Delay Analysis of Bandwidth Request in Truncated Binary Exponential Backoff Mechanism over Error-Free/Error-Prone Channels in IEEE 802.16e

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Abstract We investigate the delay of the bandwidth request under the truncated binary exponential backoff (TBEB) mechanism in the IEEE 802.16e, considering error-free/error-prone wireless channels. We derive the distribution of delay of bandwidth request packets in the TBEB by an analytic method on the assumption of Bernoulli request arrival process and error-free channel conditions, and extend the analytic results to the error-prone channel condition where the request transmissions have error with i.i.d. By numerical analysis we can find the optimal number of transmission opportunities for transmitting bandwidth requests of the TBEB satisfying QoS on delay bound and loss bound.[1]

Keywords : IEEE 802.16e; binary exponential random backoff mechansim; bandwidth request; probability generating function.

I. Introduction

As an enhancement of the IEEE 802.16 [1] for metropolitan broadband wireless access systems, the IEEE 802.16e [2] is recently standardized to support high capacity, high data rate and multimedia services as well as the service provisioning to the mobile stations (MSs). WiBro (Wireless Broadband) incorporated by the IEEE 802.16e is a wireless broadband access standard being developed and commercialized in 2006 by Korean telecommunication industry.

The IEEE 802.16e/WiBro provides various bandwidth request mechanisms in MAC protocol in order to reserve the bandwidth for data transmission. When an MS has data to send, it transmits a bandwidth request packet to its base station (BS) during contention period consisting of transmission opportunities. In the standard, two types of bandwidth request methods are defined for the bandwidth reservation: a contention-free method such as a polling scheme and a piggyback scheme, and a contention-based method such as the truncated binary exponential backoff (TBEB) scheme.

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In this paper, we consider the contention-based TBEB in WiBro system where orthogonal frequency-division-multiplexing (OFDMA) with time-division-duplex (TDD) mode is adopted. In the IEEE 802.16e/WiBro, transmissions between the BS and MSs are realized in two directions: uplink channel (from MSs to BS) and downlink channel (from BS to MSs). The frame of 5 $msec$ is operated in OFDMA/TDD mode and is divided into two subframes: DL-subframe and UL-subframe, as shown in Fig 1. In the horizontal axis, each slot of DL-subframe (UL-subframe) consists of two (three, respectively) OFDM symbols with one or more subchannels. The subchannel logical number, each of which contains 48 data subcarriers, is shown in the vertical axis. The DL-subframe is used by the BS for transmitting downlink data and control messages to MSs. The UL-subframe is shared by MSs for sending bandwidth requests and uplink data to the BS. The ranging subchannel in UL-subframe contains transmission opportunities scheduled for the purpose of bandwidth requests, in which the bandwidth request packet can be transmitted. In the UL-subframe the BS allocates bandwidth to each of MSs so that the MS is allowed to transmit uplink data. The bandwidth for data transmission is determined based on the bandwidth request information of MSs and the scheduling algorithm of the BS, which should take into account the QoS requirement of each MS and the available resource. The bandwidth request information is broadcasted by the BS via the UL-MAP at the beginning of the DL-subframe. After receiving the UL-MAP, each MS knows the schedule of its data transmission.

In the standard, five service classes are defined according to QoS requirements of each service: Unsolicited Grant Service (UGS), Real-time Polling Service (rtPS), Non-real-time Polling Service (nrtPS), Extended real-time Polling Service (ErtPS) and Best Effort (BE). In the UGS service, the MS generates periodically fixed size data, and thus the BS allocates a fixed amount of bandwidth periodically without any request. In the rtPS, that is delay-sensitive, the BS provides periodic unicast request opportunities to prevent collision, which is a contention-free manner. In general, the nrtPS and BE ser-

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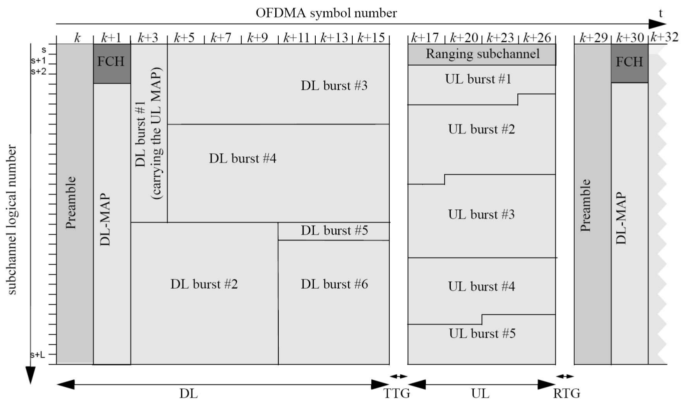


Fig. 1. OFDMA frame structure

vices use the contention-based TBEB as mandatory and the contention-free polling scheme as optional.

The contention-based TBEB in the standard is investigated in Section II. In the TBEB mechanism, the MS which has data to transmit performs the contention resolution by setting its backoff window and selecting randomly an integer number within the backoff window. This randomly chosen number indicates the number of frames that the MS defers before transmitting the request packet. One of transmission opportunities in the chosen frame is randomly selected for the request transmission. After the request is transmitted, MS waits for the bandwidth grant information in the subsequent frame. If the MS does not receive grant within pre-defined period, it regards the request to be collided or lost. Then the MS starts a new backoff process and increases the backoff window by a factor of two, as long as it is less than the maximum backoff window. The MS selects randomly a number within its new backoff window and repeat the deferring process described above.

