

Introduction to Systems Theory and Control Systems

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Course organization

- Lectures: 2h/week, Wednesday 12-14, D21
- Lab exercises: 4h / 2 weeks
- Taught by: Paula Raica (lectures)
- Lab:

Group	When	Who	Where
30421	Friday 8-12	Alexandru Codrean	Dorobanților C01
30422	Tuesday 12-16	Dora Morar	Dorobanților C12
30423	Tuesday 8-12	Dora Morar	Dorobanților C12
30424	Thursday 12-16	Paula Raica	Dorobanților C01
30425	Thursday 16-20	Alexandru Codrean	Dorobanților C01

Systems Theory. Grading

- Point accumulation
- Exams (see "Course calendar and grading"):
 - lab tests - including homework assignments (optional)
 - midterm exam (optional)
 - final exam
- Lab work policy

Prerequisites: Differential equations, Linear algebra, Laplace transform, Complex numbers

Course objective

The general objective of the course is to introduce the fundamental principles of linear system modeling, analysis and feedback control and to evaluate feedback control systems with desired behavior.

System : A set or arrangement of entities so related or connected so as to form a unity or organic whole. (Iberall)

Systems theory: interdisciplinary field which studies systems.

- Founded by Ludwig von Bertalanffy, William Ross Ashby and others between the 1940s and the 1970s on principles from physics, biology and engineering
- Grew into numerous fields including philosophy, sociology, organizational theory, management and economics among others.
- Cybernetics is a related field, sometimes considered as a part of systems theory.

- Understanding *systems*
- Control of systems
- Modeling and control of modern, complex, interrelated systems
 - traffic control systems,
 - chemical processes,
 - robotic systems
 - industrial automation systems.

Control Systems Engineering is based on the foundations of *feedback theory* and *linear systems analysis*.

Historical Background

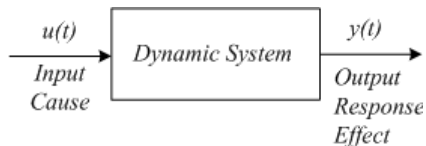
- 300 B.C, Greece : development of the float regulator mechanisms,
- mid 1860s, J.C.Maxwell: the first formal study of the "Theory of Control",
- mid 1890s, E.J. Routh and A.M. Lyapunov: "Routh Stability Test" and the "Lyapunov Stability Criteria"
- 1930s, H. Nyquist (Bell Telephone Laboratories): applied frequency analysis to control systems design
- 1930, H.W.Bode: designed electronic amplifiers using the concepts of feedback control
- 1950s onwards: control theory evolved with new mathematical techniques applied and computer technology.

Discipline of Control Systems

- Multi-disciplinary field. Covers Mechanical Engineering, Chemical Engineering, Electrical and Electronic Engineering, Environmental, Civil, Physics, Economics and Finance, Artificial Intelligence and Computer Science
- Taught in the main stream Engineering and Physics courses
- Computer-Controlled Systems: complex field in control engineering.
- Concepts overlap with the branch of Physics and Electrical Engineering known as Digital Signal Processing (DSP) and Communication Systems

Dynamical system

- A *dynamical system* is a system whose behavior changes over time, often in response to external stimulation or forcing.



- Inputs (cause) = quantities that are acting on the system from the environment
- Outputs (effect) = the results of the input acting on the system.
- Inputs, outputs = signals

Example: Helicopter



Figure: Helicopter

Inputs:

- the power produced by the engines
- the pilot control inputs
- the wind = disturbance

Outputs

- the actual position (coordinates x, y, z)
- orientation (roll, pitch, yaw)
- velocity

Terminology

Control of an inverted pendulum

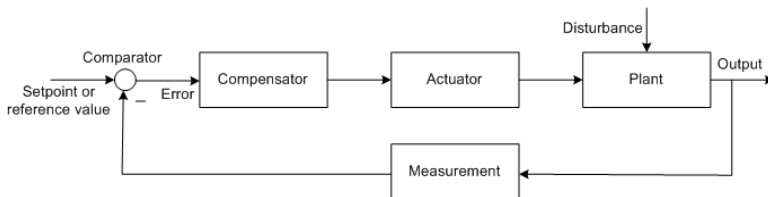


Figure: Elements in a control system

Block diagram. Input. Output. Plant (Process). Measurement. Signals. Setpoint (Reference value). Comparator. Compensator. Actuator. Disturbance. Open-loop. Closed-loop. Negative Feedback.

