

# 9. Cooperative Control

**Definition:** N agents are performing shared tasks, where the tasks depend on the relationship between the contexts of the individual agents.

**Context** = state + other information

Other information:

- constraints
- predictions
- measured or estimated disturbances
- operator recommendations or requirements
- inferenced information from the past

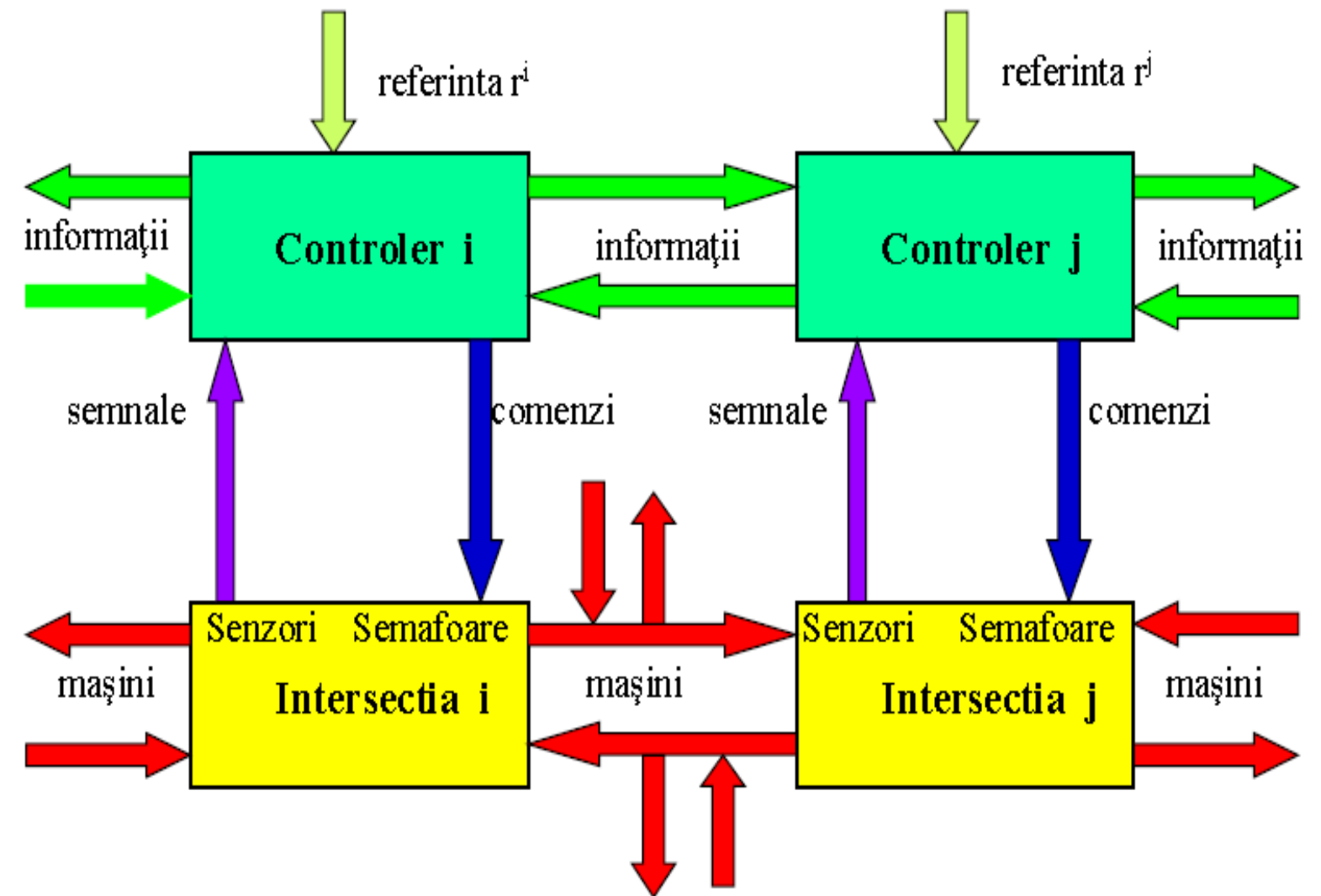
## Features:

- team vs. individual goals
- random networks
- communication delays
- asynchronous communication
- local neighbor-to-neighbor interactions
- evolve in a parallel manner

**Context-based control** vs. State-based control ← it's an evolution of the control techniques

## Utilizations:

- Power grid
- Network control (communication, traffic etc.)
  - Networking/congestion control
  - Routing/queue management
  - Servers/resource allocation
- Supply chain management
- Formation control



The group (formation) control problem can be reduced to a single-agent control problem by employing a leader-follower type control strategy.

# E. g. of Cooperative Control Systems

- birds, fish, insects moving together
- transportation problems, network systems
- human body: e. g. intestinal systems
- four wheel vehicles
  - drive systems
    - front wheel
    - rear wheel
  - steering systems
    - front wheel
    - rear wheel

Consensus dynamics = agreement dynamic:

- multi-agent systems that concerns processes by which a collection of interacting agents achieve a common goal.

Formation control: leader follower → mobile robots, unmanned air vehicles, autonomous underwater vehicles, satellites, spacecraft etc.