In the random access of TBEB, as the number of MSs increases, the collision probability of bandwidth request packet increases, and thus the BS cannot allocate the bandwidth to MSs due to the failure of the bandwidth request procedure although the BS has available bandwidth. In order to reduce the collision probability and delay of request packets, there should be many slots for transmission opportunities in UL-subframe. However, if the number of slots for transmission opportunities increases, then the amount of the bandwidth for data transmission decreases. There is a trade-off: by assigning many transmission opportunities for transmitting bandwidth requests, delay of request packets and loss probability reduce while the bandwidth for data shortens. Thus, we need to find the optimal number of transmission opportunities which are required for transmitting request packets.

The TBEB scheme in the IEEE 802.16 was first investigated by Vinel, et al. (2005) [3] under a saturation condition by using an analytical approach similar to Bianchi [4]. To deal with an unsaturated condition, Vinel, et al. (2006) [5] assumed Bernoulli process as bandwidth request arrivals and provided both simulation and analytical models for the investigations of the TBEB random access. They [5] obtained the mean delay of a bandwidth request by finding the distribution of number of active stations and by Little's law. Seo, et al. [6] analyzed queueing performance for sporadic data transmission with the TBEB mechanism in IEEE 802.16 based on OFDMA and CDMA with TDD mode, by a non-exhaustive service of $M/G/1$ queueing with set-up times. Their performance [6] presented in terms of the mean queuing length as well as the coefficient of variation of the output process. He, et al. [7] also proposed an analytical model for the TBEB scheme for IEEE 802.16 and modeled bandwidth efficiency and mean delay as functions of the network and scheme parameters such as the contention window size and the numbers of slots allocated for bandwidth request and data transmission. Kim, et al. [8] proposed an adaptive bandwidth request scheme to use bandwidth efficiently for nrtPS and BE services via a simulation-based algorithm. In their proposed scheme [8] the BS adaptively selects the bandwidth request scheme between contention-based scheme and contention-free scheme, based on the amount of remaining bandwidth.

In this paper, we find the distribution of delay request packets and loss probability of the TBEB by a new analytic model with Bernoulli request arrival assumption, under error-free/error-prone wireless channel and the unsaturated condition. Compared to existing results in the related work [5] where only the mean delay is obtained under the ideal channel condition, the contribution of this paper is that we yield the distribution of delay of the request packets under the TBEB mechanism and analyze more accurately the network system with optimization according to packet loss and delay bound. By using the distribution of delay, we can find the optimal number of transmission opportunities while satisfying QoS on delay bound and loss bound.

The rest of this paper is organized as follows. In Section II, we describe the TBEB in detail and present an analytical model of two-dimensional Markov chain with the unsaturated condition for the TBEB random mechanism under the error-free wireless channel. As a main result of this paper, we derive the PGF of delay of bandwidth requests. Furthermore, we extend the results into the error-prone wireless channel condition where the request transmissions have error with i.i.d. The delay is defined by time duration from the instant that the request is generated to the instant that the request is transmitted successfully or gotten loss. In Section III, numerical examples evaluate the probability mass distribution of the delay in the TBEB, as well as the mean delays of the TBEB and the polling schemes are compared on arrival probability of requests. We find the optimal number of transmission opportunities for transmitting bandwidth request of two mechanisms satisfying QoS on delay bound and loss bound, and thus we can determine which scheme performs better than the other depending on the arrival probability of requests. Numerical results show that the TBEB performs better than the polling as the request arrival probability is small, and vice versa as it is large.

II. Mathematical Analysis for TBEB

A. Assumptions

We assume that n MSs are associated with one BS. If a station has data to send, (coincidently it generates a request packet), it is referred to as an *active* station, otherwise, it is called an *inactive* station. An active station sends a request packet to reserve the bandwidth for data transmission as we mention above. We assume that a request arrival to each inactive MS follows Bernoulli process with probability λ, i.e., each (inactive) MS generates a request packet with probability λ during one frame.

Let K be the number of transmission opportunities in the request contention period of a frame. The size of one transmission opportunity is enough for a bandwidth request transmission. The number K of transmission opportunities in one frame is determined by the BS in order to make a trade-off between the size of request contention period and that of a data payload period within the whole frame size. The number K will be found optimally with satisfying QoS on delay bound and loss bound in numerical analysis.

In this paper, we consider the error-free wireless channel and the error-prone wireless channel. When a channel is error-prone, a bandwidth request packet may be corrupted due to poor channel conditions, e.g., path-loss, multipath fading, thermal noise or interference from other emitting sources nearby. Let q denote the probability that a transmitted request packet is corrupted due to poor channel conditions. If $q = 0$, it is the case of error-free wireless channel.

Let M be the restricted number for retransmissions in the TBEB when the request packet is collided or corrupted due to bad channel conditions. After more than M retransmissions of a request packet, it will be discarded and regarded as packet loss, for which loss probability is obtained.

