Aplicaţia 1

Pentru sistemul de forţe din figura de mai jos, să se calculeze torsorul faţă de polul O $\tau_O(\mathbf{R}, \mathbf{M_O})$.

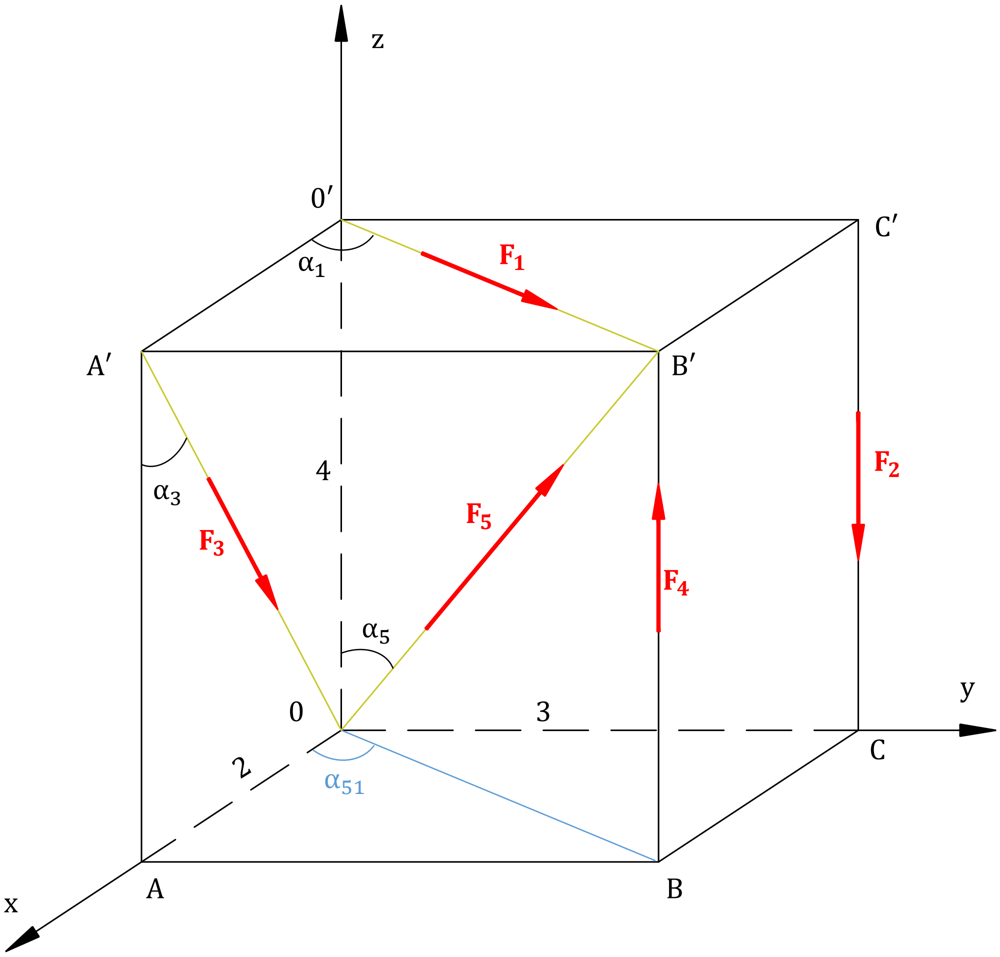

Figura 1 – Reducerea canonică a sistemelor generale de forţe

Date numerice:

Forţa	Modulul forţei [N]
F_1	$2\sqrt{13}$
F_2	3
F_3	$3\sqrt{5}$
F_4	3
F_5	5

Rezolvare:

1. Calculul rezultantei

$\mathbf{F_1} = F_{x1} \cdot \mathbf{i} + F_{y1} \cdot \mathbf{j} + F_{z1} \cdot \mathbf{k}$	$\mathbf{F_2} = F_{x2} \cdot \mathbf{i} + F_{y2} \cdot \mathbf{j} + F_{z2} \cdot \mathbf{k}$
$F_{x1} = F_1 \cdot \cos(\alpha_1) = 2\sqrt{13} \cdot \frac{2}{\sqrt{2^2+3^2}} = 4$ $F_{y1} = F_1 \cdot \sin(\alpha_1) = 2\sqrt{13} \cdot \frac{3}{\sqrt{2^2+3^2}} = 6$ $F_{z1} = 0$ $\mathbf{F_1} = 4 \cdot \mathbf{i} + 6 \cdot \mathbf{j}$	$F_{x2} = 0$ $F_{y2} = 0$ $F_{z2} = -F_2 = -3$ $\mathbf{F_2} = -3 \cdot \mathbf{k}$