Closed-loop control system for HDD

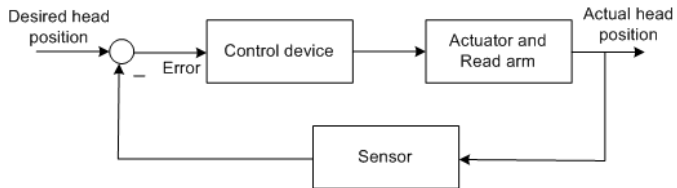
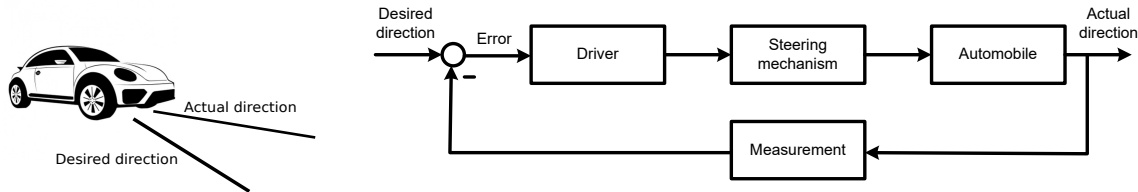


Figure: A hard disk control system

Examples

- Oven temperature control (closed-loop)
- Washing machine: (open-loop)
- Central heating: (closed-loop) vs. radiator(open-loop)
- Automobile steering control system



Applications



Figure: Maglev



Figure: Traffic control

Applications

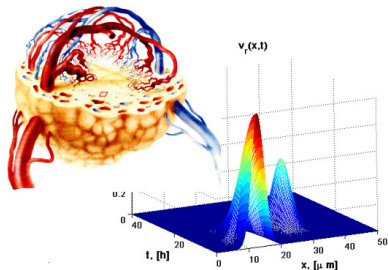


Applications



Figure: Chemical industry, energy

Applications



- Mathematical models of linear time invariant systems systems. Transfer functions, block diagram models
- Analysis of linear continuous systems. Characteristics and performance.
- Stability of linear continuous systems.
- System analysis using root locus.
- Frequency response. Bode diagrams.
- Controller design. Lead-lag compensation.
- PID control.
- Sampled-data systems. Digital control systems.

- R.C.Dorf, R.Bishop, "Modern Control Systems", Addison-Wesley, 2011;
- K.Ogata , "Modern Control Engineering", Prentice Hall, 1990.
- K.Dutton, S. Thompson, B. Barraclough, "The Art of Control Engineering", Addison-Wesley, 1997
- M. Hăngănuț, Teoria sistemelor, UTCluj, 1996
- T. Coloși, Elemente de teoria sistemelor si reglaj automat, UTCluj, 1981

Introduction to Control System Modeling

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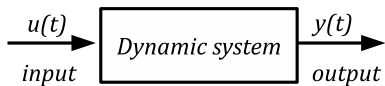
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A **mathematical model** is an equation or set of equations which adequately describes the behavior of a system.

Two approaches to finding the model:

- Lumped-parameter modeling: for each element a mathematical description is established from the physical laws.
- System identification: an experiment can be carried out and a mathematical model can be found from the results.

The important relationship is that between the manipulated inputs and measurable outputs.



Lumped-parameter models

The systems studied in this course are:

Linear - must obey the principle of superposition

Stationary (or time invariant) - the parameters inside the element must not vary with time.

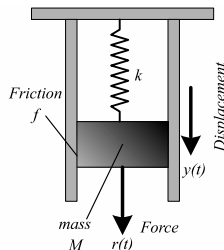
Deterministic - The outputs of the system at any time can be determined from a knowledge of the system's inputs up to that time.

Examples.