B. Analytical model for TBEB under the error-free channel

First we consider an analytical model for the random access of the TBEB under the error-free wireless channel ($q = 0$). In the collision resolution of the TBEB, before each transmission attempt, a station uniformly chooses an integer number from the interval $[0, W_i - 1]$ where W_i is the current value of its backoff stage after i collisions. The chosen value, referred to as a backoff counter, indicates the number of frames that the station has to wait before the transmission. Let W_{min} be the initial backoff window. In the case of a collision, its backoff window size is doubled. $W_i = 2^i W_{min}$ after i collisions, ($i = 0, 1, 2, \cdots$). The maximum backoff window size is $W_{max} = 2^m W_{min}$ for some m where m is called the maximum backoff exponent. If the maximum is reached, then it is not doubled, but fixed. However, since we restrict M retransmissions of a request packet, there are total $M + 1$ transmissions including the initial transmission.

As we mention above, we use two-step procedure for the random backoff: The first step is that one of the $2^w W_{min}$ frames is uniformly chosen, where w is the current backoff stage. The second step is that one transmission opportunity is uniformly chosen out of K transmission opportunities in the frame selected by the first step. Thus we adopt a discrete Markov chain with frame unit. Let t and $t+1$ be the beginnings of two consecutive *frames*. We define a Markov Chain X_t by

$$X_t = \begin{cases} e & \text{if the tagged MS is inactive at time } t, \\ (b(t), c(t)) & \text{if the tagged MS is active at time } t \end{cases}$$

where the state "e" means the inactive state of the station with no data to send, $b(t)$ is the backoff stage of the tagged MS at time t, and $c(t)$ is the backoff counter which is the number of frames during which the MS has to wait before the transmission at time t. A station chooses uniformly one of $2^w W_{min}$ frames, where w is the current backoff stage. If the backoff counter $c(t)$ reaches to zero (in frame unit), the station chooses uniformly one out of K transmission opportunities in the given frame, and attempts transmission at the chosen transmission opportunity in the frame. Assuming $m \leq M$, there are $M + 1$ backoff stages: $b(t) \in \{0, 1, \cdots, m, m+1, \cdots, M-1, M\}$, but note that if $b(t) \geq m$, then its window size is W_{max}. The state of the Markov chain is $\{e\} \bigcup \{(i, k) : 0 \leq i \leq M,\ 0 \leq k \leq W_M - 1\}$, and for $l = m, m+1, \cdots, M$, $W_l = W_{max} = 2^m W_{min}$. See Fig. 2 for the Markov chain model under the error-free channel.

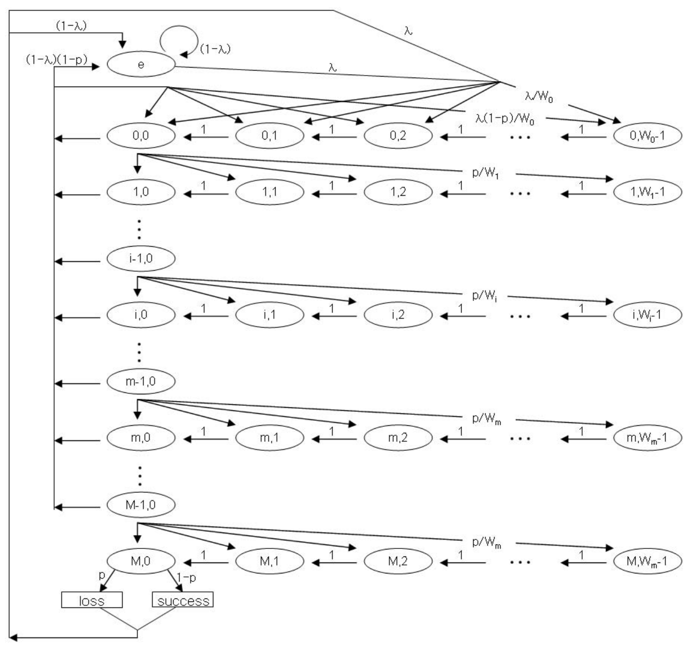


Fig. 2. Markov chain model

One-step transition probabilities are as follows:

$$\begin{array}{lll} P(i,k|i,k+1) & = 1, & k = 0,1,\cdots,W_i - 2, \quad i = 0,1,\cdots,M \\ P(0,k|i,0) & = \frac{\lambda(1-p)}{W_0}, & k = 0,1,\cdots,W_0 - 1, \quad i = 0,1,\cdots,M-1 \\ P(i+1,k|i,0) & = \frac{p}{W_{i+1}}, & k = 0,1,\cdots,W_{i+1} - 1, \quad i = 0,1,\cdots,M-1 \\ P(e|i,0) & = (1-p)(1-\lambda), & i = 0,1,\cdots,M-1 \\ P(0,k|M,0) & = \frac{\lambda}{W_0}, & k = 0,1,\cdots,W_0 - 1 \\ P(e|M,0) & = 1-\lambda, & \\ P(0,k|e) & = \frac{\lambda}{W_0}, & k = 0,1,\cdots,W_0 \\ P(e|e) & = 1-\lambda, & \end{array}$$

where p is the conditional collision probability given that the request is transmitted. The probability p is supposed to be constant, not depending on other stations' behaviors.