Cooperative monitoring of something

# Hydro Power Plant Systems

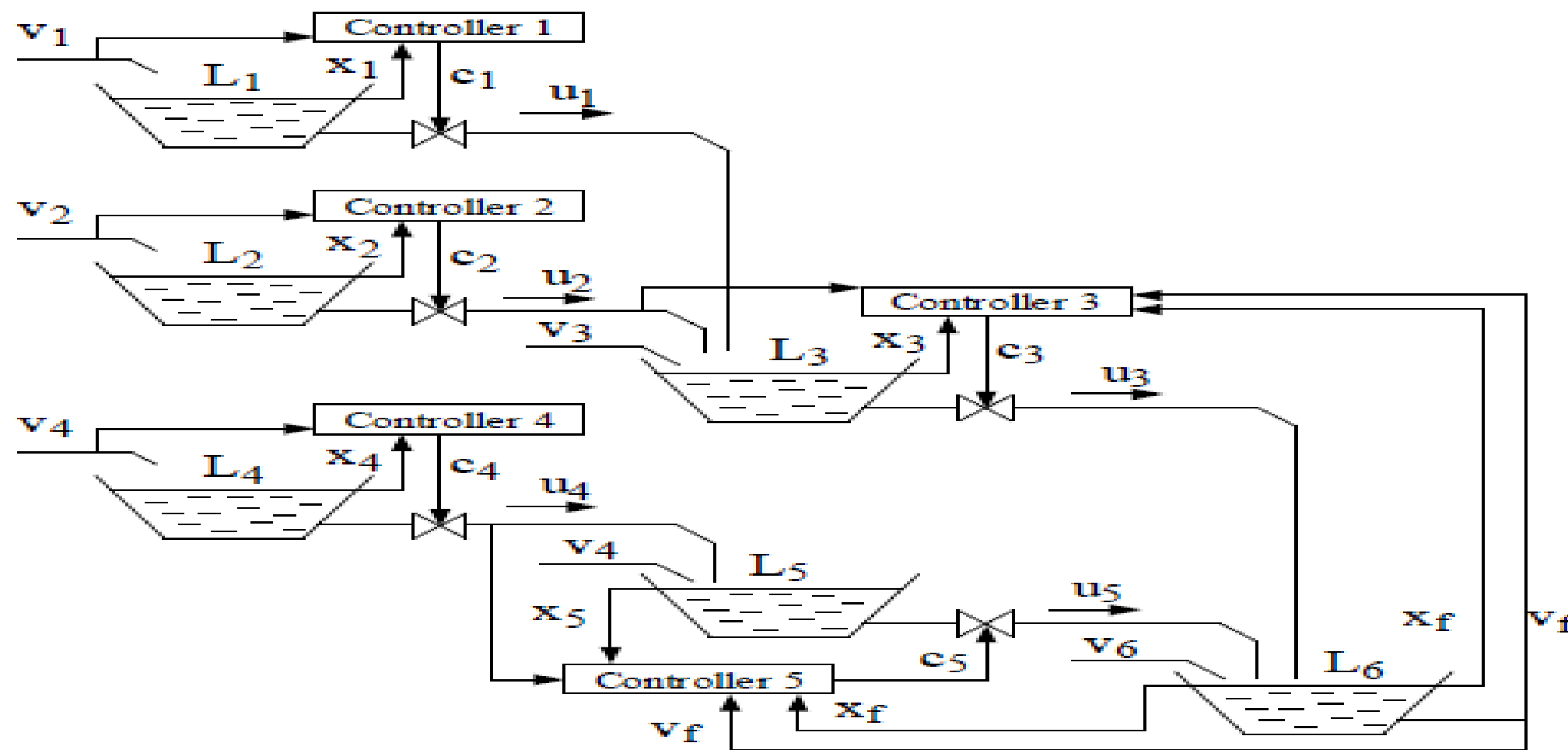


Figure 1. Lake System

**Shared goal: to maintain the power equilibrium ←  
generation = consumption**

# Cooperation and coordination

**Shared information** is a necessary condition for cooperation or coordination.

The agents must be able to communicate at least with a subset of other agents, but all must have access to a communication network.

In every cooperative control problem, there must be identifiable **cooperative objective(s)**.

⇒ Team objective(s)

**Coordination information** (variable) represents the minimal amount of information needed to perform a specific coordination objective.

Coordination information = state + other information

**Virtual structure** is a notion used to derive control strategies.

**Consensus:** A critical problem for cooperative control is to determine algorithms so that the agents can reach consensus on the values of the coordination data in the presence of

- variable agent number
- imperfect sensors
- communication dropout
- sparse communication topology
- noisy and unreliable communication links

Once a group decision for the coordination data is made, all the individual decisions are left to the individual.

The agents enter and exit asynchronously.

There are: entrance constraints and security control.

Some agents exit without noticing their neighbors.

**Resilience:** the systems works according specifications even if some agents leave the cooperation structure.

## **Motivations of cooperation:**

- distribution
- complexity
- flexibility
- reliability
- diversity

# Cooperative Control Systems Framework

## Agent dynamics

The dynamic of the  $i^{\text{th}}$  component

$$\frac{d x_i}{d t} = f_i(x_i, u_i, v_i), x_i \in R^n, u_i \in R^m, v_i \in R^p$$

$$i=1, \dots, N$$

## Shared task

The cost (objective) function:

$$J = \int_0^T L(x, u, v) dt + V(x(T))$$

$$J = \sum_{i=1}^N (\int_0^T L_i(x_i, v_i, u_i) dt + V_i(x_i(T)))$$



## **Communication graph**

- Encodes the system information flow
- Neighbor set

## **Communication channels**

- Communication information can be lost, delayed, reordered.
- There are rate constraints.

# Strategy

**Strategy:** control action of individual agents

$$u_i = k_i(x, a)$$

a – the coordination data vector

x – the state

## Decentralized strategy

$$u_i = k_i(x_i, a_i)$$

$$u_i = k_i(x_i, a_i, v_i)$$

$$i = 1, 2, \dots$$

v – the disturbance

# Consensus seeking

Input data  $X_i \rightarrow$  Coordination algorithm  $\rightarrow$  Output data  $A_i$

Input data  $X_j \rightarrow$  Coordination algorithm  $\rightarrow$  Output data  $A_j$

The consensus can be on context as input data  $X_i$ , or on coordination data  $A_i$

The consensus can be asymptotically reached.