$\mathbf{F_3} = F_{x3} \cdot \mathbf{i} + F_{y3} \cdot \mathbf{j} + F_{z3} \cdot \mathbf{k}$	$\mathbf{F_4} = F_{x4} \cdot \mathbf{i} + F_{y4} \cdot \mathbf{j} + F_{z4} \cdot \mathbf{k}$
$F_{x3} = -F_3 \cdot \sin(\alpha_3) = -3\sqrt{5} \cdot \frac{2}{\sqrt{2^2+4^2}} = -3$ $F_{y3} = 0$ $F_{z3} = -F_3 \cdot \cos(\alpha_3) = -3\sqrt{5} \cdot \frac{4}{\sqrt{2^2+4^2}} = -6$ $\mathbf{F_3} = -3 \cdot \mathbf{i} - 6 \cdot \mathbf{k}$	$F_{x4} = 0$ $F_{y4} = 0$ $F_{z4} = F_4 = 3$ $\mathbf{F_4} = 3 \cdot \mathbf{k}$

$\mathbf{F_5} = F_{x5} \cdot \mathbf{i} + F_{y5} \cdot \mathbf{j} + F_{z5} \cdot \mathbf{k}$
$F_{x5} = \underbrace{[F_5 \cdot \sin(\alpha_5)]}_{pr_{xOy}\ \mathbf{F_5}} \cdot \cos(\alpha_{51}) = 5 \cdot \frac{\sqrt{2^2+3^2}}{\sqrt{\left(\sqrt{2^2+3^2}\right)^2+4^2}} \cdot \frac{2}{\sqrt{2^2+3^2}} = \frac{10}{\sqrt{29}} = 1.86$ $F_{y5} = \underbrace{[F_5 \cdot \sin(\alpha_5)]}_{pr_{xOy}\ \mathbf{F_5}} \cdot \sin(\alpha_{51}) = 5 \cdot \frac{\sqrt{2^2+3^2}}{\sqrt{\left(\sqrt{2^2+3^2}\right)^2+4^2}} \cdot \frac{3}{\sqrt{2^2+3^2}} = \frac{15}{\sqrt{29}} = 2.79$ $F_{z5} = \underbrace{[F_5 \cdot \cos(\alpha_5)]}_{pr_{Oz}\ \mathbf{F_5}} = 5 \cdot \frac{4}{\sqrt{\left(\sqrt{2^2+3^2}\right)^2+4^2}} = \frac{20}{\sqrt{29}} = 3.71$ $\mathbf{F_5} = 1.86 \cdot \mathbf{i} + 2.79 \cdot \mathbf{j} + 3.71 \cdot \mathbf{k}$
Se poate face verificarea: $\lvert\mathbf{F_5}\rvert = \sqrt{{F_{x5}}^2 + {F_{y5}}^2 + {F_{z5}}^2} = \sqrt{1.86^2 + 2.79^2 + 3.71^2} = 5 = F_5$

$$R = R_x \cdot \mathbf{i} + R_y \cdot \mathbf{j} + R_z \cdot \mathbf{k}$$

$$R_x = \sum_{i=1}^{5} F_{x_i} = 4 - 3 + 1.86 = 2.86$$

$$R_y = \sum_{i=1}^{5} F_{y_i} = 6 + 2.79 = 8.79$$

$$R_z = \sum_{i=1}^{5} F_{z_i} = -3 - 6 + 3 + 3.71 = -2.29$$

$$\lvert\mathbf{R}\rvert = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2} = \sqrt{2.86^2 + 8.79^2 + (-2.29)^2} = 9.52\ [N]$$