Let $b_{i,k} = \lim_{t\to\infty} P(X_t = (i,k))$ and $b_e = \lim_{t\to\infty} P(X_t = e)$ be the stationary distribution of the Markov chain. Note that

$$b_{i,0} = p^i b_{0,0}, \quad i = 1, \cdots, M, \tag{1}$$

$$b_{0,k} = \lambda \frac{W_0 - k}{W_0} \left[b_e + b_{M,0} + (1-p) \sum_{j=0}^{M-1} b_{j,0} \right], \tag{2}$$

$$k = 0, 1, \cdots, W_0 - 1,$$

$$b_{i,k} = \frac{W_i - k}{W_i} p b_{i-1,0} = \frac{W_i - k}{W_i} b_{i,0}, \tag{3}$$

$$i = 1, 2, \cdots, M, \quad k = 1, 2, \cdots, W_i - 1,$$

$$b_e = \frac{1-\lambda}{\lambda} b_{0,0}. \tag{4}$$

By the normalization condition $1 = b_e + \sum_{i=0}^{M} \sum_{k=0}^{W_i - 1} b_{i,k}$ and by (1)-(4), $b_{0,0}$ is obtained by

$$b_{0,0} = \left[\frac{1-\lambda}{\lambda} + \frac{1 - p^{M+1}}{2(1-p)} + \frac{W_0}{2} \left(\frac{1 - (2p)^{m+1}}{1 - 2p} + \frac{2^m (p^{m+1} - p^{M+1})}{1-p} \right) \right]^{-1} \tag{5}$$

Now we can express the probability τ that a station transmits in a randomly chosen frame,

$$\tau = \sum_{i=0}^{M} b_{i,0} = b_{0,0} \frac{1 - p^{M+1}}{1-p}. \tag{6}$$

In terms of the transmission probability τ, we express the conditional collision probability p as follows: as one tagged station transmits in a frame, the probability that u stations from the remaining $n-1$ stations transmit in the same frame is equal to $\binom{n-1}{u} \tau^u (1-\tau)^{n-1-u}$, and the probability that all of u stations transmit in the transmission opportunities different from the one chosen by the considered station is $(1 - 1/K)^u$. Thus the conditional collision probability p is given by

$$p = 1 - \sum_{u=0}^{n-1} \binom{n-1}{u} \tau^u (1-\tau)^{n-1-u} \left(1 - \frac{1}{K}\right)^u. \tag{7}$$

The two unknown variables p and τ can be solved numerically from (6) and (7).

One of important performance measures is loss probability obtained by

$$P_{loss} = p^{M+1}. \tag{8}$$

Now we find the main result of this paper: probability generating function (PGF) of the delay of a tagged request packet in the system. Let D_T be the delay of the tagged request packet in the TBEB method, defined by the time duration from the instant that the request is generated to the instant that the request is transmitted successfully or lost. We assume that when the request is collided, the station knows the collision at the next DL-subframe and attempts retransmission at the next UL-subframe. The delay D_T is denoted by

$$D_T = \begin{cases} D_1 & \text{with probability } 1 - P_{loss} \\ & \text{if the request is successfully transmitted} \\ D_2 & \text{with probability } P_{loss} \\ & \text{if the request is not successfully transmitted} \end{cases}$$

The PGF of D_T is given by

$$E[z^{D_T}] = (1 - P_{loss}) E[z^{D_1}] + P_{loss} E[z^{D_2}] \tag{9}$$

$$E[z^{D_1}] = \sum_{k=0}^{M} E[z^{D_1} | k \text{ collisions before success}] \cdot P(k \text{ collisons before success})$$

where $P(k \text{ collisions before success})$ is given by $\frac{p^k(1-p)}{1-p^{M+1}}$.

$$E[z^{D_1} | k \text{ collisions before success}] = \prod_{i=0}^{k} T_i(z) z^{k+1}$$

where $T_i(z)$ is the PGF of the time duration of the ith backoff stage: $T_i(z) = \sum_{a=0}^{W_i - 1} \frac{1}{W_i} z^a$

$$T_i(z) = \sum_{a=0}^{2^i W - 1} \frac{1}{2^i W} z^a = \frac{z^{2^i W} - 1}{2^i W (z-1)} \quad \text{for } i \le m$$

letting $W = W_0$ for simplicity, and

$$T_i(z) = \frac{z^{2^m W} - 1}{2^m W (z-1)} \quad \text{for } m < i \le M.$$

Thus

$$\begin{aligned} E[z^{D_1}] &= \sum_{k=0}^{m} \frac{p^k (1-p)}{1-p^{M+1}} \left(\prod_{i=0}^{k} \frac{z^{2^i W} - 1}{2^i W (z-1)} \right) z^{k+1} \\ &+ \sum_{k=m+1}^{M} \frac{p^k (1-p)}{1-p^{M+1}} \left(\prod_{i=0}^{m} \frac{z^{2^i W} - 1}{2^i W (z-1)} \right) \left(\frac{z^{2^m W} - 1}{2^m W (z-1)} \right)^{k-m} z^{k+1} \\ &= \frac{(1-p) z}{1 - p^{M+1}} \left[\sum_{k=0}^{m} \frac{(pz)^k}{(z-1)^{k+1} W^{k+1} 2^{k(k+1)/2}} \prod_{i=0}^{k} (z^{2^i W} - 1) \right. \\ &+ \sum_{k=m+1}^{M} \frac{(pz)^k}{(z-1)^{k+1} W^{k+1} 2^{m(m+1)/2} 2^{m(k-m)}} \\ &\left. \cdot \prod_{i=0}^{m} (z^{2^i W} - 1)(z^{2^m W} - 1)^{k-m} \right]. \end{aligned}$$