A consensus is global reachable if there exists an information strategy for each coordination vector that achieves global consensus asymptotically.

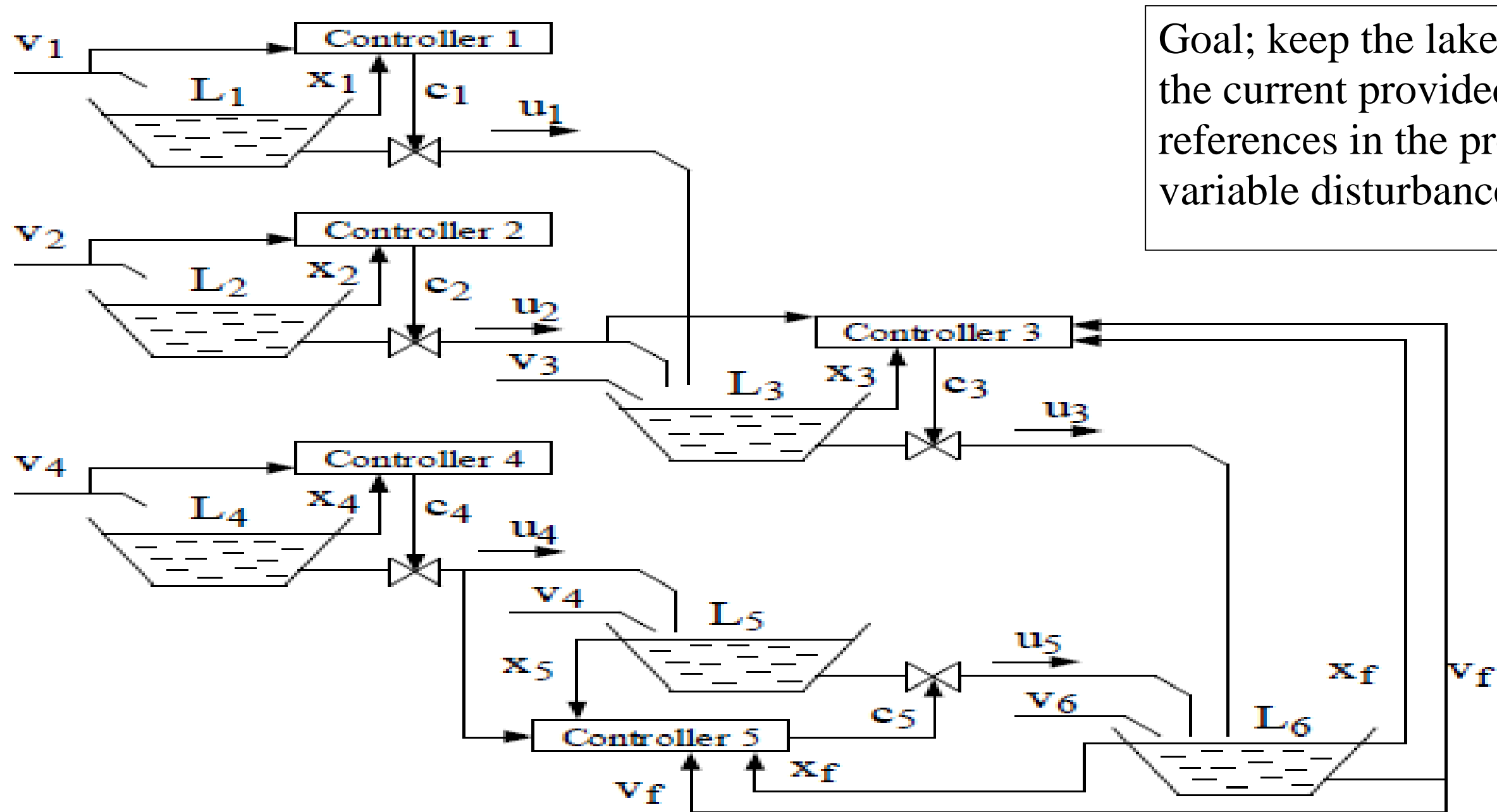
# Cooperation methods

**Implicit cooperation** is defined as the process of cooperative inference through the exchange of measurements or estimates.  
Context construction → context exchange.

**Explicit cooperation** is the process of cooperation through joint decision making or planning.

- Context construction → context exchange
- Inference of coordination data
- Agreement on coordination data
- Inference of the control decisions i

# Lake System



Goal; keep the lake level at the current provided references in the presence of variable disturbances.

Figure 1. Lake System

where:

$v_i$  – disturbance  $\rightarrow$  uncontrolled input ( $\frac{m^3}{s}$ );

$x_i$  – current level ( $m$ );

$u_i$  – output flow ( $\frac{m^3}{s}$ )  $\rightarrow$  controlled output;

$c_i$  – control signal ( $\frac{m^3}{s}$ );

One lake model:

–nominal values:

$$v_i^0 = u_i^0 = u_{prev}^0 + d_i^0$$

–disturbance error:

$$v_i(k) - v_i^0 = d_i(k) - v_i^0 + u_{prev}(k) - d_i^0$$

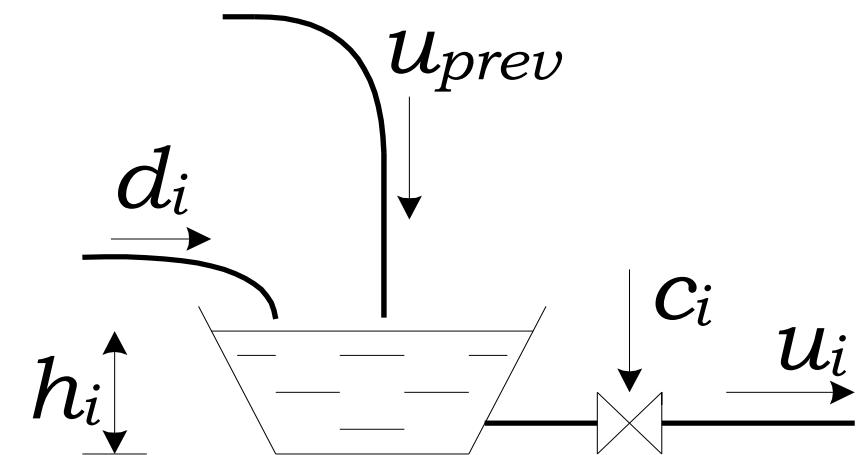
–level error:

$$x_i(k) = r_i^0 - h_i(k)$$

–surface:

$$S_i(k) = S_i^0 + g_i(r_i + x_i(k)) \bullet (r_i + x_i(k))$$

$$\Delta V_i(k+1) = S_i(k) \cdot (x_i(k+1) - x_i(k)) = v_i(k) - u_i(k)$$



Discrete model: nonlinear recurrence

$$x_i(k+1) = x_i(k) + \frac{u_{prev}(k) + d_i(k) - u_i(k)}{S_i^0 + g_i(r_i + x_i(k)) \cdot (r_i + x_i(k))}$$

$$u_i(k) = k \cdot c_i(k)$$

## Homework:

- . conceive the OER-TPN model of a lake
- . conceive the OER-TPN model the 6 lakes system
- . Conceive the OER-TPN model of an independent controller of the lake level
- .

# Independent control

How can fuzzy logic context base-control be applied to the level of an independent lake?

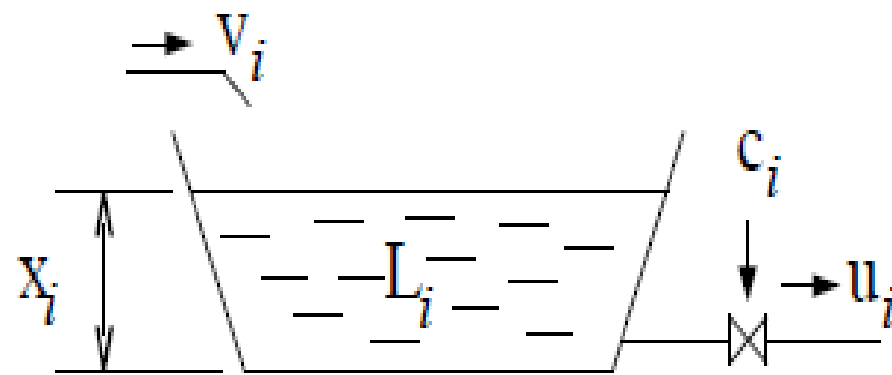
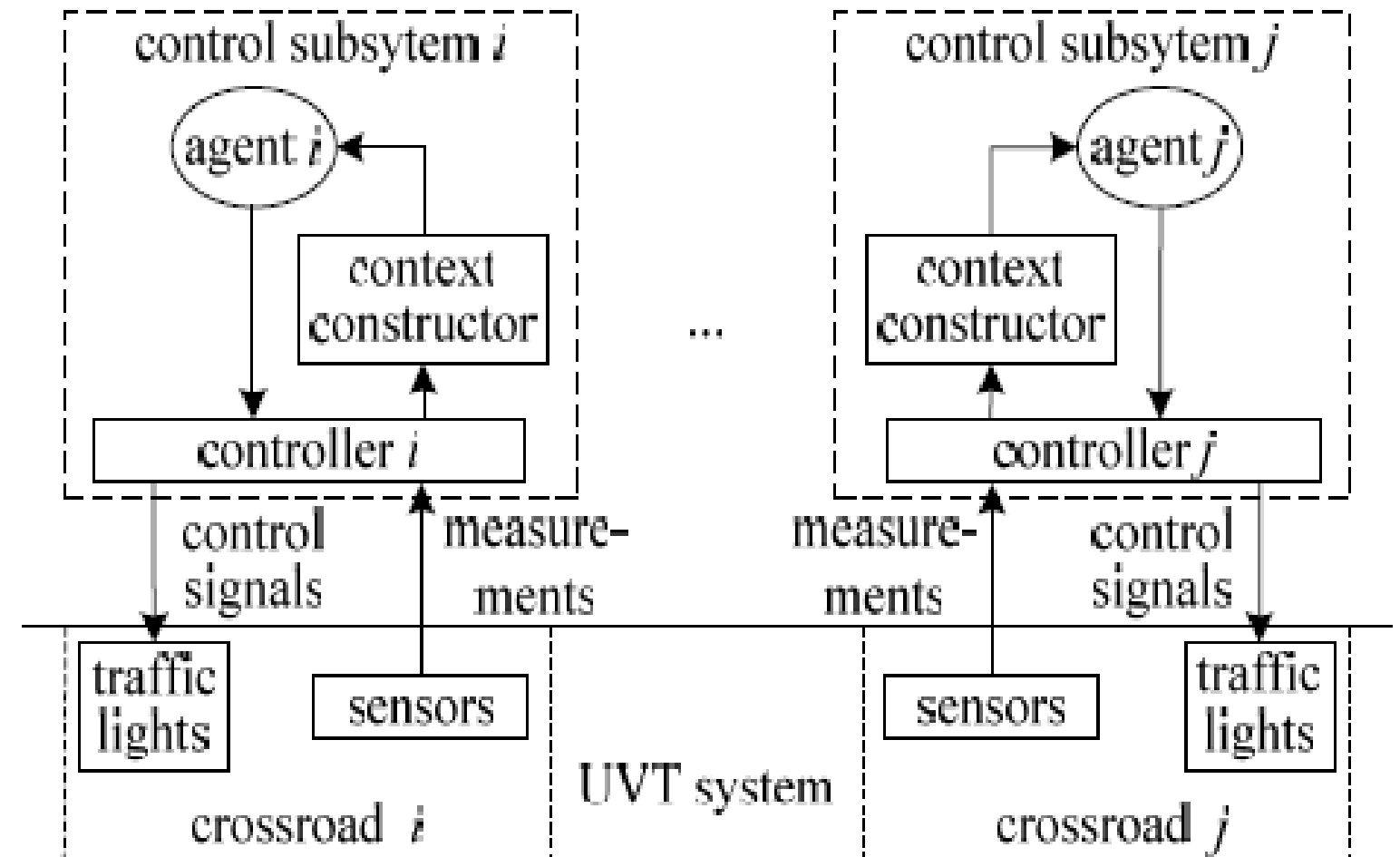


