Amplitude Shift Keying - ASK

- the Amplitude Shift Keying (ASK) is a particular case of DSB-SC modulation, see the lectures on LM, where the modulating signal has a finite number of amplitude levels, according to a modulation alphabet. The modulating signal is actually the RRC-filtered PAM signal, see the lecture on PAM.
- the particular nature of the modulating signal requires some additional operations, compared to the LM transmissions with analog modulating signals.
- the PAM signal's infinite frequency bandwidth should be limited by an RRC filtering, see the data Filtering lecture, and so the expression of the ASK signal is given by (1), where $h_{FFE}(t)$ is the impulse response of the RRC characteristic and m_k is the amplitude level of the M-PAM:

$$s_{ASK}(t) = \sum_{k=0}^{\infty}\left(m_k \cdot u_T(t-kT_S)\right) * h_{FFE}(t) \cdot V_0 \cdot \cos\omega_c t / V_{ref} = m(t) \cdot V_0 \cdot \cos\omega_c t / V_{ref} \tag{1}$$

- since the PAM modulating levels have zero mean value, the modulated signal is a DSB-SC signal.
- the ASK's frequency band and bandwidth are expressed by (2) , where f_s is the symbol frequency and α is the roll-off factor of the RRC filter.

$$FB = [f_c - f_N(1+\alpha); f_c + f_N(1+\alpha)]; \quad BW = f_s(1+\alpha) \tag{2}$$

Generation of the ASK signal

- the generation of the ASK signal involves serial to parallel conversion of the data stream to obtain the n-bit group (multibit), $n = \log_2 M$, and multibit-to-PAM-level (symbol) mapping which is performed either by D/A conversion or by look-up table, depending on the mapping rule that is used. Then, the PAM resulted signal is band limited by an RRC-shaping filter and becomes the modulating signal of a DSB-SC modulation, i.e. it is multiplied to the channel carrier signal. Finally, an output BP filtering, with a passing band expressed by (2) is applied to remove any undesired spectral components outside the allowed frequency band. Fig.1 shows the block diagram of an ASK transmitter; it doesn't contain the output BPF.

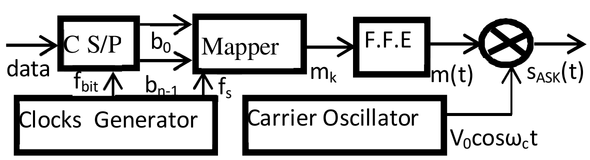

Figure 1 The block diagram of the ASK transmitter

Demodulation of the ASK signal

- the demodulation of the ASK signal involves the following steps
- Band-pass filtering meant to increase the SNR at demodulator's input. The passing band of the filter should equal the frequency band of the modulated signal
- Conversion to the base band by means of a coherent product demodulation of the DSB-SC signal; This operation provides the base-band modulated signal P'(t) which is the modulating signal affected by the channel's distortions and noise. The coherent product demodulation involves multiplication of the filtered received signal to the a local synchronized carrier followed by a low-pass filtering with a cut-off frequency $f_t = f_N(1+\alpha)$, see the LM lectures
- Filtering the demodulated signal with a LP-RRC characteristic, the FFR filter, which completes the RC filtering in order to remove the ISI
- Probing the P'(t) signal at the $t=kT_s$ instants with a locally synchronized symbol-clock signal
- Decision of the estimated permitted level, which is obtained by determining the permitted level that is closest to the received level (one-dimensional Euclidian distance)
- Demapping of the multibit mapped on the decided symbol
- Parallel to serial conversion of the multibit

- figure 2 presents the block scheme of the ASK receiver.

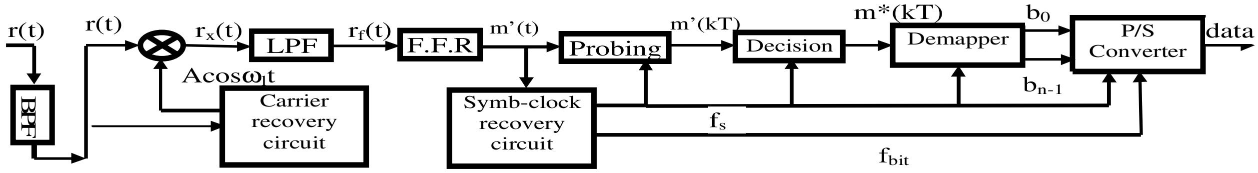

Figure 2 Block diagram of the ASK receiver

- the demodulation of the ASK signal should consider the imperfect recovery and synchronization of the local carrier, which is expressed by.

$$\omega_1 t = \omega_c t + d\omega t + \Theta_0 = \omega_c t + \Theta(t) \tag{3}$$

- assuming an imperfect synchronization, i.e., $\Theta(t) \neq 0°$, the signal expression at the multiplier's output is:

$$r_x(t) = \frac{r(t)A\cos\omega_l t}{K} = \frac{A \cdot V \cdot P'(t)}{2K}[\cos\theta(t) + \cos(2\omega_c t + \theta(t))] \quad (4)$$

- the spectral components placed around the 2[nd] harmonic of the carrier are removed by the LP filter and at its output the signal's expression is given by in the assumption the A=K:

$$r_f(t) = \frac{V}{2}(P'(t)\cos\theta(t) \to \frac{V}{2} \cdot P'(t) \text{ for } \theta(t) \to 0 \quad (5)$$

- if the phase-shift ensured by the carrier recovery circuit is very small, $\theta(t) \to 0$, the signals at the output of the shaping filters would have values proportional to the modulating signal, in the probing time instants.
- the incorrect carrier-recovery leads to the occurrence of a "parasitic amplitude modulations" of the P'(t) signal, which modifies the values in the probing instants and, finally, leads to errors after the decision block.