On the other hand, for $E[z^{D_2}]$, in this case, the request experiences total $M+1$ collisions and so gets loss at the Mth backoff stage.

$$\begin{aligned} E[z^{D_2}] &= \prod_{i=0}^{M} T_i(z) z^{M+1} = \left(\prod_{i=0}^{m} \frac{z^{2^i W} - 1}{2^i W (z-1)} \right) \left(\frac{z^{2^m W} - 1}{2^m W (z-1)} \right)^{M-m} z^{M+1} \\ &= \frac{z^{M+1}}{(z-1)^{M+1} W^{M+1} 2^{m(m+1)/2} 2^{m(M-m)}} \prod_{i=0}^{m} (z^{2^i W} - 1)(z^{2^m W} - 1)^{M-m}. \end{aligned}$$

By (9), we obtain the PGF of D_T:

$$E[z^{D_T}] = (1-p) z \left[\sum_{k=0}^{m} \frac{(pz)^k}{(z-1)^{k+1} W^{k+1} 2^{k(k+1)/2}} \prod_{i=0}^{k} (z^{2^i W} - 1) \right.$$

$$+\sum_{k=m+1}^{M} \frac{(pz)^k (z^{2^m W}-1)^{k-m}}{(z-1)^{k+1} W^{k+1} 2^{m(m+1)/2} 2^{m(k-m)}} \prod_{i=0}^{m} (z^{2^i W}-1) \Bigg]$$

$$+\frac{(pz)^{M+1} (z^{2^m W}-1)^{M-m}}{(z-1)^{M+1} W^{M+1} 2^{m(m+1)/2} 2^{m(M-m)}} \prod_{i=0}^{m} (z^{2^i W}-1). \quad (10)$$

C. Extension for the error-prone wireless channel

Now we assume that the probability q of a transmitted request packet being corrupted due to poor channel conditions is nonzero. Let σ be the probability that the tagged MS transmits successfully its request packet under the error-prone channel. The probability σ depends on the collision probability p and corrupted probability q:

$$\sigma = (1-p)(1-q). \quad (11)$$

In Fig. 2 of the Markov Chain, we replace p and $1-p$ by $1-\sigma$ and σ, respectively, to obtain for a new Markov Chain under the error-prone wireless channel. In other words, one-step transition probabilities in this case are as follows:

$$\begin{array}{lll}
P(i,k|i,k+1) & = 1, & k=0,1,\cdots,W_i-2, \quad i=0,1,\cdots,M \\
P(0,k|i,0) & = \frac{\lambda\sigma}{W_0}, & k=0,1,\cdots,W_0-1, \quad i=0,1,\cdots,M-1 \\
P(i+1,k|i,0) & = \frac{1-\sigma}{W_{i+1}}, & k=0,1,\cdots,W_{i+1}-1, \quad i=0,1,\cdots,M-1 \\
P(e|i,0) & = \sigma(1-\lambda), & i=0,1,\cdots,M-1 \\
P(0,k|M,0) & = \frac{\lambda}{W_0}, & k=0,1,\cdots,W_0-1 \\
P(e|M,0) & = 1-\lambda, & \\
P(0,k|e) & = \frac{\lambda}{W_0}, & k=0,1,\cdots,W_0 \\
P(e|e) & = 1-\lambda. &
\end{array}$$

Thus the transmission probability τ^* is given by

$$\tau^* = b^*_{0,0} \frac{1-(1-\sigma)^{M+1}}{\sigma}, \quad (12)$$

where $b^*_{0,0}$ is of the same form in (5), but with $1-\sigma$ instead of p. For the same reason in (7), the conditional probability p is given by

$$p = 1 - \sum_{u=0}^{n-1} \binom{n-1}{u} (\tau^*)^u (1-\tau^*)^{n-1-u} \left(1-\frac{1}{K}\right)^u. \quad (13)$$