Figure 2. Model for a lake.





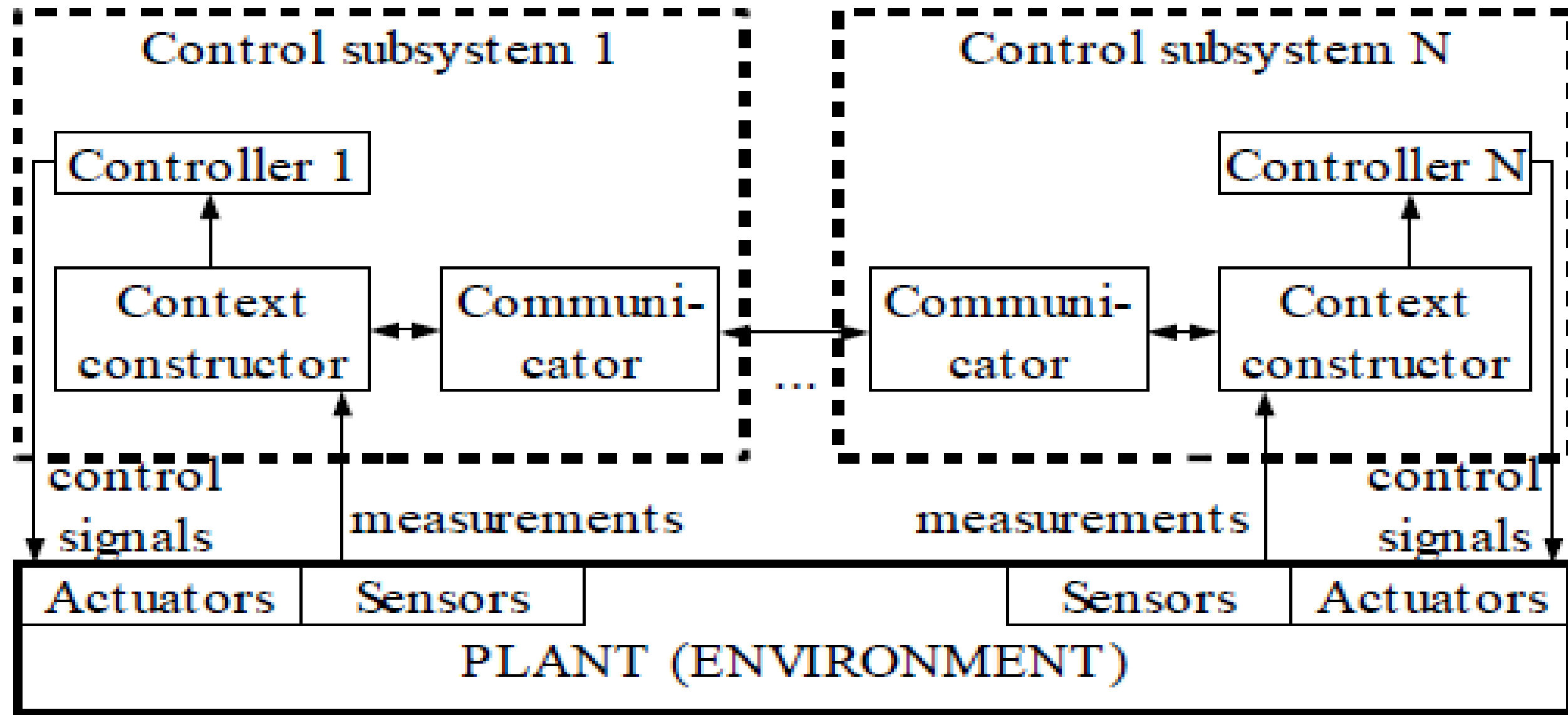


Figure 3. Implicit cooperative control.

How can fuzzy logic be applied to implement the implicit cooperative (lake level) control of a set of lakes?

