Quadrature Amplitude Modulation - QAM
- this chapter presents the Quadrature Amplitude Modulation for the case when the amplitude of the modulating signal has a finite number of discrete values, i.e. they are RRC-filtered PAM signals.
- besides its employment as such to transmit two independent data flows, the QAM is used as a method of modulation-demodulation of other amplitude, frequency or phase modulations, as will be shown in the subsequent chapters.

Quadrature Amplitude Modulation – QAM

- the two independent PAM signals $m_1(kT)$, $m_2(kT)$, of symbol-period T, are modulated on the orthogonal carriers $\cos\omega_c t$ and $\sin\omega_c t$
- the expression of a QAM modulated signal is:
$$s_{QAM}(t) = m_1(t) \cdot \cos\omega_c t - m_2(t) \cdot \sin\omega_c t; \qquad (1)$$

Generation of the QAM signals
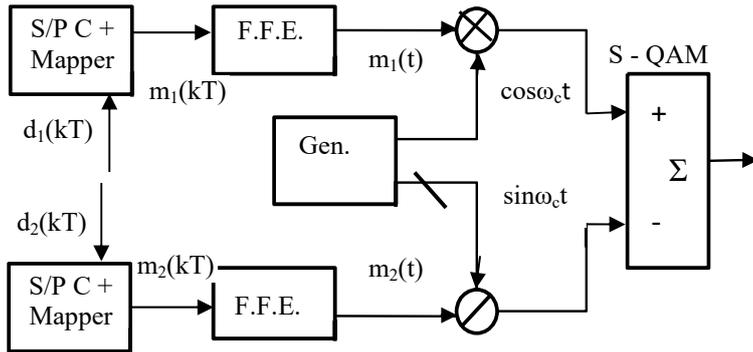- the block diagram of the QAM modulator is presented in figure 1.

Figure 1. Block diagram of the QAM modulator

- the modulating PAM signals may take 2 values (1bit/symb) or N > 2 values (multibit/symb).
- the $m_1(kT)$ and $m_2(kT)$ modulating signals should be filtered with a RRC low-pass characteristic (2), to ensure (after the completion RRC-filtering in the receiver) ISI = 0 at the sampling moments and better performances in the presence of noise:

$$N_E(\omega) = N_R(\omega) = N(\omega)^{\frac{1}{2}} = \begin{cases} 1; & \omega \in [0, \omega_N(1-\alpha)]; \\ \cos(\dfrac{\pi(\omega)}{4\omega_N\alpha} - \dfrac{\pi(1-\alpha)}{4\alpha}); & \omega \in [\omega_N(1-\alpha), \omega_N(1+\alpha)]; \end{cases} \qquad (2)$$

- the filtering delivers the modulating signals of the two branches, $m_1(t)$ and $m_2(t)$ in (1), which have continuous amplitude variations
– the bandwidth and frequency band of the modulated signals are given by (3), for LP shaping filters with the passing band up to $f_N(1+\alpha)$:
$$BW = 2f_N(1+\alpha) = f_s(1+\alpha); \quad FB = [f_c - f_N(1+\alpha); f_c + f_N(1+\alpha)]; \qquad (3)$$
- the QAM signal may be regarded as a superposition of two independent ASK signals which are transmitted in the same frequency band.

Demodulation of the QAM signals
- because the QAM signal is a superposition of two ASK signals modulated on the two orthogonal subcarriers, its demodulation is made using two ASK demodulators which use the corresponding local carrier, i.e. $\cos\omega_l t$ and $\sin\omega_l t$,
- we assume here as well that the relation between the phases of the transmitter and local receiver carrier signals, with pulsations ω_c and ω_l respectively, is:
$$\omega_l t = \omega_c t + \Delta\omega t + \Theta_0 = \omega_c t + \theta(t); \qquad (4)$$
- the block diagram of the QAM receiver is shown in figure 2; it does not figure the signals used by the local carrier and symbol-clock recovery blocks, which will described in the PSK chapter of the course, and the demapping and parallel to serial conversion blocks on each arm, which are identical to the ones used in the ASK receiver.
- the equations that describe the demodulation on the two branches i(t), the in-phase branch, and c(t), the quadrature one are (5) and (6). $i_x(t)$ and $c_x(t)$ denote the signals at the output of the multipliers and $i_F(t)$ and $c_F(t)$, denote the signals at the output of the LP-filters of the two branches.
- in the above mentioned equations $m'_1(t)$ and $m'_2(t)$ denote the modulated signals affected by the channel distortions.
- The QAM demodulation is similar to separate product-coherent demodulations of two independent DSB-SC signals.
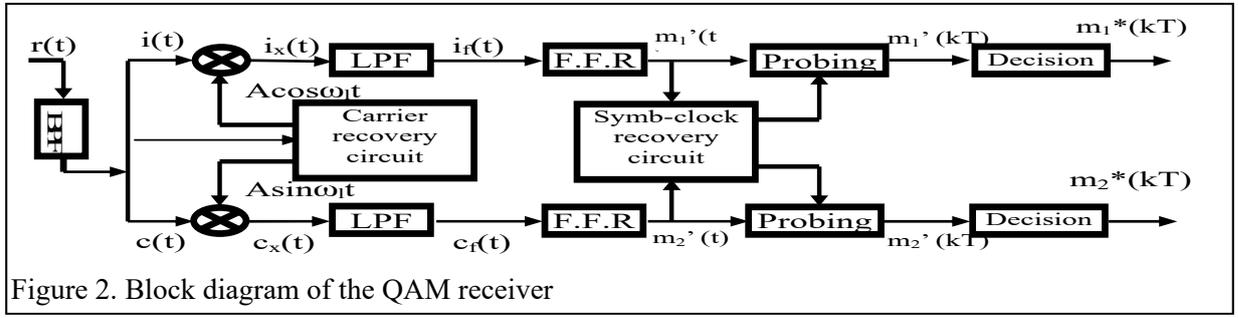
Figure 2. Block diagram of the QAM receiver