Carrier and symbol clock recovery

- the local carrier can be recovered using the Quadratic method, presented in the second lecture on LM.
- the signal recovered by quadratic method is used as a phase reference to synchronize a local oscillator, either by means of an analog PLL circuit or by using a digital synchronization circuit, similar to those presented in second lecture on BB transmissions, where $f_{local} = f_c$.

Error Performance of ASK

- if we assume a perfect carrier synchronization, the coherent product demodulation of ASK does not change the noise's p.d.f. and power and so it does not modify the SNR of the received signal. Moreover, as shown in the Data Filtering lecture, the RRC filtering and the probing in every symbol period also does not modify the noise's p.d.f. and power.
- therefore the symbol-error and bit-error probabilities of ASK, p_e and BER, can be computed similarly to the ones of PAM, but we have to consider that the power of the cosine carrier equals $1/2 \cdot V_0^2$ and therefore the signal power the receiver's input is decreased by 1/2. Considering that the modulated signal has a bandwidth $BW = f_s \cdot (1+\alpha)$, the noise power equals $\sigma^2 = N_0 \cdot f_s \cdot (1+\alpha)$. But after the coherent demodulation and RRC filtering, the signal power remains the same, while the noise power decreases to $\sigma^{2'} = N_0 \cdot f_s \cdot (1+\alpha)/2$, i.e. by a factor of 1/2, due to the $f_N(1+\alpha)$ bandwidth of the filtered modulating sugnal . So, actually, the SNR at the input of the probing block would be the same as the ones of a PAM transmission. So, the p_e of ASK is expressed by (6).a. In the assumption of the Gray mapping, the BER is given by (6).b.

$$p_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6P_m}{(M^2-1)\sigma^2}}\right) \approx \frac{2(M-1)}{M} \cdot \frac{\sqrt{M^2-1}}{\sqrt{6}} \cdot \frac{e^{-\frac{6}{M^2-1}\cdot\frac{\rho}{2}}}{\sqrt{2\pi\rho}}; \quad P_m = P_{m\_PAM}; a. \quad BER = \frac{p_e}{\log_2 M}; b. \quad (6)$$

- considerations regarding the numerical values of the rather high bit-error rate vs. SNR provided by the M-ASK will be presented in the second lecture on A+PSK.

Considerations regarding the definition of the signal-to-noise ratio

- the error-probabilities are usually expressed in terms of the signal to noise ratio, SNR
- but this approach is not always suitable for the comparisons between the error-performance provided by modulations that have different numbers of bits/symbol and/or occupy different frequency bandwidths.
- therefore, the error performance is (are) sometimes expressed in terms of the ratio of the energy-per-bit over the noise power spectral density, E_b/N_0
- in the SNR, the noise power is computed by multiplying the power spectral density of the noise, e.g. the constant N_0 $[V^2/(R \cdot Hz)]$ (or N_0[dBm/kHz] in logarithmic representation) of the Gaussian noise, to the bandwidth of the input filter.
- Considering the two bandwidths usually employed for the RC filtered signals, (7), the power can be expressed by (8), also using its variance σ.

$$BW = f_s \text{ – for RC } \alpha = 0 \text{ - ideal;} \qquad BW = f_s(1+\alpha) \text{ – for RC } \alpha \neq 0 \text{ - real;} \quad (7)$$

$$P_z = \sigma^2/R = N_0 \cdot f_s/R \text{ – for RC } \alpha = 0; \qquad P_z = \sigma^2/R = N_0 \cdot f_s(1+\alpha)/R \text{ – for RC } \alpha \neq 0; \quad (8)$$

- but the noise power and the value of the SNR are depending of the bandwidth of the modulated signal.
- to avoid this dependency, the SNR may be expressed by the ratio of the energy/bit to the noise power spectral density, denoted by E_b/N_0.
- the relation between the two ratios is derived in (9) for $\alpha = 0$ and in (10) for $\alpha \neq 0$. In both relations E_s and E_b denote the average energy per symbol and per bit, respectively, of the modulated signal; A denotes the carrier amplitude and *n* the number of bits/symbol, while ρ denotes the signal/noise ratio in linear values and SNR, the signal/noise ratio in logarithmic values, i.e. in dB.

$$\rho = \frac{P_s}{P_z} = \frac{A^2 \cdot R}{2R \cdot \sigma^2} = \frac{P_s \cdot T_s}{\sigma^2 \cdot T_s} = \frac{E_s}{N_0} = \frac{n \cdot E_b}{N_0}; \quad SNR[dB] = \frac{E_b}{N_0}[dB] + 10\lg n[dB]; \quad \text{for } \alpha = 0 \tag{9}$$

$$\rho = \frac{P_s}{P_z} = \frac{A^2 \cdot R}{2R \cdot \sigma^2} = \frac{P_s \cdot T_s(1+\alpha)}{\sigma^2 \cdot T_s(1+\alpha)} = \frac{E_s}{N_0(1+\alpha)} = \frac{n \cdot E_b}{N_0(1+\alpha)}; \quad SNR[dB] = \frac{E_b}{N_0}[dB] + 10\lg \frac{n}{(1+\alpha)}[dB]; \quad \text{for } \alpha \neq 0 \tag{10}$$