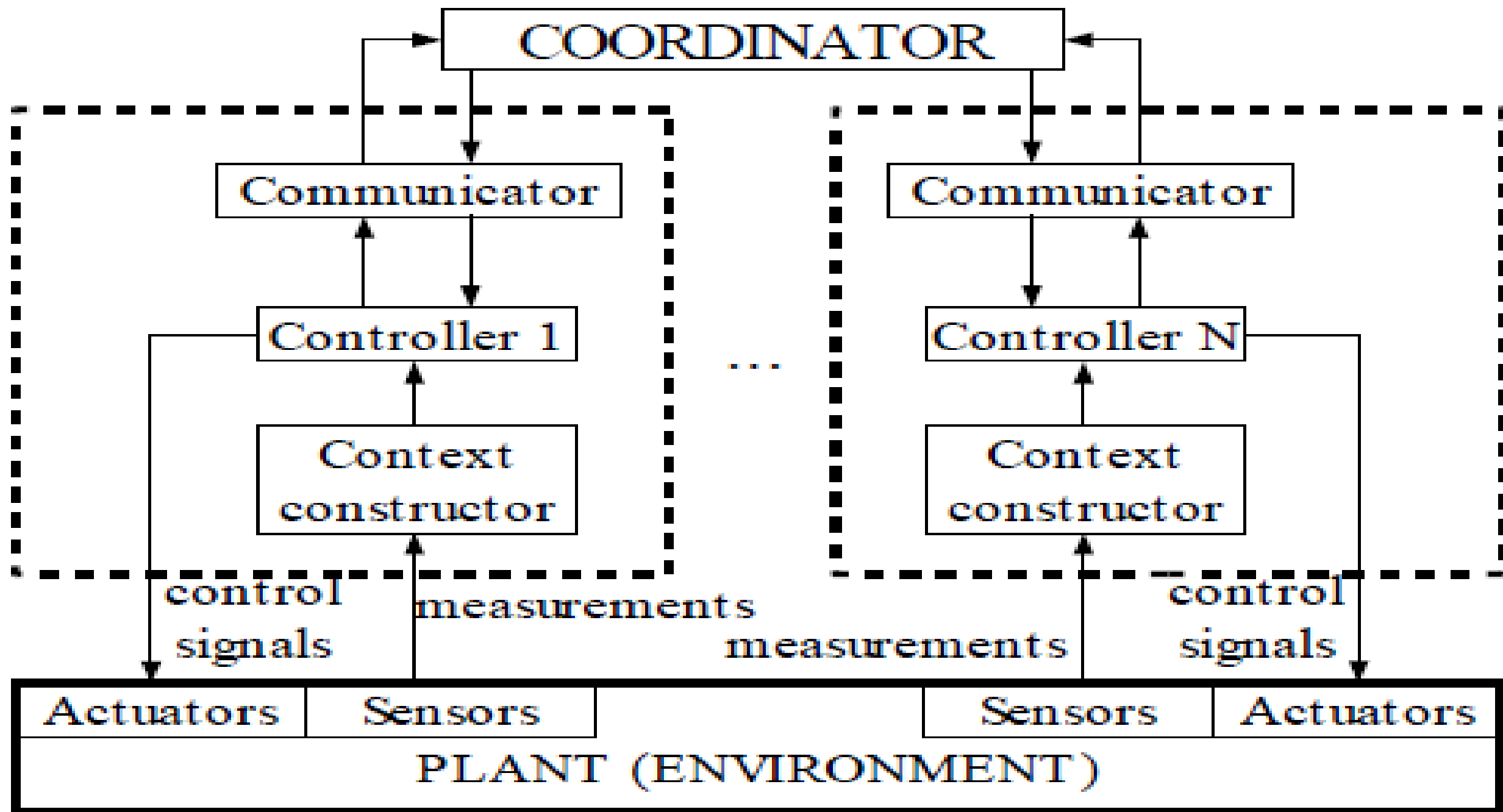


Figure 4. Coordinated Control.

How can fuzzy logic be applied to implement the coordinated (lake level) control of a set of lakes?