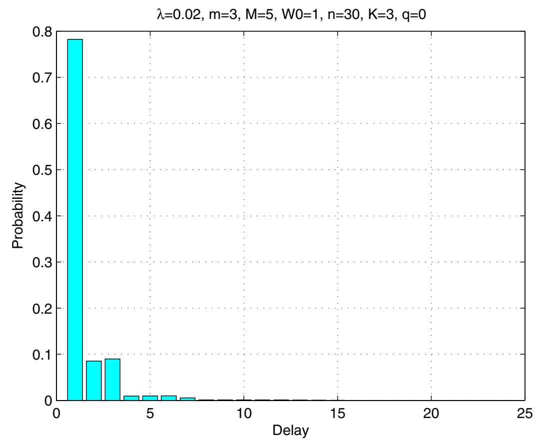
The two unknown variables p and τ^* can be solved numerically from (12) and (13) with the relation in (11) for the given value of q. Therefore, the loss probability in this case is given by

$$P^*_{loss} = (1-\sigma)^{M+1}. \quad (14)$$

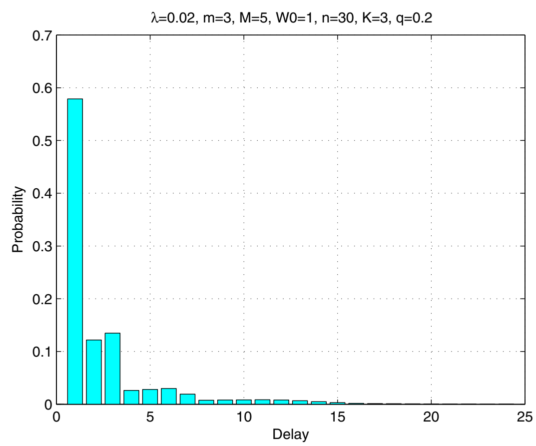
The PGF $E[z^{D_T}]$ of delay of the tagged request packet under the error-prone channel is obtained, similarly to (10), using $1-\sigma$ instead of p.

III. Numerical Analysis

In this section we give numerical examples for probability mass distribution of delay of request packets in the TBEB mechanism under the error-free/error-prone channels (Fig. 3) and compare the mean delay of the TBEB and the polling schemes vs. request arrival probability (Fig. 5). We also find the optimal number of transmission opportunities by considering various system parameters (Fig. 4 and Fig. 7) and determine which scheme among the TBEB and the polling performs better than the other in view of the optimal number of transmission opportunities, depending on the request arrival probability (Fig. 7). Finally, we find loss probability vs. arrival probability in the TBEB (Fig. 8).



(a) error-free channel, $q=0$

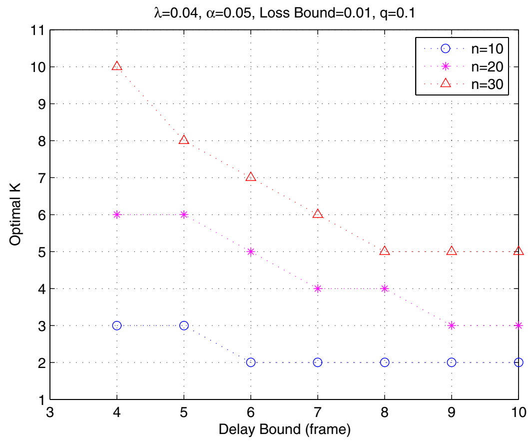


(b) error-prone channel, $q=0.2$

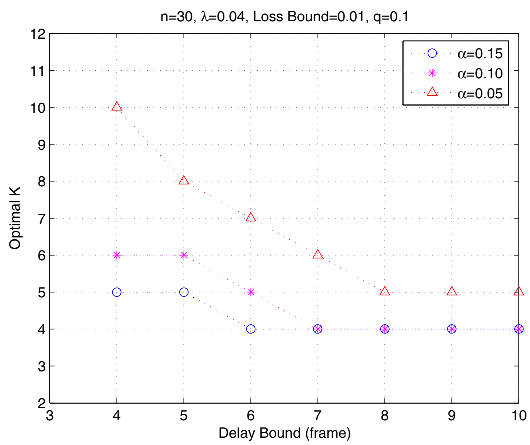
Fig. 3. *Probability mass distribution of delay in the TBEB under error-free/error-prone channels, with* $\lambda = 0.02$, $n = 30$, $m = 3$, $M = 5$, $W_0 = 1$, $K = 3$

Fig. 3 presents examples of probability mass distribution of delay in the TBEB mechanism under error-free/error-prone channels. In Fig. 3, we consider the scenario with $n = 30$ stations, the arrival probability $\lambda = 0.02$ per frame, the number $K = 3$ of transmission opportunities, the maximum backoff exponent $m = 3$, the retransmission number $M = 5$ and the initial window size $W_0 = 1$. On x-axis of Fig. 3, delay is measured in a discrete time scale of frame unit. As compared to Fig. 3(a) and (b), the delay distribution in case of the error-prone channel condition in (b) have longer tails as we expect.

The number K of transmission opportunities in one frame affects on the distribution of delay of request packets. We find the optimal number K with satisfying QoS on loss bound and delay bound in the TBEB. The optimal number K is obtained depending on a delay bound as follows: For each given value of system parameters such as λ, n, W_0, m, M, q