- The resistor: $i(t) = \frac{1}{R} v(t)$
- The inductor: $i(t) = \frac{1}{L} \int v(t) dt$ or $v(t) = L \frac{di(t)}{dt}$
- The capacitor: $i(t) = C \frac{dv(t)}{dt}$

Examples

Spring-mass-damper system



$$M \frac{d^2 y(t)}{dt^2} + f \frac{dy(t)}{dt} + ky(t) = r(t)$$

where: f is a damping coefficient, M - the mass, k - the stiffness of the linear spring.

Principle of superposition

A system is defined as linear in terms of the system excitation and response.

Additivity

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) \rightarrow y_2(t)$$

$$x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$$

Homogeneity

$$x(t) \rightarrow y(t)$$

$$mx(t) \rightarrow my(t)$$

Linear Approximation

Nonlinear system

$$y = x^2$$

Nonlinear system

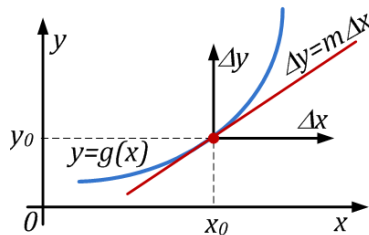
$$y = mx + b$$

Linear about an operating point x_0, y_0 for small changes Δx and Δy . When $x = x_0 + \Delta x$ and $y = y_0 + \Delta y$:

$$y_0 + \Delta y = mx_0 + m\Delta x + b$$

and therefore

$$\Delta y = m\Delta x$$



Linear Approximation

If the dependent variable y depends upon several excitation variables $x_1, x_2, \dots, x_n$:

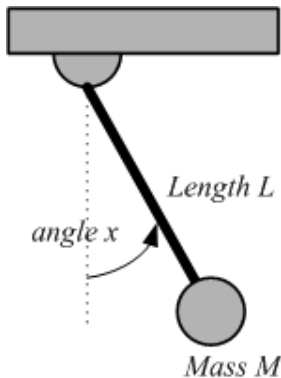
$$y = g(x_1, x_2, \dots, x_n).$$

The Taylor series expansion about the operating point $x_{10}, x_{20}, \dots, x_{n0}$ (the higher-order terms are neglected):

$$\begin{aligned} y = & g(x_{10}, x_{20}, \dots, x_{n0}) + \frac{\partial g}{\partial x_1} \Big|_{x=x_0} (x_1 - x_{10}) + \\ & + \frac{\partial g}{\partial x_2} \Big|_{x=x_0} (x_2 - x_{20}) + \dots + \frac{\partial g}{\partial x_n} \Big|_{x=x_0} (x_n - x_{n0}) \end{aligned}$$

where x_0 is the operating point.

Example - Pendulum oscillator



The torque on the mass:

$$T = MgL\sin(x)$$

The equilibrium condition for the mass is $x_0 = 0^\circ$.

$$T - T_0 \cong MgL \frac{\partial \sin x}{\partial x} \Big|_{x=x_0} (x - x_0),$$

where $T_0 = 0$.

$$T = MgL(\cos 0^\circ)(x - 0^\circ) = MgLx$$

The approximation is reasonably accurate for $-\pi/4 \leq x \leq \pi/4$.

Linear Approximation

Input $x(t)$ and a response $y(t)$: $y(t) = g(x(t))$

Taylor series expansion about the operating point x_0 :

$$y = g(x) = g(x_0) + \left. \frac{dg}{dx} \right|_{x=x_0} \frac{x - x_0}{1!} + \text{higher order terms}$$

The slope at the operating point,

$$m = \left. \frac{dg}{dx} \right|_{x=x_0},$$

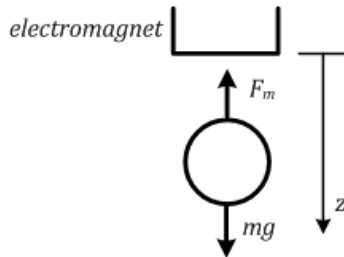
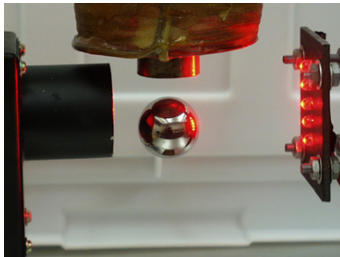
$$y = g(x_0) + \left. \frac{dg}{dx} \right|_{x=x_0} (x - x_0) = y_0 + m(x - x_0),$$

Finally, this equation can be rewritten as the linear equation

$$(y - y_0) = m(x - x_0) \text{ or } \Delta y = m\Delta x$$

Example. Magnetic levitation

- The system: an iron-core electromagnet and the steel ball levitated by the electromagnet.