$$i_x(t) = \frac{r(t)A\cos\omega_L t}{K} = \frac{Am_1'(t)}{2K}[\cos\theta(t) + \cos(2\omega_p t + \theta(t))] - \frac{Am_2'(t)}{2K}[-\sin\theta(t) + \sin(2\omega_p t + \theta(t))] \quad (5)$$

$$c_x(t) = \frac{r(t)A\sin\omega_L t}{K} = \frac{Am_1'(t)}{2K}[\sin\theta(t) + \sin(2\omega_p t + \theta(t))] - \frac{Am_2'(t)}{2K}[\cos\theta(t) + \cos(2\omega_p t + \theta(t))] \quad (6)$$

- the LP-filters suppress the spectral components centered around $2\omega_c$ and the signals at their outputs are:

$$i_F(t) = \frac{A}{2K}(m_1'(t)\cos\theta(t) + m_2'(t)\sin\theta(t)) \rightarrow \frac{A}{2K} \cdot m_1'(t) \text{ for } \theta(t) \rightarrow 0 \quad (7)$$

$$c_F(t) = \frac{A}{2K}(m_1'(t)\sin\theta(t) - m_2'(t)\cos\theta(t)) \rightarrow \frac{A}{2K} \cdot m_2'(t) \text{ for } \theta(t) \rightarrow 0 \quad (8)$$

- if the carrier recovery circuit ensures a phase-shift $\theta(t) \rightarrow 0$, then the signals at the outputs of the shaping filters would take, in the probing time instants, values proportional to the modulating signals.
- the effects of the incorrect carrier-recovery may be derived from relations (7) and (8), see the LM coherent demodulation, and consist, for each branch, in the occurrence of a „parasitic" AM and the addition of the modulating signal of the quadrature branch, which also has a „parasitic" AM. The second undesired term in each equation is specific to the QAM transmission and is generated by the presence of a second modulated signal in the same frequency band, which affect the desired signal due to the loss of orthogonality between the channel carriers, $\theta(t) \neq 0$.
- therefore, the condition $\theta(t) \rightarrow 0$ should be imposed.
- The symbol-clock recovery circuit is intended to extract the phase-reference and synchronize the local symbol-clocks, f_{si} and f_{sc}, with the demodulated signals so that they are probed at the right time-instants.
- using the synchronized symbol-clocks, the demodulated signals $m_1'(t)$ and $m_2'(t)$ are „read" in the probing moments, when they are not affected by ISI, generating the m_{1k}' and m_{2k}' levels, that have constant values during the k-th symbol period.
- the probed levels are then delivered to the decision circuits, which decide which of the permitted PAM levels is closer to the received one, thus delivering the estimated (decided) PAM levels $m_{1k}*$ and $m_{2k}*$. If the transmission employs more bits/symbol, then from the decided levels, the corresponding multibits are demapped, according to the inverse rule of the mapping performed at the transmitter. Then the resulted bits are serialized and provided to the data sink with the clock of frequency $f_{bit} = n \cdot f_s$, where n denotes the number of bits per PAM level, i.e. $n = \log_2 M$.
- in the assumption of a perfect carrier recovery, which would preserve the orthogonality between the signals modulated on the $\cos\omega_c t$ and $\sin\omega_c t$ channel carriers, the symbol and bit error probabilities of QAM are identical to the ones of ASK.