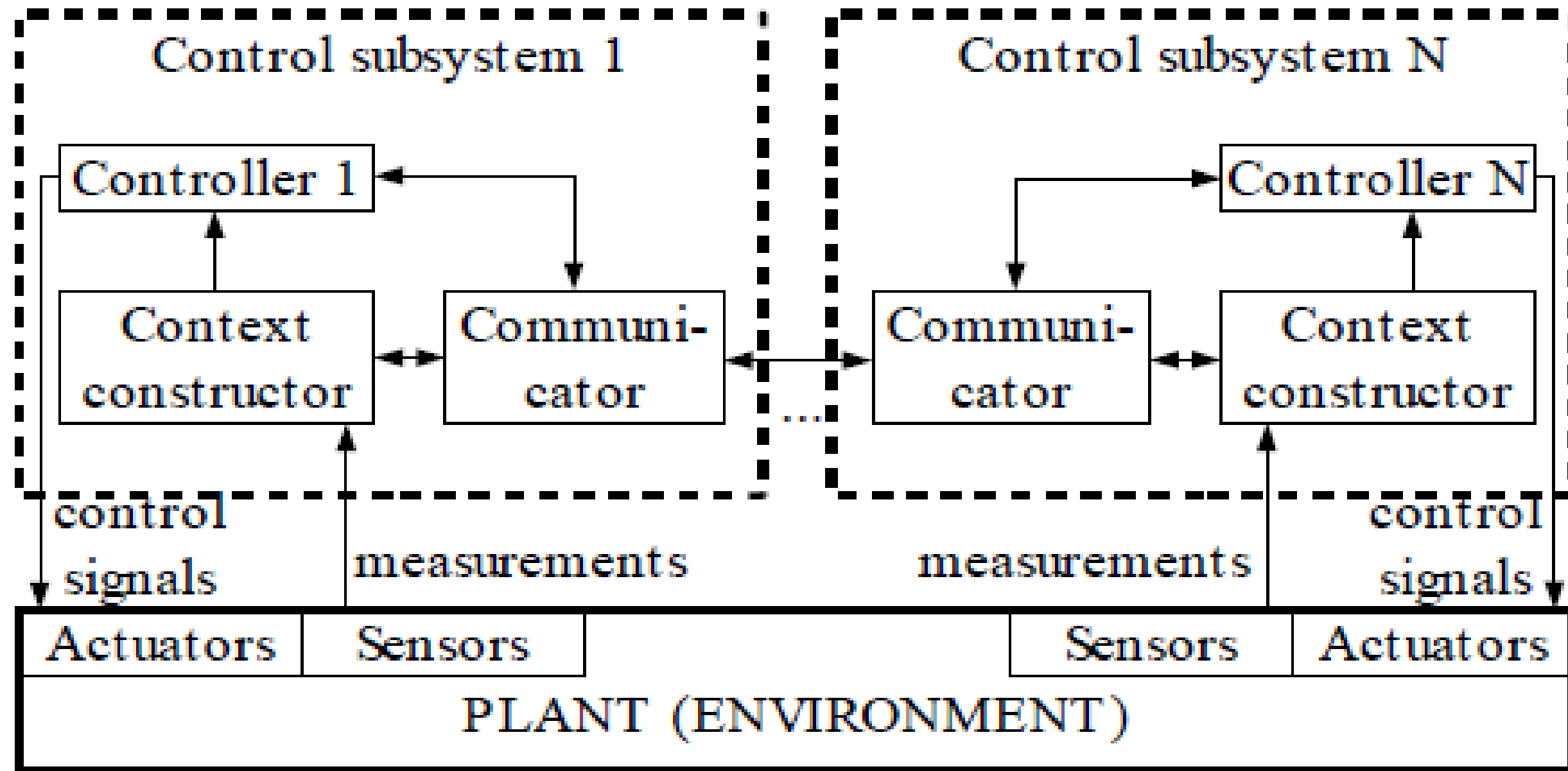


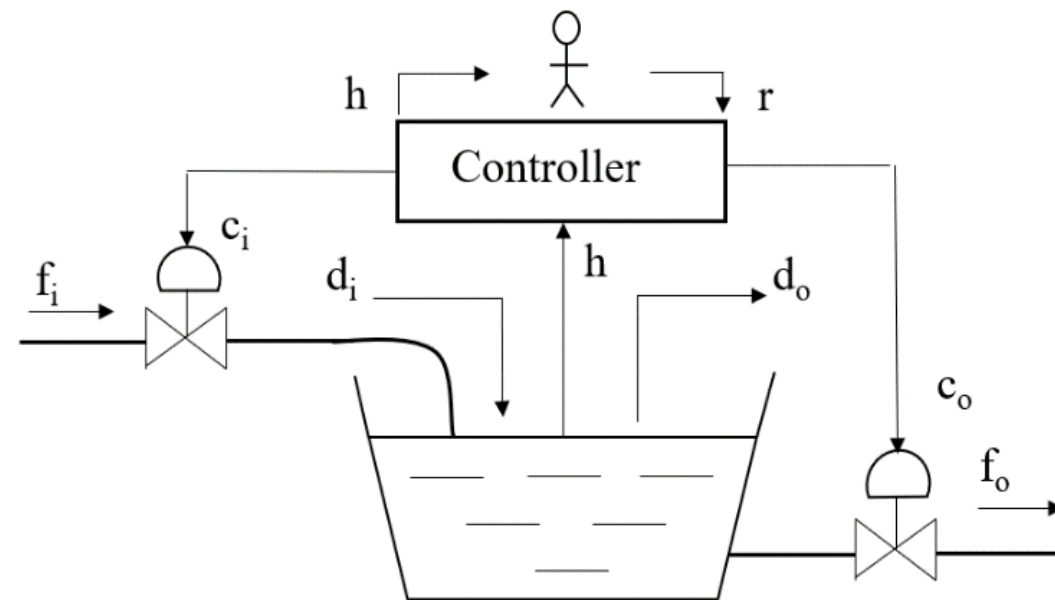
Figure 5. Explicit cooperative control.

How can fuzzy logic be applied to implement the explicit cooperative (lake level) control of a set of lakes?

## Homework: Controlul nivelului unui lac

Notății:

- $h$ : nivelul apei din lac
- $f_i$  flux de intrare controlat
- $f_o$  flux de ieșire controlat
- $d_i$  perturbație de intrare
- $d_o$  perturbație de ieșire
- $c_i$  semnal de control al fluxului de intrare
- $c_o$  semnal de control al fluxului de ieșire



Evolution relations: ???

Ecuția dinamică a lacului (model mathematic):

$$h(\tau) = h(\tau-1) + f_i(\tau-1) \cdot c_i(\tau-1) - h(\tau-1) \cdot c_o(\tau-1) + d_i(\tau-1) - d_o(\tau-1)$$

Condiție inițială:  $h(0) = h_0$ ,  $c_i = c_{i0}$ ;  $c_o = c_{o0}$ ; (date).