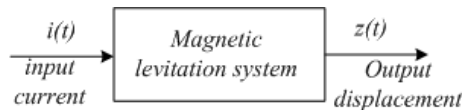


- Electromagnetic force F_m :

$$F_m = C \frac{i^2(t)}{z^2(t)}$$

Example. Magnetic levitation

- Input: the current through the coils of the electromagnet $i(t)$
- Output: the displacement of the ball $z(t)$



- The equation of motion for the ball:

$$m\ddot{z}(t) = mg - C \frac{i^2(t)}{z^2(t)}$$

- Nonlinear model !!

Example. Magnetic levitation - linearization

- Rewrite the equation:

$$g(\ddot{z}(t), z(t), i(t)) = m\ddot{z}(t) - mg + C \frac{i^2(t)}{z^2(t)} = 0$$

- Choose an operating point: $(\ddot{z}_0, z_0, i_0)$ such that

$$m\ddot{z}_0 = mg - C \frac{i_0^2}{z_0^2}$$

- Write the truncated Taylor series around the operating point:

$$\begin{aligned} 0 = g(\ddot{z}(t), z(t), i(t)) &\approx g(\ddot{z}_0, z_0, i_0) + \frac{\partial g}{\partial \ddot{z}}|_{(\ddot{z}_0, z_0, i_0)}(\ddot{z}(t) - \ddot{z}_0) + \\ &+ \frac{\partial g}{\partial z}|_{(\ddot{z}_0, z_0, i_0)}(z(t) - z_0) + \\ &+ \frac{\partial g}{\partial i}|_{(\ddot{z}_0, z_0, i_0)}(i(t) - i_0) \end{aligned}$$

Example. Magnetic levitation - linearization

- Compute the partial derivatives and evaluate them at the operating point. The Taylor series expansion is:

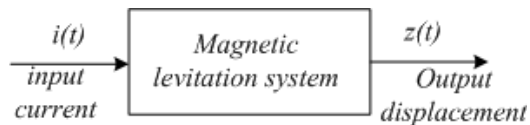
$$0 \approx 0 + m \cdot (\ddot{z}(t) - \ddot{z}_0) - 2C \frac{i_0^2}{z_0^3} \cdot (z(t) - z_0) + 2C \frac{i_0}{z_0^2} \cdot (i(t) - i_0)$$

- Denote the variations around the operating point by: $\Delta\ddot{z}(t) = \ddot{z}(t) - \ddot{z}_0$, $\Delta z(t) = z(t) - z_0$ and $\Delta i(t) = i(t) - i_0$
- Linear differential equation in terms of $\Delta\ddot{z}(t)$, $\Delta z(t)$, $\Delta i(t)$:

$$m\Delta\ddot{z}(t) = 2C \frac{i_0^2}{z_0^3} \Delta z(t) - 2C \frac{i_0}{z_0^2} \Delta i(t)$$

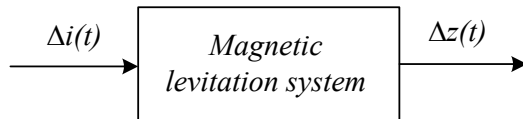
Example. Magnetic levitation - linearization

Nonlinear model



$$m\ddot{z}(t) = mg - C \frac{i^2(t)}{z^2(t)}$$

Linear approximation



$$m\Delta\ddot{z}(t) = 2C \frac{i_0^2}{z_0^3} \Delta z(t) - 2C \frac{i_0}{z_0^2} \Delta i(t)$$

To do

- Review:
 - Differential equations
 - Complex numbers
 - **Laplace transform**