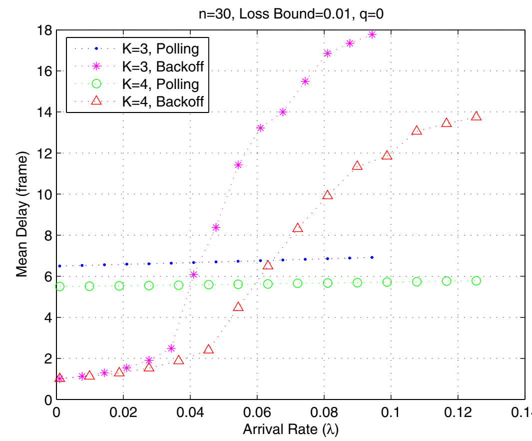


(a) Comparison with $n = 10$, $n = 20$, $n = 30$ under $\alpha = 0.05$

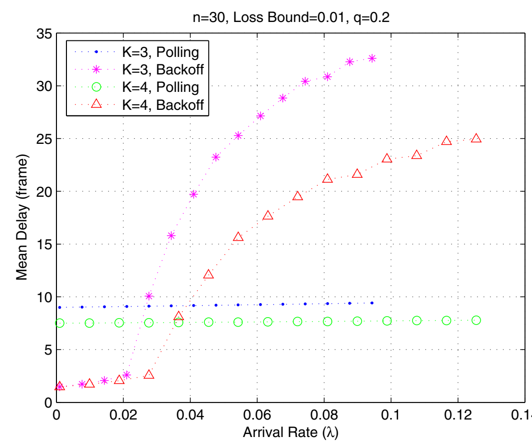


(b) Comparison with $\alpha = 0.15,\ 0.10,\ 0.05$ under $n = 30$

Fig. 4. *Optimal K vs. delay bound in the TBEB*



(a) error-free channel, $q = 0$



(b) error-prone channel, $q = 0.2$

Fig. 5. *Comparison of mean delay vs. arrival probability between the TBEB and the polling under error-free/error-prone channels*

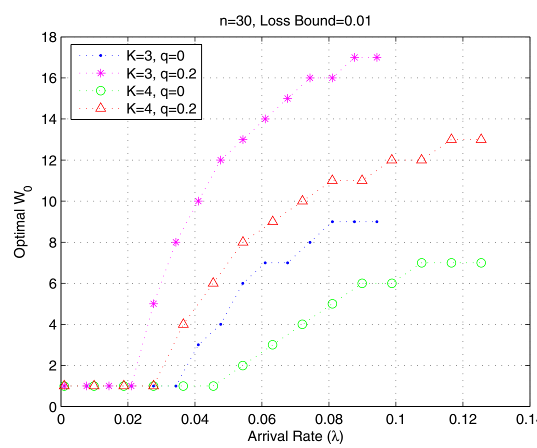
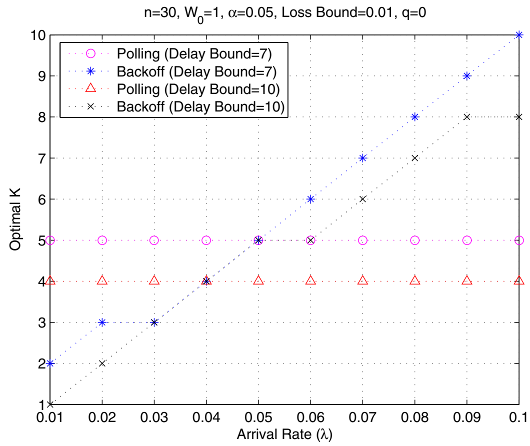


Fig. 6. *Optimal initial window size W_0 vs. arrival probability in the TBEB under error-free/error-prone channels and $K = 3,\ K = 4$*

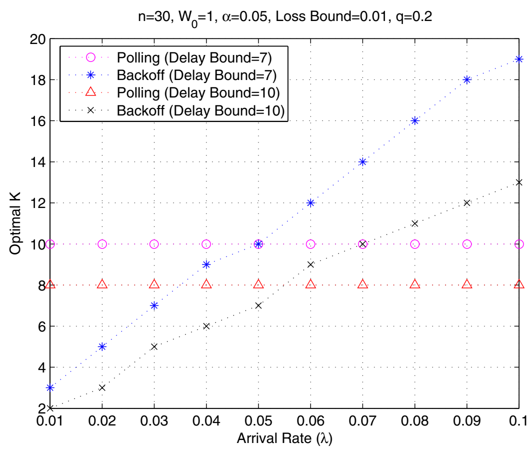
and loss probability, and for the probability α exceeding the delay bound satisfying $P(delay > x) \leq \alpha$ where x is a delay bound, we find the minimum number K among all numbers satisfying the delay bound. In other words, the optimal K is the minimum number of transmission opportunities such that $100(1-\alpha)\%$ of request packets can be successfully transmitted within the delay bound.

Fig. 4 depicts the graph of the optimal number K on y-axis, vs. the delay bound on x-axis, where the following default parameters are used: $\lambda = 0.04,\ W_0 = 1,\ m = 3,\ M = 5,\ q = 0.1$, and loss probability is 0.01. In Fig. 4, we compare the optimal K, respectively, as the parameters vary on (a) $n = 10,\ 20,\ 30$, and (b) $\alpha = 0.15,\ 0.10,\ 0.05$. For example, for the default parameters $\lambda = 0.04,\ n = 30,\ W_0 = 1,\ m = 3,\ M = 5,\ \alpha = 0.05,\ q = 0.1$ and loss probability 0.01, and for the delay bound $x = 7$, the optimal K is 6, and for the delay bound $x = 10$, the optimal K is 5, as seen in (a)(b) of Fig. 4.