2. Calculul momentului rezultant

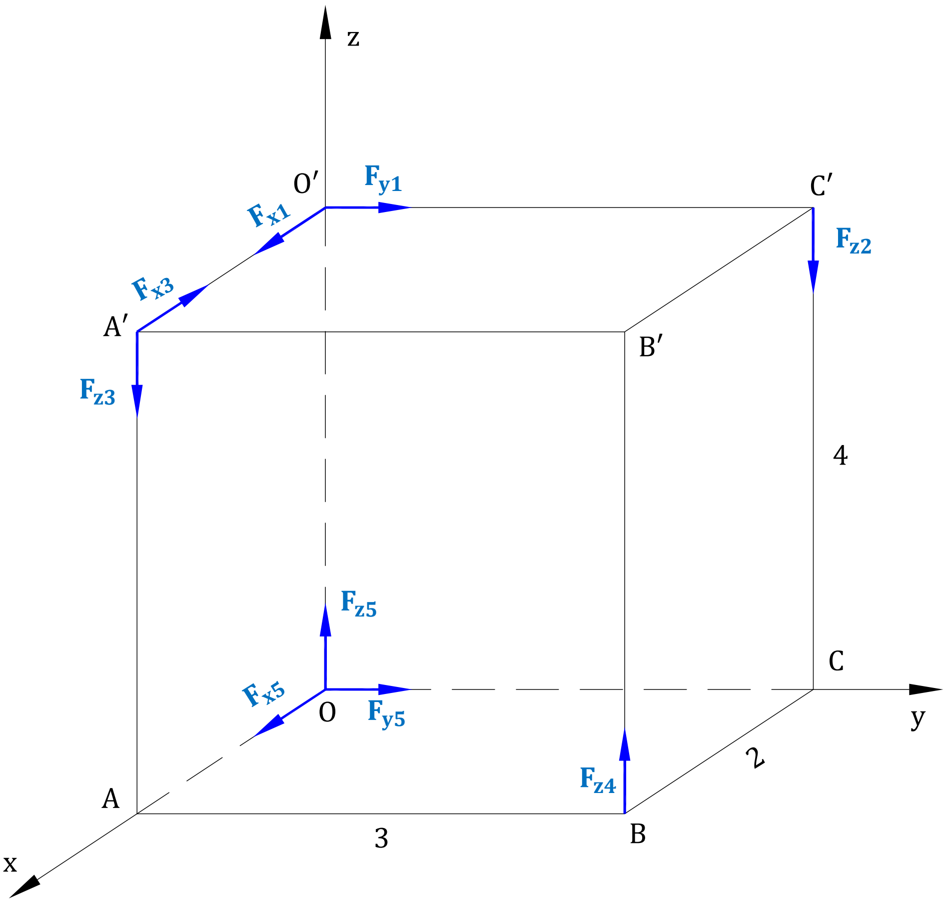

Figura 2. Schema echivalentă a sistemului de forțe

$$M_{Ox} = F_{z4} \cdot \overset{AB}{\overbrace{3}} - F_{y1} \cdot \overset{OO'}{\overbrace{4}} - F_{z2} \cdot \overset{O'C'}{\overbrace{3}} = 3 \cdot 3 - 6 \cdot 4 - 3 \cdot 3 = -24$$

$$M_{Oy} = F_{z3} \cdot \overset{AO}{\overbrace{2}} - F_{x3} \cdot \overset{OO'}{\overbrace{4}} + F_{x1} \cdot \overset{OO'}{\overbrace{4}} - F_{z4} \cdot \overset{BC}{\overbrace{2}} = \overbrace{6 \cdot 2 - 3 \cdot 4}^{=0\ (\mathbf{M_0}(\mathbf{F_3})=0)} + 4 \cdot 4 - 3 \cdot 2 = 10$$

$$M_{Oz} = 0$$

$$|\mathbf{M_0}| = \sqrt{{M_{ox}}^2 + {M_{oy}}^2 + {M_{oz}}^2} = \sqrt{(-24)^2 + 10^2 + 0^2} = 26\ [\mathrm{N \cdot m}]$$

Aplicația 2

Pentru sistemele de forțe din figurile 2 și 3, de mai jos, să se calculeze torsorul în polul 0, $\tau_0(\mathbf{R}, \mathbf{M_0})$.

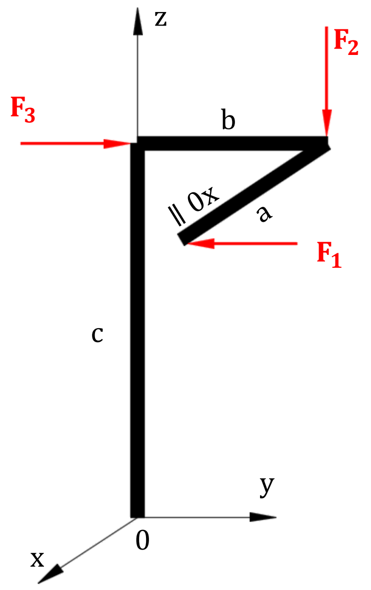

Figura 2 – Reducerea a sistemelor generale de forțe

Date numerice:

Forța	Modulul forței [N]	Dimensiuni geometrice [m]
F_1	2	a = 2
F_2	3	b = 2
F_3	4	c = 4

Rezolvare:

1. Calculul rezultantei

$$\mathbf{F_1} = -2 \cdot \mathbf{j}$$

$$\mathbf{F_2} = -3 \cdot \mathbf{k}$$

$$\mathbf{F_3} = 4 \cdot \mathbf{j}$$

$$\mathbf{R} = R_x \cdot \mathbf{i} + R_y \cdot \mathbf{j} + R_z \cdot \mathbf{k} = 0 \cdot \mathbf{i} + 2 \cdot \mathbf{j} - 3 \cdot \mathbf{k}$$

$$|\mathbf{R}| = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2} = \sqrt{0^2 + 2^2 + (-3)^2} = 3.61 \text{ [N]}$$

2. Calculul momentului rezultant

$$M_{0x} = -F_3 \cdot \overbrace{4}^{c} - F_2 \cdot \overbrace{2}^{b} + F_1 \cdot \overbrace{4}^{c} = -4 \cdot 4 - 3 \cdot 2 + 2 \cdot 4 = -14$$

$$M_{0y} = 0$$

$$M_{0z} = -F_1 \cdot \overbrace{2}^{c} = -2 \cdot 2 = -4$$

$$|\mathbf{M_0}| = \sqrt{{M_{ox}}^2 + {M_{oy}}^2 + {M_{oz}}^2} = \sqrt{(-14)^2 + 0^2 + (-4)^2} = 14.56 \text{ [N} \cdot \text{m]}$$

Aplicația 3

Pentru sistemele de forțe din figurile 2 şi 3, de mai jos, să se calculeze torsorul în polul 0, $\tau_0(\mathbf{R}, \mathbf{M_0})$.

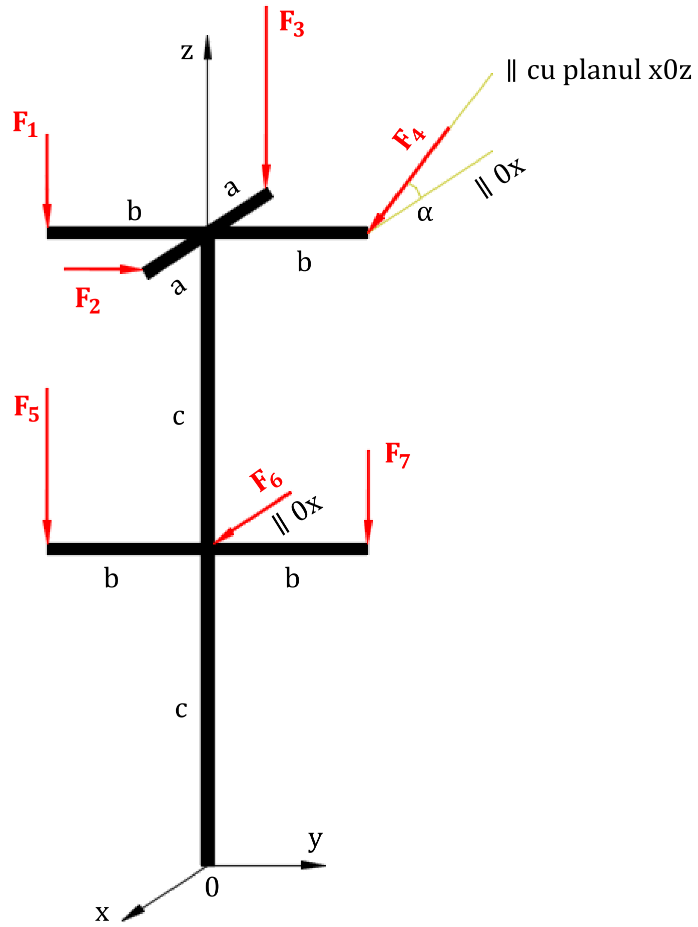

Figura 3 – Reducerea a sistemelor generale de forțe

Date numerice:

Forța	Modulul forței [N]	Dimensiuni geometrice [m]
F_1	100	$a = 2$ $b = 3$ $c = 4$ $\alpha = 12.3°$
F_2	50	
F_3	300	
F_4	75	
F_5	350	
F_6	150	
F_7	175	

