (but it uses a half-bandwidth). The value of η for AM is $\eta \leq 2/3$. (*its value is also depending on the modulating signal's shape).*

References

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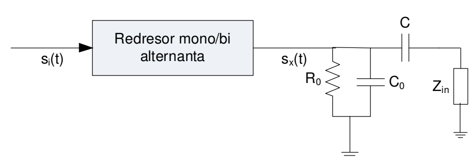
Annex 2

Envelope detectors **- this annex is not required for the exam**

- the task of the envelope detector is to extract the baseband signal, "contained" in the envelope of the modulated signal
- the task is accomplished in two steps: a non-linear processing, in this case a one-way or a two-way rectification, followed by a low-pass filtering;
- a HPF can be inserted to suppress the d.c. component, if needed,– see the neighbouring figure;



AM non-coherent detector with a rectifier envelope detector

Averaging detector

- it usually employs a one-way rectification of the input signal (a diode) followed by a LP filtering.
- since the diode is controlled by the amplitude of the input signal, the ideal diode acts like an interrupter;
- rewriting the Fourier series of the interruption function, see (34) in the first LM lecture, *for an ideal diode!,* we get (57) because the diode is open with the frequency of the carrier signal**:**

$$f_i(s_i(t))=\begin{cases}1 & \text{if } s_i(t)>0;\\ 0 & \text{if } s_i(t)\leq 0;\end{cases}\qquad f_i(t)=\frac{1}{2}+\frac{2}{\pi}\sin\omega_c t+\frac{2}{3\pi}\sin 3\omega_c t\ldots \tag{57}$$

- the output signal is the product of the input signal with the interruption function (controlled by the input signal); in (59) A(t) denotes the envelope of the modulated signal, i.e. (59).
- if the spectra of the first two terms of (59) are separable, i.e. : $f_{mM} < f_c - f_{mM} \leftrightarrow 2f_{mM} < f_c$ (58)

the low-pass filtering extracts only the baseband signal.

$$s_r(t) = s_i(t)\cdot f_i(s_i(t)) = A(t)\sin(\omega_c t - \Theta(t))\cdot[\frac{1}{2}+\frac{2}{\pi}\sin(\omega_c t-\Theta(t))+\frac{2}{3\pi}\sin 3(\omega_c t-\Theta(t))...] =$$
$$= \frac{A(t)}{2}\sin(\omega_c t-\Theta(t))+\frac{2A(t)}{\pi}\sin(\omega_c t-\Theta(t))\sin(\omega_c t-\Theta(t))+\frac{2A(t)}{3\pi}\sin(\omega_c t-\Theta(t))\sin 3(\omega_c t-\Theta(t))... \quad (59)$$
$$= \frac{2A(t)}{2\pi}+\frac{A(t)}{2}\sin(\omega_c t-\Theta(t))-\frac{A(t)}{\pi}K\cdot\cos 2(\omega_c t-\Theta(t))+..$$

- as seen in (59), the one-way rectification (due to its non-linearity) inserts more spectra that contain the modulating signal, out of which one is the modulating signal's baseband spectrum and the others care centered around the harmonics of the carrier frequency.
- to perform the LP filtering of the signal (59) the R_0C_0 constant should observe:

$$R_0C_0 < \frac{1}{2\pi f_{mM}}; \quad R_0C_0 > \frac{1}{2\pi f_c}; \quad (60)$$

- the output signal is expressed by the first term of the last line in (59), where $\eta_e = 1/\pi$ is called the detector's efficiency (constant).
- then, the d.c. removal is performed by the right-hand RC group of the above figure
- the above considerations hold for an ideal diode;
- for the real diode the analysis is more complex [see ref. E. Nicolau], due to the opening voltage of the diode.
- denoting by U_o the opening voltage of the diode, the modulated signal's amplitude A(t) should observe (61) to ensure the operation of detector, for a modulation index equaling *m*:

$$A(t) > 4U_o/(1-m) \quad (61)$$

- *if condition* (61) *is fulfilled, the envelope detector is considered to be an average detector*
- if condition (61) is not fulfilled all the time by the input signal → the detector acts like *a peak detector*
- due to the fact that for some alternations of the input signal (the modulated signal), the diode is not open (due to their small amplitudes), the output signal does not follow the envelope of the modulated signal, generating the "non-pursuit" distortion, see ref. E. Nicolau
- as shown in the above-mentioned reference, this distortion could be decreased if:

$2R_e(C+C_0) < 1/(2\pi f_{mM})$; $R_e = Z_{in} \parallel R_0/2$; - condition which ensures $f_t > f_{mM}$ (62)

$2Z_{in}C > R_0C_0$ – condition for the non-occurrence of the "non-pursuit" distortion (63)

- this shortcoming could be avoided (decreased) if an active OpAmp-based 2-way rectifier is employed.

Annex 3 - this annex is not required for the exam

SNR performance of DSB-C demodulated with an envelope detector on AWGN channels

b) ***Envelope level significantly smaller than the noise level***, i.e. the probability that the signal level is smaller than noise level is close to unity, $P\left(g_c\left[1+m\cdot f(t)\right] << n(t)\right)\approx 1$

- in this case the signal at the envelope-detector's output can be approximated by:

$$\begin{aligned} A_r(t) &= \sqrt{\left\{g_c\left[1+m\cdot f(t)\right]+n_I(t)\right\}^2+n_Q^2(t)} = \\ &= \sqrt{g_c^2\left[1+m\cdot f(t)\right]^2+2g_c\left[1+m\cdot f(t)\right]n_I(t)+n_I^2(t)+n_Q^2(t)} = \\ & \sqrt{\left(n_I^2(t)+n_Q^2(t)\right)\cdot\left[1+\frac{2g_c n_I(t)}{n_I^2(t)+n_Q^2(t)}\left[1+m\cdot f(t)\right]\right]+g_c^2\left[1+m\cdot f(t)\right]^2} \end{aligned} \quad (64)$$

- since the noise is much greater than the signal's envelope, the term $g_c^2\left[1+m\cdot f(t)\right]^2$ is significantly smaller than the first term, so it can be neglected. Then, taking into account (34), relation (64) is approximated by:

$$A_r(t)\approx\sqrt{\left(n_I^2(t)+n_Q^2(t)\right)\cdot\left[1+\frac{2g_c n_I(t)}{n_I^2(t)+n_Q^2(t)}\right]} == R(t)\sqrt{1+\frac{2g_c n_I(t)}{R^2(t)}\left[1+m\cdot f(t)\right]} \quad (65)$$

- if we approximate $\sqrt{1+\varepsilon}\approx 1+\frac{\varepsilon}{2}$ for ε sufficiently small, (65) becomes (66) which proves (56):

$$A_r(t)\approx R(t)\left[1+\frac{g_c n_I(t)}{R^2(t)}\left[1+m\cdot f(t)\right]\right] == R(t)+\frac{g_c n_I(t)}{R(t)}\left[1+m\cdot f(t)\right] \quad (66)$$