Now we compare the TBEB and the polling in two points of view: (i) mean delay (Fig. 5) and (ii) the optimal number of transmission opportunities satisfying QoS on delay bound and loss bound by applying the delay distributions (Fig. 7). For the comparisons in Fig. 5, under the error-prone channel



(a) error-free channel, $q = 0$



(b) error-prone channel, $q = 0.2$

Fig. 7. *Comparison of the optimal number K vs. arrival probability between the TBEB and the polling under error-free/error-prone channels*

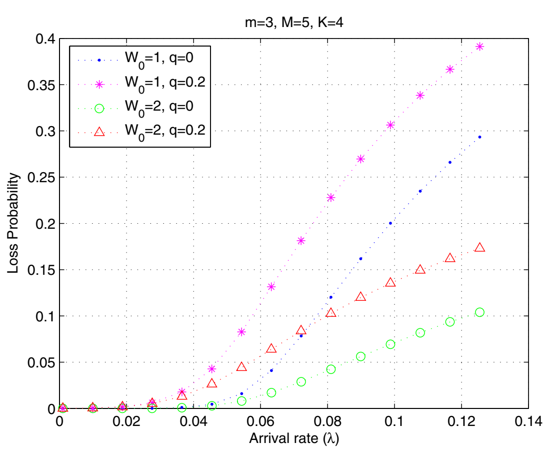


Fig. 8. *Loss probability vs. arrival probability in the TBEB under error-free/error-prone channels and $W_0 = 1$, $W_0 = 2$*

in the polling, $q = 0.2$ and $M = 5$ is used. In the TBEB, the maximum backoff exponent $m = 3$ and the retransmission number $M = 5$ are used for the system parameters and the optimal window size W_0 is selected for each given value of n, K and q, satisfying QoS on loss bound 0.01, (see Fig. 6). The optimal initial window size is chosen as the minimum number among all values of initial window size W_0 that satisfy the loss bound such that $P_{loss} \leq 0.01$. The optimal W_0 in Fig. 6 is used for each λ in the comparison of Fig. 5. We see that the TBEB has smaller delay than the polling if the arrival probability λ of request packets is small, and vice versa if λ is large. For example, for $\lambda = 0.1$ and $K = 4$, under the error-free channel, delays of the TBEB and the polling are respectively 12 and 6 frames. For the $\lambda = 0.1$ and $K = 4$, under the error-prone channel, delays of the TBEB and the polling respectively 23 and 8 frames.

In Fig. 7, we compare two schemes in view of the optimal number of transmission opportunities by applying the delay distributions, respectively, like Fig. 4. Fig. 7 depicts the optimal number K vs. the arrival probability. Its default parameters are $n = 30$, $m = 3$, $M = 5$, $W_0 = 1$, $\alpha = 0.05$, and loss bound 0.01. Under the error-free channel in Fig. 7(a), for the delay bound 10, the TBEB has the optimal $K = 2$ for $\lambda = 0.02$, and it has the optimal $K = 8$ for $\lambda = 0.09$, where as the polling has the optimal $K = 4$ for both $\lambda = 0.02$ and $\lambda = 0.09$. Under the error-prone channel in Fig. 7(b), for the delay bound 10, the TBEB has the optimal $K = 3$ for $\lambda = 0.02$, and it has the optimal $K = 12$ for $\lambda = 0.09$, where as the polling has the optimal $K = 8$ for both $\lambda = 0.02$ and $\lambda = 0.09$. As seen in Fig. 7, as the arrival probability is small, the TBEB has smaller K and so we prefer the TBEB, while the arrival probability is large, the TBEB has more K and so we prefer the polling.

Finally, Fig. 8 illustrates loss probability, given by (8) and (14), vs. arrival probability in cases of $W_0 = 1$ and $W_0 = 2$ of the TBEB under the error-free/error-prone channels, where the default parameters stated above are used. Obviously, $W_0 = 2$ gives longer delay than $W_0 = 1$, but we see in Fig. 8 that the case of $W_0 = 2$ has smaller loss probability than the case of $W_0 = 1$. For example, for $\lambda = 0.1$, if $W_0 = 1$ then loss probability is 0.20, and if $W_0 = 2$ then it is 0.07 under the error-free channel, and if $W_0 = 1$ then loss probability is 0.31, and if $W_0 = 2$ then it is 0.14 under the error-prone channel ($q = 0.2$). By the analytic results, we can evaluate, but omit here, the performance measures for other values of parameters under other circumstances in the IEEE 802.16e bandwidth request access of the TBEB.

IV. Conclusion

In conclusions, we find probability distributions of delay of bandwidth request packets in truncated binary exponential random backoff mechanism in IEEE 802.16e network with OFDMA/TDD mode, considering error-free/error-prone wireless channel conditions. We present a new approach on mathematical model for the bandwidth request in the TBEB over error-free/error-prone channel, compared to those in existing results of the related work, where only mean delay is obtained over an ideal channel condition. Numerical analysis gives examples of probability mass distribution of delay under error-free/error-prone channels, and compare the mean delays of the TBEB and the polling according to the arrival probability of packets. By the analytical results, we find the optimal parameters such as the initial window size and the number of transmission opportunities satisfying QoS on delay bound and loss bound.

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