Rezolvare:

1. Calculul rezultantei

$$\mathbf{F_1} = -100 \cdot \mathbf{k}$$

$$\mathbf{F_2} = 50 \cdot \mathbf{j}$$

$$\mathbf{F_3} = -300 \cdot \mathbf{k}$$

$$\mathbf{F_4} = F_{x4} \cdot \mathbf{i} + F_{y4} \cdot \mathbf{j} + F_{z4} \cdot \mathbf{k}$$

$$F_{x4} = F_4 \cdot \cos 12.3^0 = 75 \cdot \cos 12.3^0 = 73.28$$

$$F_{y4} = 0$$

$$F_{z4} = -F_4 \cdot \sin 12.3^0 = -75 \cdot \sin 12.3^0 = -15.98$$

$$\mathbf{F_4} = 73.28 \cdot \mathbf{i} - 15.98 \cdot \mathbf{k}$$

$$\mathbf{F_5} = -350 \cdot \mathbf{k}$$

$$\mathbf{F_6} = 150 \cdot \mathbf{i}$$

$$\mathbf{F_7} = -175 \cdot \mathbf{k}$$

$$\mathbf{R} = R_x \cdot \mathbf{i} + R_y \cdot \mathbf{j} + R_z \cdot \mathbf{k} = 223.28 \cdot \mathbf{i} + 50 \cdot \mathbf{j} - 940.98 \cdot \mathbf{k}$$

$$|\mathbf{R}| = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2} = \sqrt{223.28^2 + 50^2 + (-940.98)^2} = 968.40 \text{ [N]}$$

2. Calculul momentului rezultant

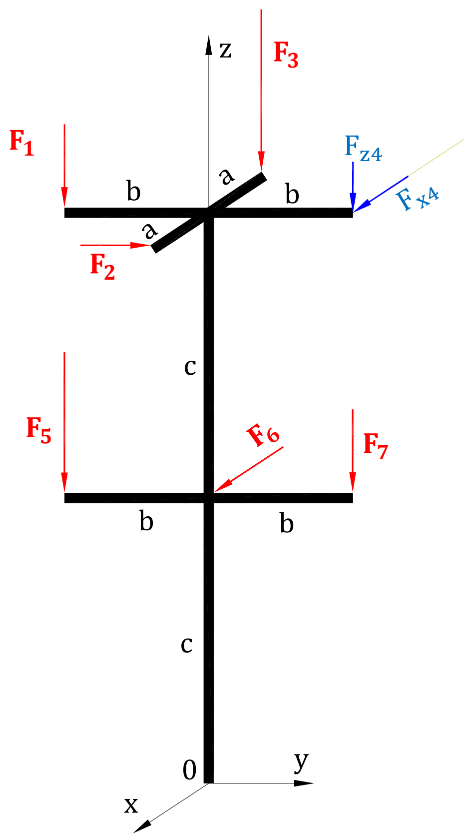

Figura 4 – Schema echivalentă pentru calculul momentului

$$M_{0x} = F_1 \cdot \overbrace{3}^{b} - F_2 \cdot \overbrace{8}^{2\cdot c} - F_{z4} \cdot \overbrace{3}^{b} + F_5 \cdot \overbrace{3}^{b} - F_7 \cdot \overbrace{3}^{b} = 100 \cdot 3 - 50 \cdot 8 - 15.98 \cdot 3 + 350 \cdot 3 - 175 \cdot 3 = 377.06\ [N \cdot m]$$

$$M_{Oy} = -F_3 \cdot \overbrace{2}^{a} + F_{x4} \cdot \overbrace{8}^{2\cdot c} + F_6 \cdot \overbrace{4}^{c} = -300 \cdot 2 + 73.28 \cdot 8 + 150 \cdot 4 = 585.24$$

$$M_{Oz} = F_2 \cdot \overbrace{2}^{a} - F_{x4} \cdot \overbrace{3}^{b} = 50 \cdot 2 - 73.28 \cdot 3 = -119.84$$

$$\mathbf{M_0} = 377.06 \cdot \mathbf{i} + 585.24 \cdot \mathbf{j} - 119.84 \cdot \mathbf{k}$$

$$|\mathbf{M_0}| = \sqrt{{M_{ox}}^2 + {M_{oy}}^2 + {M_{oz}}^2} = \sqrt{377.06^2 + 585.24^2 + (-119.84)^2} = 706.43 [N \cdot m]$$