$p_7$  furnizează  $h(\tau)$

$$f_i(\tau-1), f_o(\tau-1), d_i(\tau-1), d_o(\tau-1) \geq 0.$$

$$c_i(\tau-1), c_o(\tau-1) \in [0, 1]; \quad h(\tau) \geq 0.$$

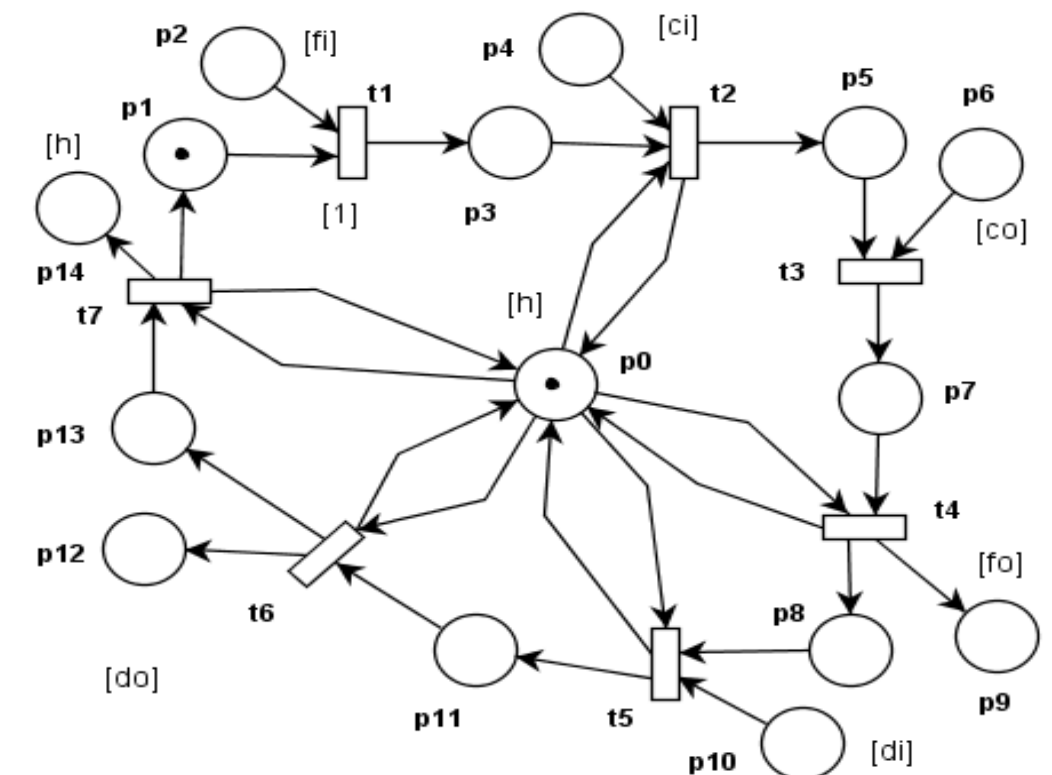
This equation corresponds to a sequential OER-TPN model.

The lake sequential model is presented in the second figure.

The input and output flows are independent so the previous model is closer to the real behavior.

Transition significances:

- $t_2$  input flow (controlled)
- $t_4$  output flow (controlled)
- $t_5$  input disturbance flow
- $t_6$  output flow disturbance
- $t_7$  level sensor measurement



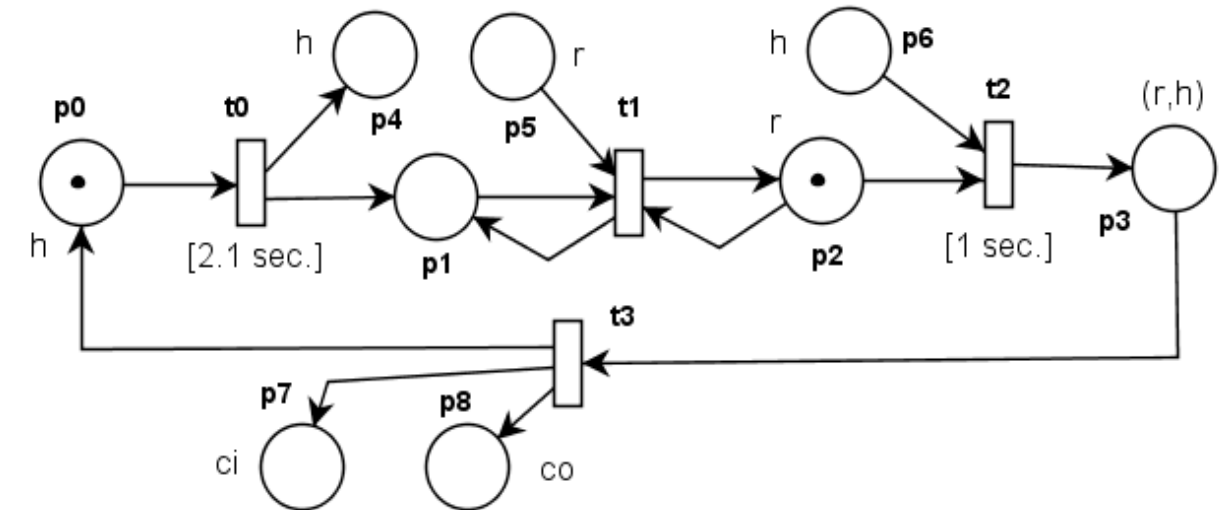
## Control synthesis

Control OETPN model

Inp={p5, p6}: input channel set

Out={p4, p7, p8}: output channel set

- t0 displays the measured lake level on p4
- t1 reads the reference and sets it in the place p2 when the reference  $r > 0$ .  
When  $r=0$  it does not set there zero (i.e.  $r=0$ ) meaning no control is required.
- t2 reads the reference value (when it exist) and the measured lake level from p5. It sets the values r and h in p3.
- t3 sets the lake level in p0. If  $r > 0$  it calculates the lake level deviation.



When  $r > h$   $ci = K \cdot (r - h)$  and  $co = 0$ . sets the corresponding control signals (ci or co) in p7 or p8.

When  $r < h$   $ci = 0$  and  $co = K \cdot (h - r)$ .

When  $h = r$  t3 sets  $ci = 0$  and  $co = 0$ .

The evolution rules of transitions  $t_i$  ( $i=0,1,2, \dots$ ) will be denoted by  $e_i$ . When the transition  $t_i$  has more than one rule they are denoted by  $e_i^k$ ,  $k=1,2, \dots$ . So an evolution rule has the form  $e_i^k = (\text{grd}_i^k; \text{map}_i^k)$ .

Conceive the OER-TPN model of a 3 such lakes linked in cascade.

Conceive the cooperative (implicit, explicit, coordinated) OER-TPN models of the lake system controllers working with Fuzzy Logic (implement the FLETPNs). Use your intuition for the Fuzzy Logic Rules.

*How can be determined if the consensus were attained in all the cases?*

*Determine the disturbances limits until the levels can be maintained.*

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