

Digital control systems. Applications

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Digital controllers

The general form of the pulse transfer function between an input $X(z)$ and an output $Y(z)$ is given by:

$$\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_m z^{-m}}{1 + a_1 z^{-1} + \dots + a_n z^{-n}}$$

where a_i 's and b_i 's are bl coefficients. By cross multiplication we obtain:

$$Y(z)(1 + a_1 z^{-1} + \dots + a_n z^{-n}) = X(z)(b_0 + b_1 z^{-1} + \dots + b_m z^{-m})$$

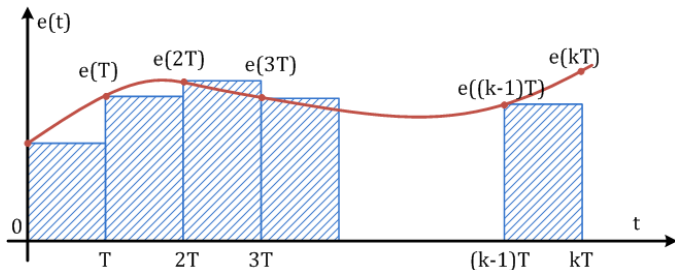
In terms of difference equation (where $kT \rightarrow k$):

$$y(k) = a_1 y(k-1) + \dots + a_n y(k-n) = b_0 x(k) + b_1 x(k-1) + \dots + b_m x(k-m)$$

Digital PID from the time domain

Consider an ideal PID controller in continuous time domain, where $u(t)$ is the control signal and $e(t)$, the error signal:

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt}$$



- Approximate the integral and derivative terms

Digital PID from the time domain

- Sampling time is T :

$$\left. \frac{de(t)}{dt} \right|_{t=kT} \approx \frac{e(kT) - e((k-1)T)}{T}$$

$$\int_0^t e(\tau) d\tau \big|_{t=kT} \approx \sum_{n=0}^{k-1} T \cdot e(nT)$$

Positional PID algorithm

$$u(k) = K_P e(k) + K_I T \sum_{n=0}^{k-1} e(n) + K_D \frac{e(k) - e(k-1)}{T}$$

Digital PID from the s-domain

The ideal PID algorithm is written as:

$$\frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s$$

We can apply $s \rightarrow z$ transformations to get an equivalent discrete PID controller. If, for example, the transformation is:

$$s = \frac{1 - z^{-1}}{T}$$

then, the discrete PID pulse transfer function is:

$$\frac{U(z)}{E(z)} = K_P + \frac{K_I T}{1 - z^{-1}} + K_D \frac{1 - z^{-1}}{T}$$

Digital PID from the s-domain

$$\frac{U(z)}{E(z)} = K_P(1 - z^{-1}) + K_I T + \frac{K_D}{T}(1 - z^{-1})^2$$

$$(1 - z^{-1})U(z) = K_P(1 - z^{-1})E(z) + K_I T E(z) + \frac{K_D}{T}(1 - z^{-1})^2 E(z)$$

Velocity PID algorithm

$$\begin{aligned} u(k) &= u(k-1) + K_P(e(k) - e(k-1)) + K_I T e(k) + \\ &+ \frac{K_D}{T} (e(k) - 2e(k-1) + e(k-2)) \end{aligned}$$

Pulse transfer function and system stability Consider a first order system having the transfer function:

$$G(s) = \frac{1}{s + 4}$$

Compute the pulse transfer function with the simple substitution and the ZOH method and analyze the system stability.

Pulse transfer function and system stability Consider a critically stable system having the transfer function:

$$G(s) = \frac{1}{s^2 + 1}$$

The system has two poles on the imaginary axis, $s_{1,2} = \pm j$, thus it is critically stable. Determine the pulse transfer function $G(z)$ with the simple substitution and the ZOH method and analyze the stability of the discrete system.

Pulse transfer function from a difference equation Consider a system with input u and output y whose behavior can be described with a difference equation:

$$y(k) + 2y(k-1) + \frac{3}{4}y(k-2) = u(k-1)$$

- 1 Determine the pulse transfer function of the system
- 2 Is the system stable? Justify your answer.